

# Neural networks and optimization

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- 1 Introduction
- 2 Linear classifier
- 3 Convolutional neural networks
- 4 Stochastic gradient descent

# Foreword

- I'm here for you, I already know that stuff
- It's better to look silly than to stay so
- Ask questions if you don't understand !

# Goal : classification and regression

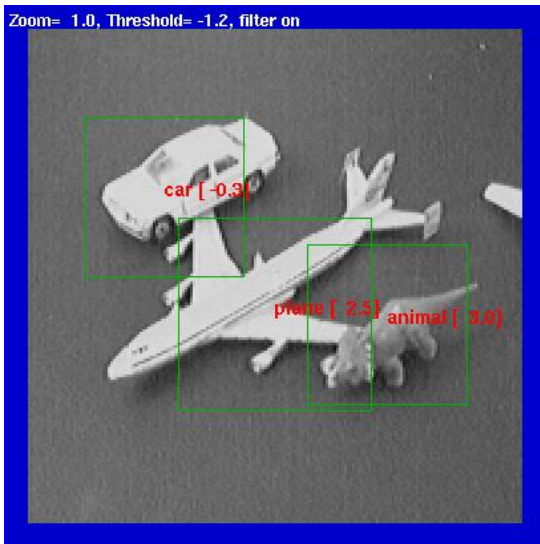
- Medical imaging : cancer or not ? **Classification**
- Autonomous driving : optimal wheel position **Regression**
- Kinect : where are the limbs ? **Regression**
- OCR : what are the characters ? **Classification**

# Goal : classification and regression

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- Kinect : where are the limbs ? **Regression**
- OCR : what are the characters ? **Classification**

Regression and classification are similar problems

# Goal : real-time object recognition

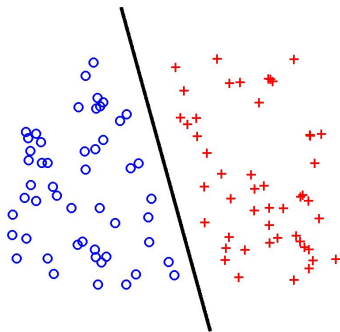


# Linear classifier

- Dataset :  $(X^{(i)}, Y^{(i)})$  pairs,  $i = 1, \dots, N$ .
- $X^{(i)} \in \mathbb{R}^n$ ,  $Y^{(i)} \in \{-1, 1\}$ .
- Goal : Find  $w$  and  $b$  such that  $\text{sign}(w^\top X^{(i)} + b) = Y^{(i)}$ .

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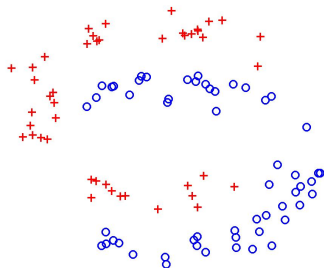


# Perceptron algorithm (Rosenblatt, 57)

- $w_0 = 0, b_0 = 0$
- $\hat{Y}^{(i)} = \text{sign}(w^\top X^{(i)} + b)$
- $w_{t+1} \leftarrow w_t + \sum_i (Y^{(i)} - \hat{Y}^{(i)}) X^{(i)}$
- $b_{t+1} \leftarrow b_t + \sum_i (Y^{(i)} - \hat{Y}^{(i)})$

Movie `linearly_separable_perceptron.avi`

# Some data are not separable



The Perceptron algorithm is **NOT** convergent for non linearly separable data.

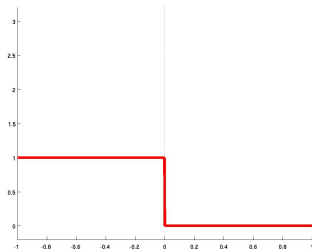
# Non convergence of the perceptron algorithm

Movie `non_linearly_separable_perceptron.avi`

- We need an algorithm which works both on separable and non separable data.

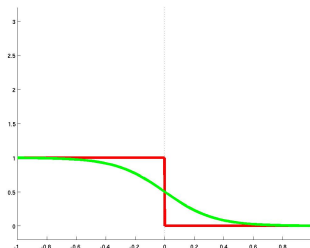
# Cost function

- Classification error is not smooth.



# Cost function

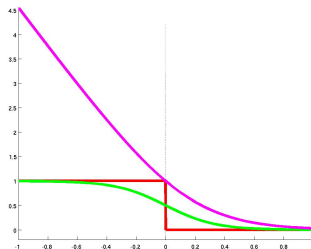
- Classification error is not smooth.
- Sigmoid is smooth but not convex.



- Convexity guarantees the same solution every time.
- In practice, it is not always crucial.

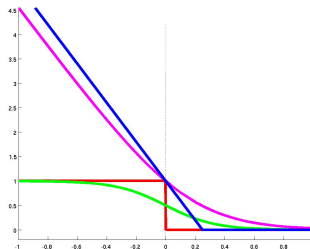
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- Logistic loss is a convex upper bound.



# Convex cost functions

- Classification error is not smooth.
- Sigmoid is smooth but not convex.
- Logistic loss is a convex upper bound.
- Hinge loss (SVMs) is very much like logistic.



# Solving separable AND non-separable problems

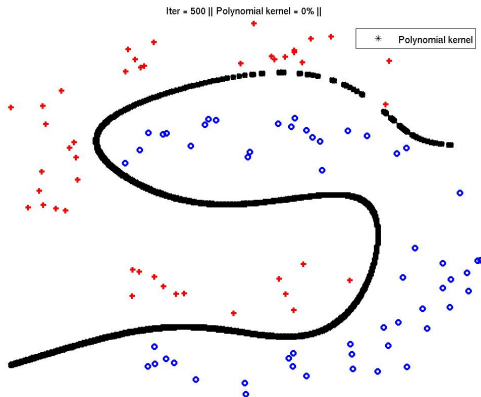
**Movie** non\_linearly\_separable\_logistic.avi **Movie**  
linearly\_separable\_logistic.avi



# Non-linear classification

**Movie** `non_linearly_separable_poly_kernel.avi`

# Non-linear classification



- Features :  $X_1, X_2 \rightarrow$  linear classifier
- Features :  $X_1, X_2, X_1X_2, X_1^2, \dots \rightarrow$  non-linear classifier

# Choosing the features

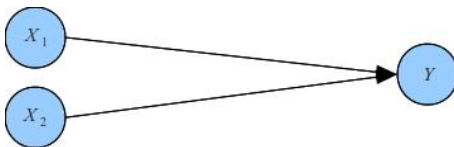
- To make it work, I created lots of extra features :
- $(X_1, X_2, X_1X_2, X_1^2X_2, X_1X_2^2)^{(1,2,3,\dots,10)}$

# Choosing the features

- To make it work, I created lots of extra features :
- $(X_1, X_2, X_1 X_2, X_1^2 X_2, X_1 X_2^2)^{(1,2,3,\dots,10)}$
- Would it work with fewer features ?
- Test with  $(X_1, X_2, X_1 X_2, X_1^2 X_2, X_1 X_2^2)^{(1,2)}$

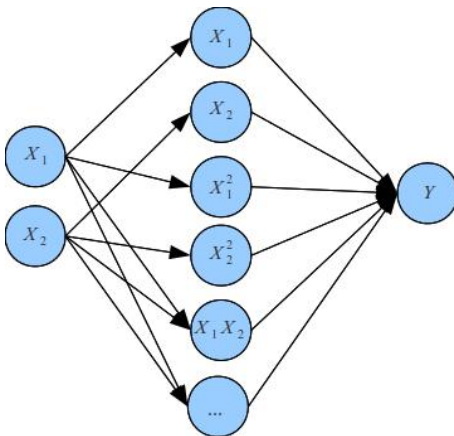
Movie `non_linearly_separable_poly_2.avi`

# A graphical view of the classifiers



$$f(X) = w_1 X_1 + w_2 X_2 + b$$

# A graphical view of the classifiers



$$f(X) = w_1 X_1 + w_2 X_2 + w_3 X_1^2 + w_4 X_2^2 + w_5 X_1 X_2 + \dots$$

# Non-linear features

- A linear classifier on a non-linear transformation is non-linear.
- A non-linear classifier relies on non-linear features.
- Which ones do we choose ?

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- Example :  $H_j = X_1^{p_j} X_2^{q_j}$



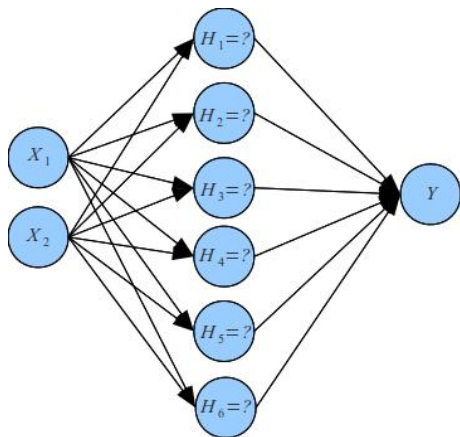
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# Non-linear features

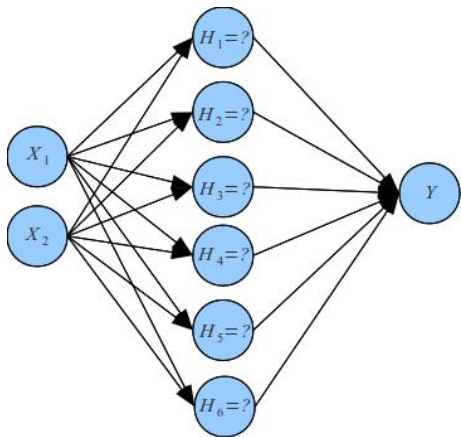
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- Which ones do we choose ?
- Example :  $H_j = X_1^{p_j} X_2^{q_j}$
- SVM :  $H_j = K(X, X^{(j)})$  with  $K$  some kernel function
- Do they have to be predefined ?

# Neural networks



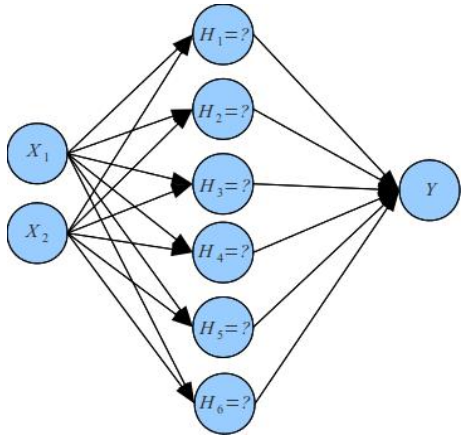
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# Neural networks



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# Neural networks

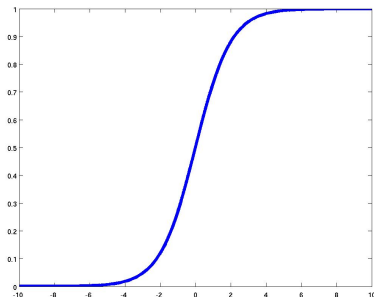


- A neural network will learn the  $H_j$ 's
- **Usually**, we use  $H_j = g(v_j^\top X)$
- $H_j$  : Hidden unit
- $v_j$  : Input weight
- $g$  : Transfer function

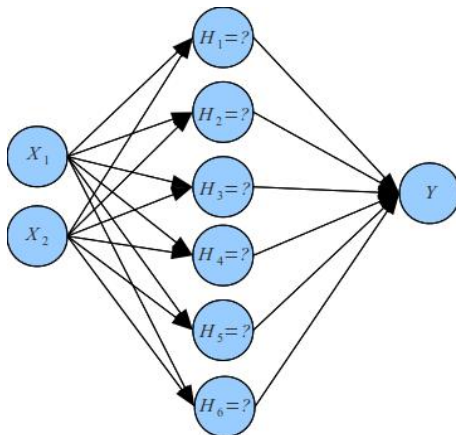
# Transfer function

$$f(X) = \sum_j w_j H_j(X) + b = \sum_j w_j g(v_j^\top X) + b$$

- $g$  is the *transfer function*.
- Usually,  $g$  is the sigmoid or the tanh.



# Neural networks



$$f(X) = \sum_j w_j H_j(X) + b = \sum_j w_j g(v_j^\top X) + b$$

# Example on the non-separable problem

**Movie** `non_linearly_separable_mlp_3.avi`



# Training a neural network

- Dataset :  $(X^{(i)}, Y^{(i)})$  pairs,  $i = 1, \dots, N$ .
- Goal : Find  $w$  and  $b$  such that

$$\text{sign}(w^\top X^{(i)} + b) = Y^{(i)}$$

# Training a neural network

- Dataset :  $(X^{(i)}, Y^{(i)})$  pairs,  $i = 1, \dots, N$ .
- Goal : Find  $w$  and  $b$  **to minimize**

$$\sum_i \log (1 + \exp (-Y^{(i)} (w^\top X^{(i)} + b)))$$

# Training a neural network

- Dataset :  $(X^{(i)}, Y^{(i)})$  pairs,  $i = 1, \dots, N$ .
- Goal : Find  $v_1, \dots, v_k$ ,  $w$  and  $b$  to minimize

$$\sum_i \log \left( 1 + \exp \left( -Y^{(i)} \left[ \sum_j w_j g(v_j^\top X^{(i)}) \right] \right) \right)$$

# Neural network - 8 hidden units

Movie `non_linearly_separable_mlp_8.avi`

# Neural network - 5 hidden units

Movie `non_linearly_separable_mlp_5.avi`

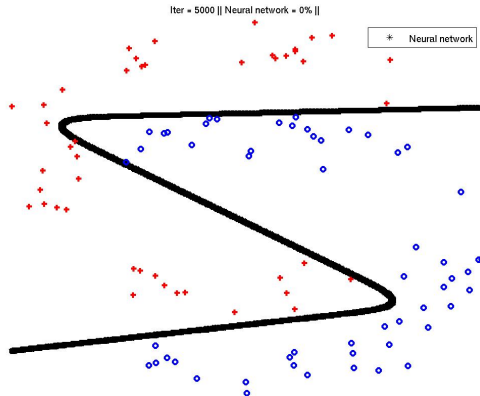
# Neural network - 3 hidden units

Movie `non_linearly_separable_mlp_3.avi`

# Neural network - 2 hidden units

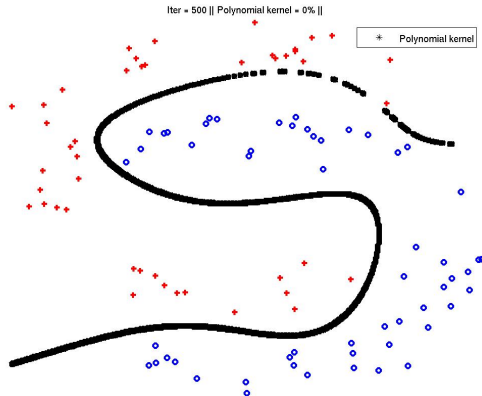
Movie `non_linearly_separable_mlp_2.avi`

# Non-linear classification

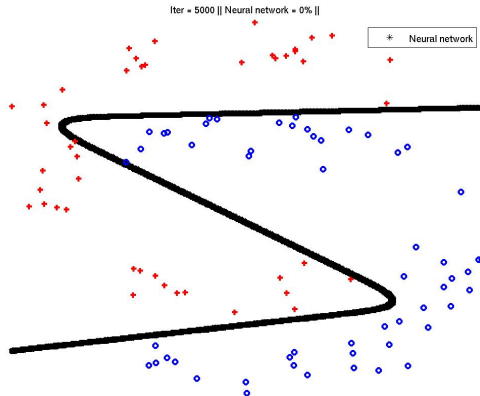




# Non-linear classification



# Non-linear classification



# Cost function

$s$  = cost function (logistic loss, hinge loss, ...)

$$\begin{aligned}\ell(v, w, b, X^{(i)}, Y^{(i)}) &= s\left(\hat{Y}^{(i)}, Y^{(i)}\right) \\ &= s\left(\sum_j w_j H_j(X^{(i)}), Y^{(i)}\right) \\ &= s\left(\sum_j w_j g\left(v_j^\top X^{(i)}\right), Y^{(i)}\right)\end{aligned}$$

# Backpropagation - Output weights

$s$  = cost function (logistic loss, hinge loss, ...)

$$\hat{Y}^{(i)} = \sum_j w_j H_j(X^{(i)})$$

$$\begin{aligned} \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial w_j} &= \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial \hat{Y}^{(i)}} \frac{\partial \hat{Y}^{(i)}}{\partial w_j} \\ &= \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial \hat{Y}^{(i)}} H_j(X^{(i)}) \end{aligned}$$

# Backpropagation - Input weights

$s$  = cost function (logistic loss, hinge loss, ...)

$$\hat{Y}^{(i)} = \sum_j w_j H_j(X^{(i)})$$

$$H_j(X^{(i)}) = g(v_j^\top X^{(i)})$$

$$\begin{aligned} \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial v_j} &= \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial H_j(X^{(i)})} \frac{\partial H_j(X^{(i)})}{\partial v_j} \\ &= \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial \hat{Y}^{(i)}} \frac{\partial \hat{Y}^{(i)}}{\partial H_j(X^{(i)})} \frac{\partial H_j(X^{(i)})}{\partial v_j} \\ &= \frac{\partial \ell(v, w, b, X^{(i)}, Y^{(i)})}{\partial \hat{Y}^{(i)}} w_j X^{(i)} g'(v_j^\top X^{(i)}) \end{aligned}$$

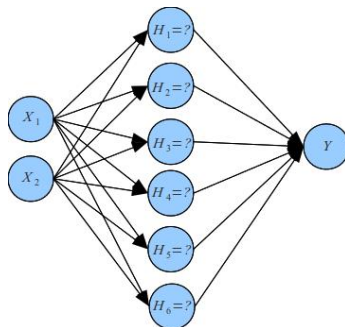
# Training neural networks - Summary

- For each datapoint, compute the gradient of the cost with respect to the weights.
- Done using the backpropagation of the gradient.
- Convex with respect to the output weights (linear classifier).
- **NOT** convex with respect to the input weights : **POTENTIAL PROBLEMS !**

# Neural networks - Summary

- A linear classifier in a feature space can model non-linear boundaries.
- Finding a good feature space is essential.
- One can design the feature map by hand.
- One can learn the feature map, using fewer features than if it done by hand.
- Learning the feature map is potentially **HARD** (non-convexity).

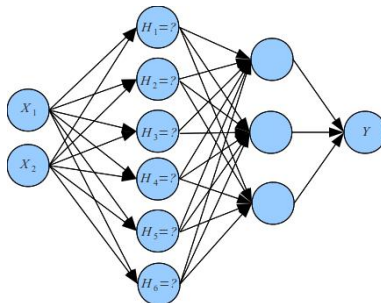
# Neural networks - Not summary



- Linear combination of the output of soft classifiers.
- This is a non-linear classifier.
- One can take a linear combination of these.



# Neural networks - Not summary



- Linear combination of the output of soft classifiers.
- This is a non-linear classifier.
- One can take a linear combination of these.
- **This becomes a neural network with two hidden layers.**

# Advantages of neural networks

- They can learn anything.
- Extremely fast at test time (computing the answer for a new datapoint) because fewer features.
- Complete control over the power of the network (by controlling the hidden layers sizes).

# Problems of neural networks

- Highly non-convex  $\rightarrow$  many local minima
- Can learn anything but have more parameters  $\rightarrow$  need tons of examples to be good.

# Take-home messages

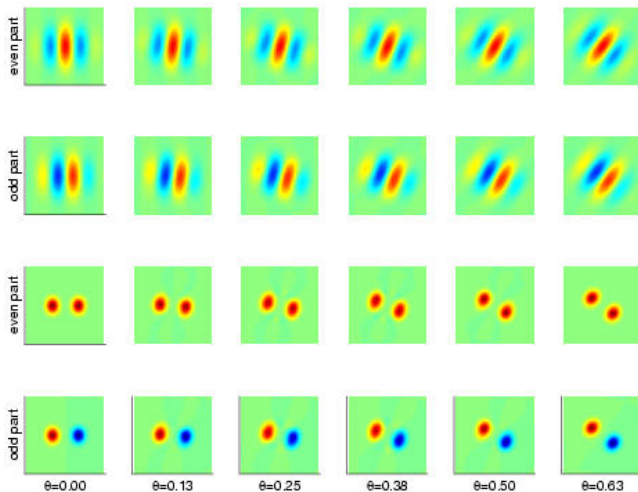
- Neural networks are potentially extremely efficient.
- But it is HARD to train them !
- If you wish to use them, be smart (or ask someone who knows) !
- If you have a huge dataset, they CAN be awesome !

# $v_j$ 's for images

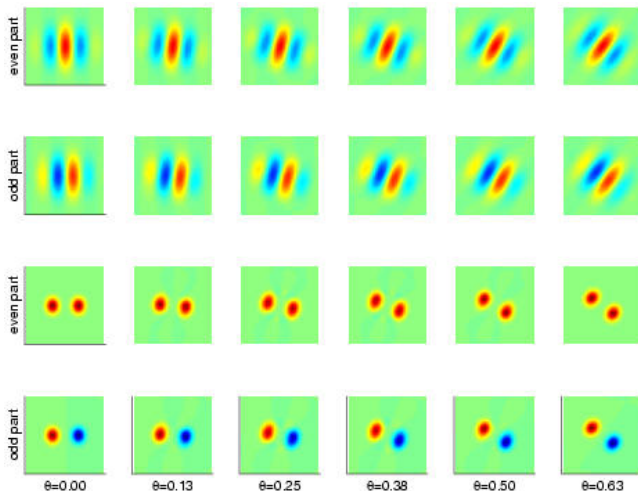
$$f(X) = \sum_j w_j H_j(X) + b = \sum_j w_j g(v_j^\top X) + b$$

- If  $X$  is an image,  $v_j$  is an image too.
- $v_j$  acts as a **filter** (presence or absence of a pattern).
- What does  $v_j$  look like ?

# $v_j$ 's for images - Examples



# $v_j$ 's for images - Examples



- Filters are mostly local

# Basic idea of convolutional neural networks

- Filters are mostly local.
- Instead of using image-wide filters, use small ones over patches.
- Repeat for every patch to get a response image.
- Subsample the response image to get local invariance.



# Filtering - Filter 1

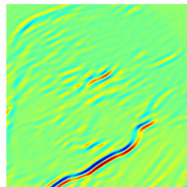
Original image



Filter



Output image



# Filtering - Filter 2

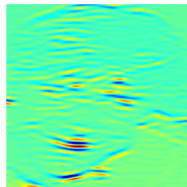
Original image



Filter



Output image



# Filtering - Filter 3

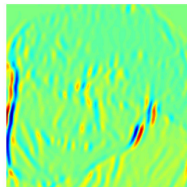
Original image



Filter



Output image

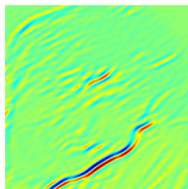


# Pooling - Filter 1

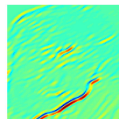
Original image



Output image



Subsampled image



How to do 2x subsampling-pooling :

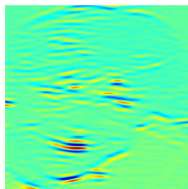
- Output image =  $O$ , subsampled image =  $S$ .
- $S_{ij} = \max_{k \text{ over window around } (2i, 2j)} O_k$ .

# Pooling - Filter 2

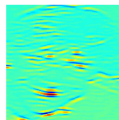
Original image



Output image



Subsampled image



How to do 2x subsampling-pooling :

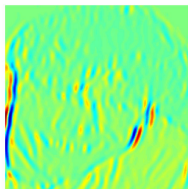
- Output image =  $O$ , subsampled image =  $S$ .
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# Pooling - Filter 3

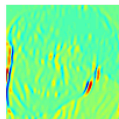
Original image



Output image



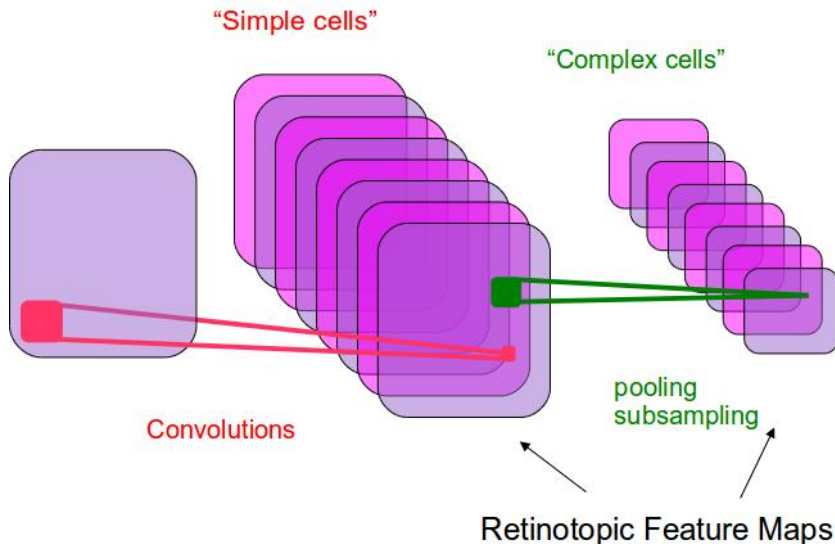
Subsampled image



How to do 2x subsampling-pooling :

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# A convolutional layer

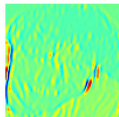
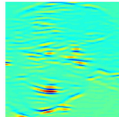
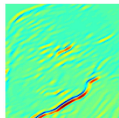


# Transforming the data with a layer

Original datapoint

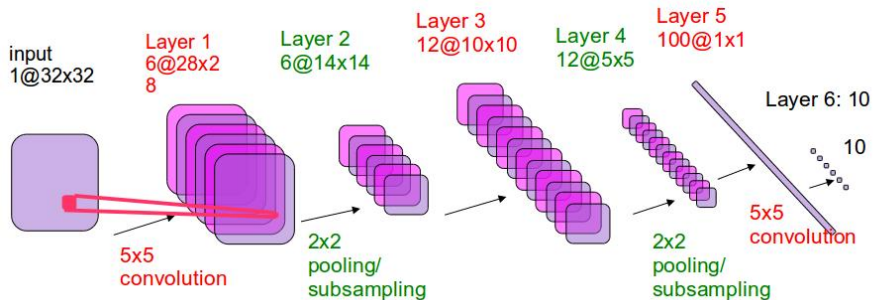


New datapoint





# A convolutional network





# Face detection



# Face detection



# NORB dataset

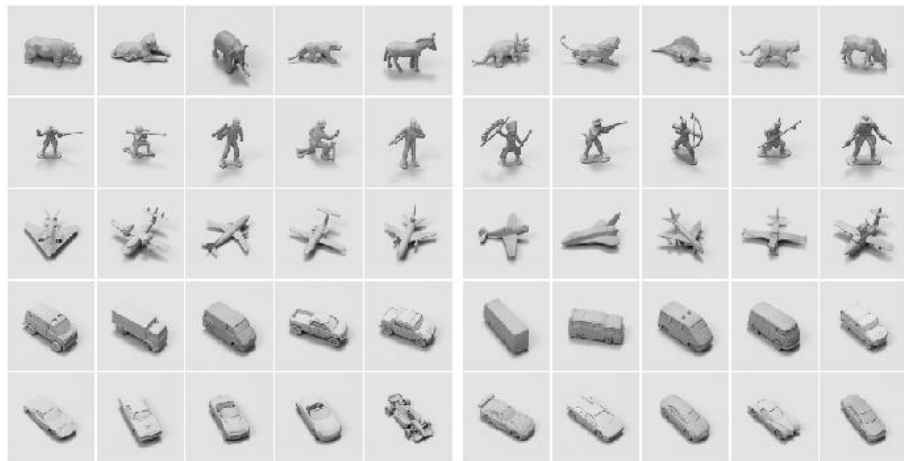
- 50 toys belonging to 5 categories
  - ▶ animal, human figure, airplane, truck, car
- 10 instance per category
  - ▶ 5 instances used for training, 5 instances for testing
- Raw dataset
  - ▶ 972 stereo pairs of each toy. 48,600 image pairs total.

# NORB dataset - 2

For each instance :

- 18 azimuths
- 0 to 350 degrees every 20 degrees
- 9 elevations
- 30 to 70 degrees from horizontal every 5 degrees
- 6 illuminations
- on/off combinations of 4 lights
- 2 cameras (stereo), 7.5 cm apart
- 40 cm from the object

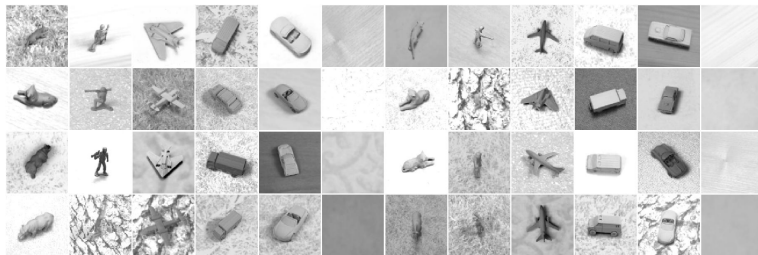
# NORB dataset - 3



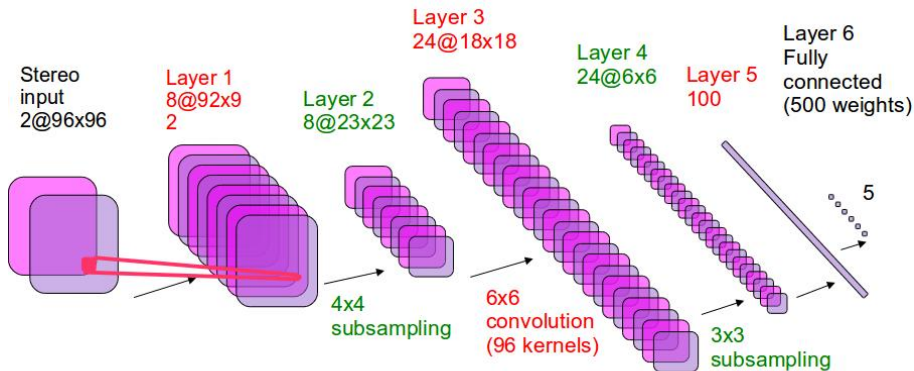
**Training instances**

**Test instances**

# Textured and cluttered versions







- 90,857 free parameters, 3,901,162 connections.
- The entire network is trained end-to-end (all the layers are trained simultaneously).

# Normalized-Uniform set

Method	Error
Linear Classifier on raw stereo images	30.2%
K-Nearest-Neighbors on raw stereo images	18.4%
K-Nearest-Neighbors on PCA-95	16.6%
Pairwise SVM on 96x96 stereo images	11.6%
Pairwise SVM on 95 Principal Components	13.3%
Convolutional Net on 96x96 stereo images	5.8%

# Jittered-Cluttered Dataset



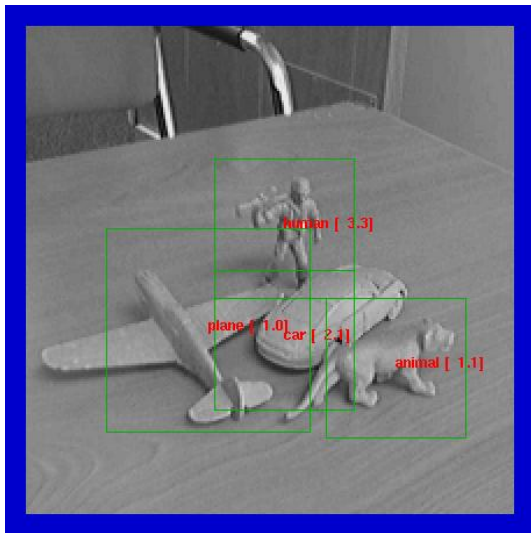
- 291,600 stereo pairs for training, 58,320 for testing
- Objects are jittered
  - ▶ position, scale, in-plane rotation, contrast, brightness, backgrounds, distractor objects,...
- Input dimension : 98x98x2 (approx 18,000)

# Jittered-Cluttered Dataset - Results

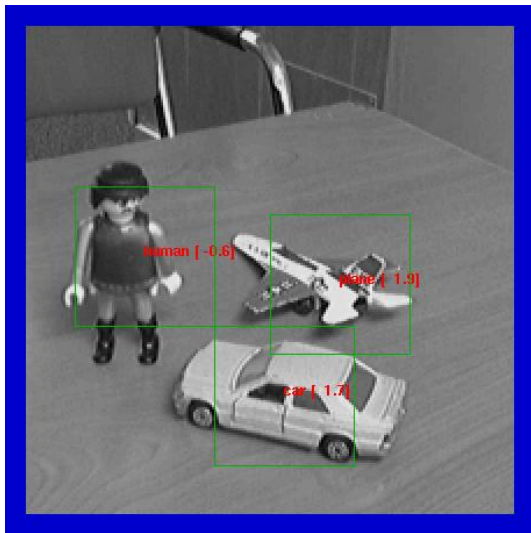
Method	Error
SVM with Gaussian kernel	43.3%
Convolutional Net with binocular input	7.8%
Convolutional Net + SVM on top	5.9%
Convolutional Net with monocular input	20.8%
Smaller mono net (DEMO)	26.0%

Dataset available from <http://www.cs.nyu.edu/~yann>

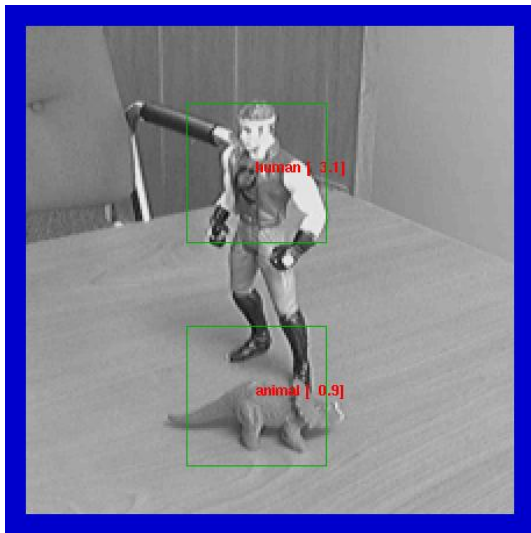
# NORB recognition - 1



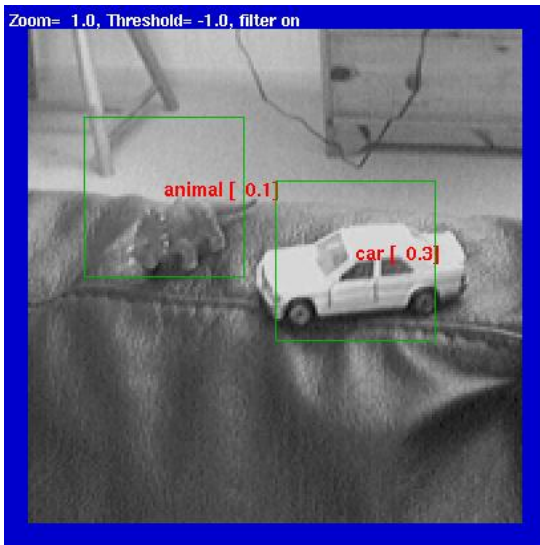
# NORB recognition - 2



# NORB recognition - 3

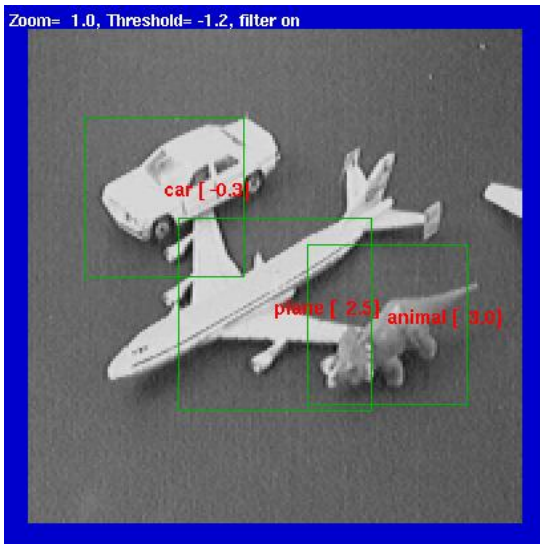


# NORB recognition - 4

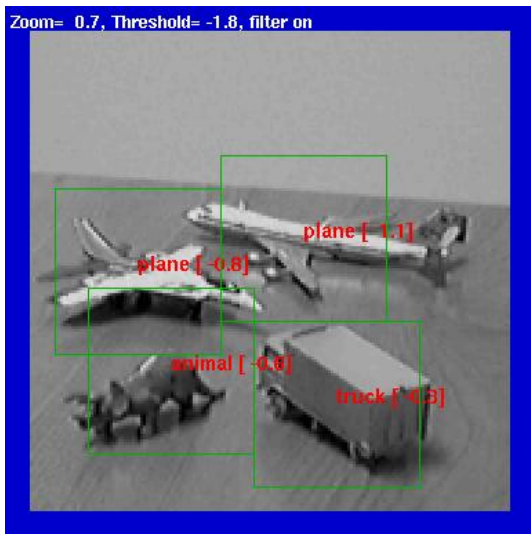




# NORB recognition - 5



# NORB recognition - 6



# Summary

- With complex problems, it is hard to design features by hand.
- Neural networks circumvent this problem.
- They can be hard to train (again...).
- Convolutional neural networks use knowledge about locality in images.
- They are much easier than standard networks.
- And they are FAST (again...).

# What has not been covered

- In some cases, we have lots of data, but without the labels.
- *Unsupervised* learning.
- There are techniques to use these data to get better performance.
- E.g. : *Task-Driven Dictionary Learning*, Mairal et al.

# The need for fast learning

- Neural networks may need many examples (several millions or more).
- We need to be able to use them quickly.

# Batch methods

$$L(\theta) = \frac{1}{N} \sum_i \ell(\theta, X^{(i)}, Y^{(i)})$$
$$\theta_{t+1} \rightarrow \theta_t - \frac{\alpha_t}{N} \sum_i \frac{\partial \ell(\theta, X^{(i)}, Y^{(i)})}{\partial \theta}$$

- To compute one update of the parameters, we need to go through all the data.
- This can be very expensive.
- What if we have an infinite amount of data ?

# Potential solutions

- 1 Discard data.
  - ▶ Seems stupid
  - ▶ Yet many people do it

# Potential solutions

- 1 Discard data.
  - ▶ Seems stupid
  - ▶ Yet many people do it
- 2 Use approximate methods.
  - ▶ Update = average of the updates for all datapoints.
  - ▶ Are these update really different ?
  - ▶ If not, how can we learn faster ?



# Stochastic gradient descent

$$L(\theta) = \frac{1}{N} \sum_i \ell(\theta, X^{(i)}, Y^{(i)})$$

$$\theta_{t+1} \rightarrow \theta_t - \frac{\alpha_t}{N} \sum_i \frac{\partial \ell(\theta, X^{(i)}, Y^{(i)})}{\partial \theta}$$

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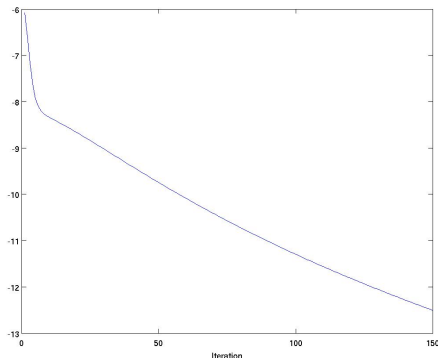
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# Stochastic gradient descent

$$L(\theta) = \frac{1}{N} \sum_i \ell(\theta, \mathbf{X}^{(i)}, \mathbf{Y}^{(i)})$$
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What do we lose when updating the parameters to satisfy just one example ?

# Disagreement



- $\|\mu\|^2/\sigma^2$  during optimization (log scale)
- As optimization progresses, disagreement increases
- Early on, one can pick one example at a time
- What about later ?

# Training vs test

Standard learning paradigm :

- We want to solve a task on new datapoints.
- We have a training set.
- We **hope** that the performance on the training set is informative of the performance on new datapoints.

Can we know when we start overfitting ?

# Overfitting

- When all gradients disagree, stochastic error stalls.
- When all gradients disagree, training and test error part.

**IT DOES NOT MATTER IF ONE DOES NOT REACH THE  
MINIMUM OF THE TRAINING ERROR !**

# Decomposition of the error

$$\begin{aligned} E(\tilde{f}_n) - E(f^*) &= E(f_{\mathcal{F}}^*) - E(f^*) \text{ Approximation error} \\ &+ E(f_n) - E(f_{\mathcal{F}}^*) \text{ Estimation error} \\ &+ E(\tilde{f}_n) - E(f_n) \text{ Optimization error} \end{aligned}$$

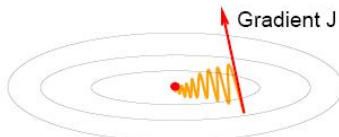
Questions :

- 1 Do we optimize the training error to decrease  $E(\tilde{f}_n) - E(f_n)$  ?
- 2 Do we increase  $n$  to decrease  $E(f_n) - E(f_{\mathcal{F}}^*)$  ?

# Gradient Descent (GD)

Iterate

$$\bullet \quad w_{t+1} \leftarrow w_t - \eta \frac{\partial E_n(f_{w_t})}{\partial w}$$



Best speed achieved with fixed learning rate  $\eta = \frac{1}{\lambda_{\max}}$ .  
(e.g., Dennis & Schnabel, 1983)

	Cost per iteration	Iterations to reach $\rho$	Time to reach accuracy $\rho$	Time to reach $E(\tilde{f}_n) - E(f_{\mathcal{F}}^*) < \varepsilon$
<b>GD</b>	$\mathcal{O}(nd)$	$\mathcal{O}\left(\kappa \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(nd\kappa \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2 \kappa}{\varepsilon^{1/\alpha}} \log^2 \frac{1}{\varepsilon}\right)$

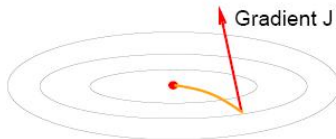
- In the **last column**,  $n$  and  $\rho$  are chosen to reach  $\varepsilon$  as fast as possible.
- Solve for  $\varepsilon$  to find the **best error rate achievable in a given time**.
- Remark: abuses of the  $\mathcal{O}()$  notation



# Second Order Gradient Descent (2GD)

Iterate

- $w_{t+1} \leftarrow w_t - H^{-1} \frac{\partial E_n(f_{w_t})}{\partial w}$



We assume  $H^{-1}$  is known in advance.

Superlinear optimization speed (e.g., Dennis & Schnabel, 1983)

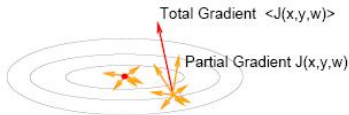
	Cost per iteration	Iterations to reach $\rho$	Time to reach accuracy $\rho$	Time to reach $E(\tilde{f}_n) - E(f_{\mathcal{F}}^*) < \varepsilon$
<b>2GD</b>	$\mathcal{O}(d(d+n))$	$\mathcal{O}\left(\log \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(d(d+n) \log \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2}{\varepsilon^{1/\alpha}} \log \frac{1}{\varepsilon} \log \log \frac{1}{\varepsilon}\right)$

- Optimization speed is much faster.
- Learning speed only saves the condition number  $\kappa$ .

# Stochastic Gradient Descent (SGD)

Iterate

- Draw random example  $(x_t, y_t)$ .
- $w_{t+1} \leftarrow w_t - \frac{\eta}{t} \frac{\partial \ell(f_{w_t}(x_t), y_t)}{\partial w}$



Best decreasing gain schedule with  $\eta = \frac{1}{\lambda_{\min}}$ .  
(see Murata, 1998; Bottou & LeCun, 2004)

	Cost per iteration	Iterations to reach $\rho$	Time to reach accuracy $\rho$	Time to reach $E(\tilde{f}_n) - E(f_{\mathcal{F}}^*) < \varepsilon$
<b>SGD</b>	$\mathcal{O}(d)$	$\frac{\nu k}{\rho} + o\left(\frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d \nu k}{\rho}\right)$	$\mathcal{O}\left(\frac{d \nu k}{\varepsilon}\right)$

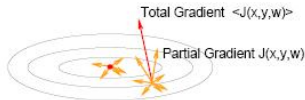
With  $1 \leq k \leq \kappa^2$

- Optimization speed is *catastrophic*.
- Learning speed does not depend on the statistical estimation rate  $\alpha$ .
- Learning speed depends on condition number  $\kappa$  but *scales very well*.

# Second order Stochastic Descent (2SGD)

Iterate

- Draw random example  $(x_t, y_t)$ .
- $w_{t+1} \leftarrow w_t - \frac{1}{t} H^{-1} \frac{\partial \ell(f_{w_t}(x_t), y_t)}{\partial w}$



Replace scalar gain  $\frac{\eta}{t}$  by matrix  $\frac{1}{t} H^{-1}$ .

	Cost per iteration	Iterations to reach $\rho$	Time to reach accuracy $\rho$	Time to reach $E(\tilde{f}_n) - E(f_{\mathcal{F}}^*) < \varepsilon$
<b>2SGD</b>	$\mathcal{O}(d^2)$	$\frac{\nu}{\rho} + o\left(\frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2 \nu}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2 \nu}{\varepsilon}\right)$

- Each iteration is  $d$  times more expensive.
- The number of iterations is reduced by  $\kappa^2$  (or less.)
- Second order only changes the constant factors.

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In practice :

- You will **ALMOST ALWAYS** have enough data.
- You will **ALMOST ALWAYS** lack time.
- You must **ALMOST ALWAYS** use stochastic methods.
- How to use accelerated techniques remains to be seen.