Neural networks for vision

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Outline

- Linear classifiers
- Combining linear classifiers
- Learning a neural network
- Convolutional neural networks
- The power of sloppiness

Foreword

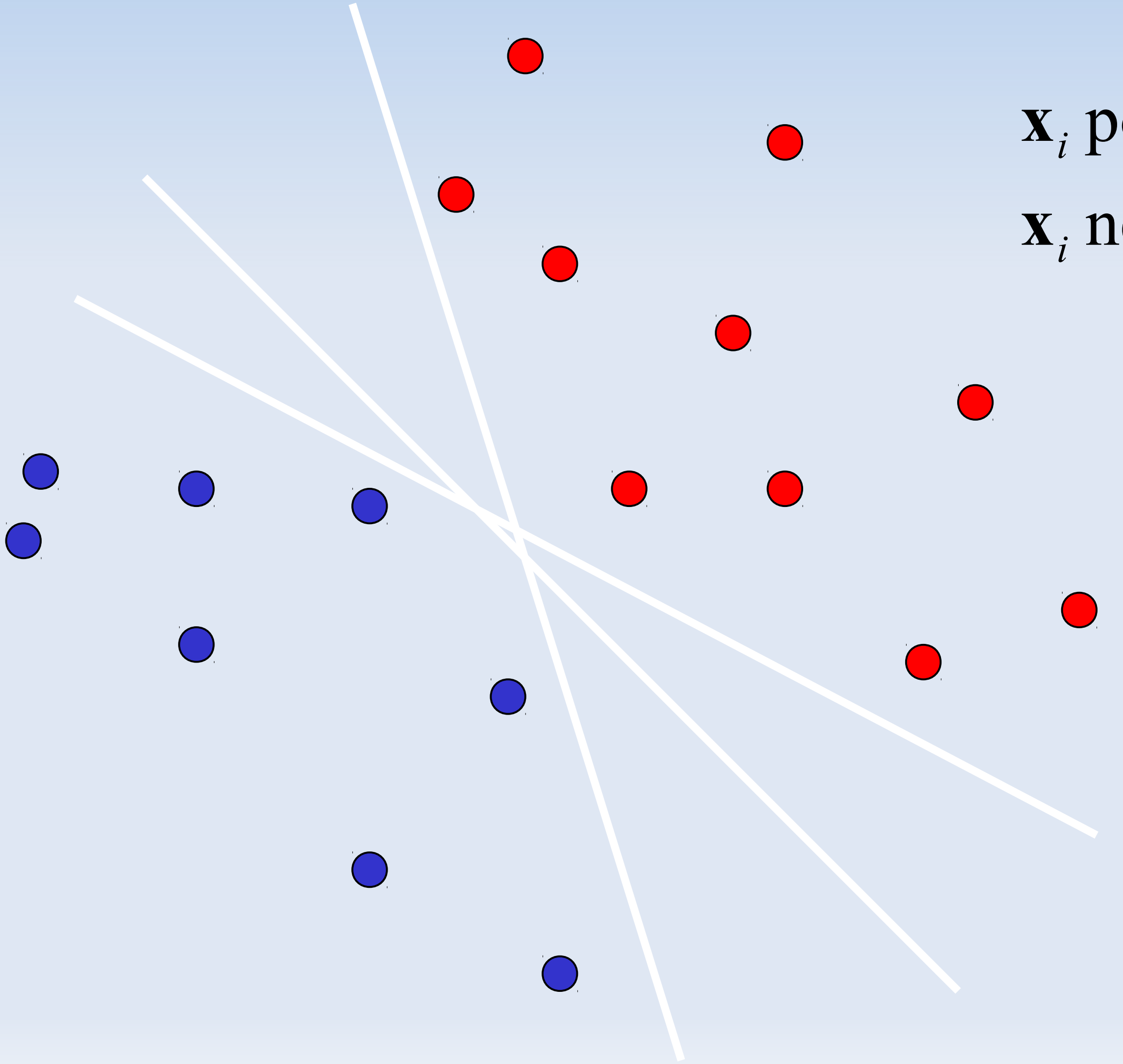
- I'm here for you, I already know that stuff
- It's better to look silly than to stay so
- Ask questions if you don't understand!

Linear classifiers

Find linear function (*hyperplane*) to separate positive and negative examples

$$\mathbf{x}_i \text{ positive}: \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 0$$

$$\mathbf{x}_i \text{ negative}: \quad \mathbf{x}_i \cdot \mathbf{w} + b < 0$$



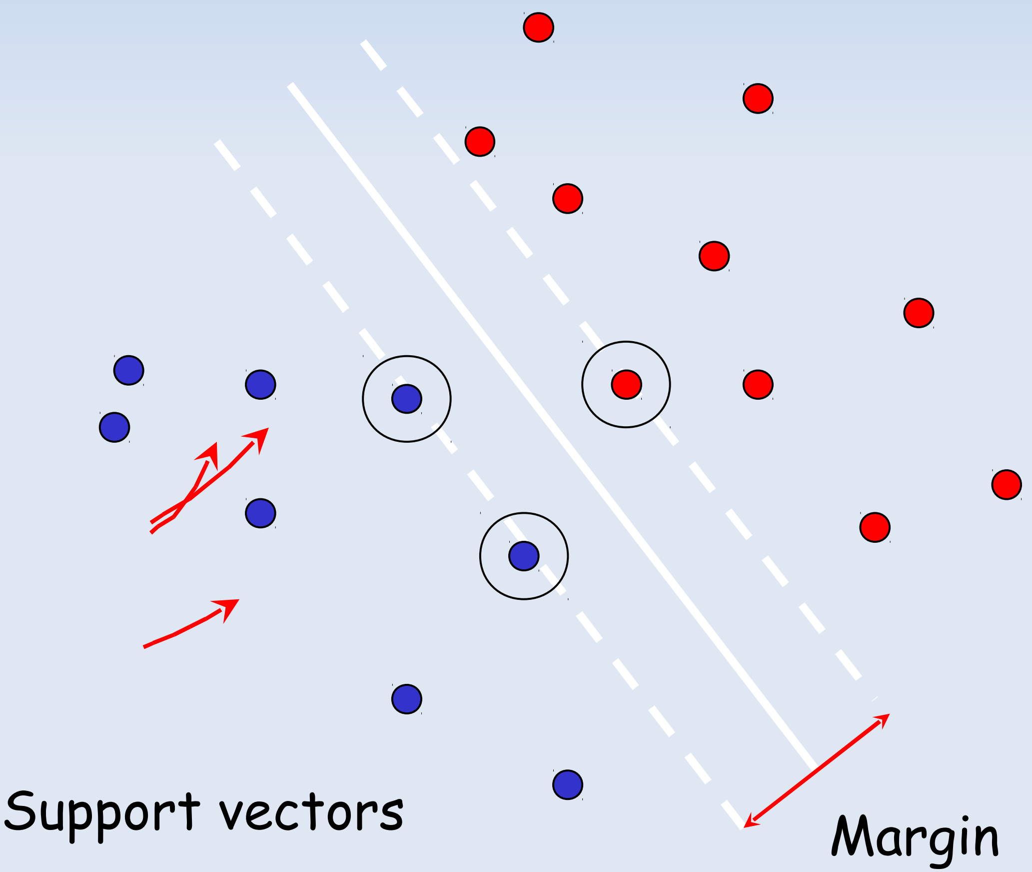
Which hyperplane is best?

Support vector machines

- Find hyperplane that maximizes the *margin* between the positive and negative examples

$$\mathbf{x}_i \text{ positive } (y_i = 1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \geq 1$$

$$\mathbf{x}_i \text{ negative } (y_i = -1): \quad \mathbf{x}_i \cdot \mathbf{w} + b \leq -1$$



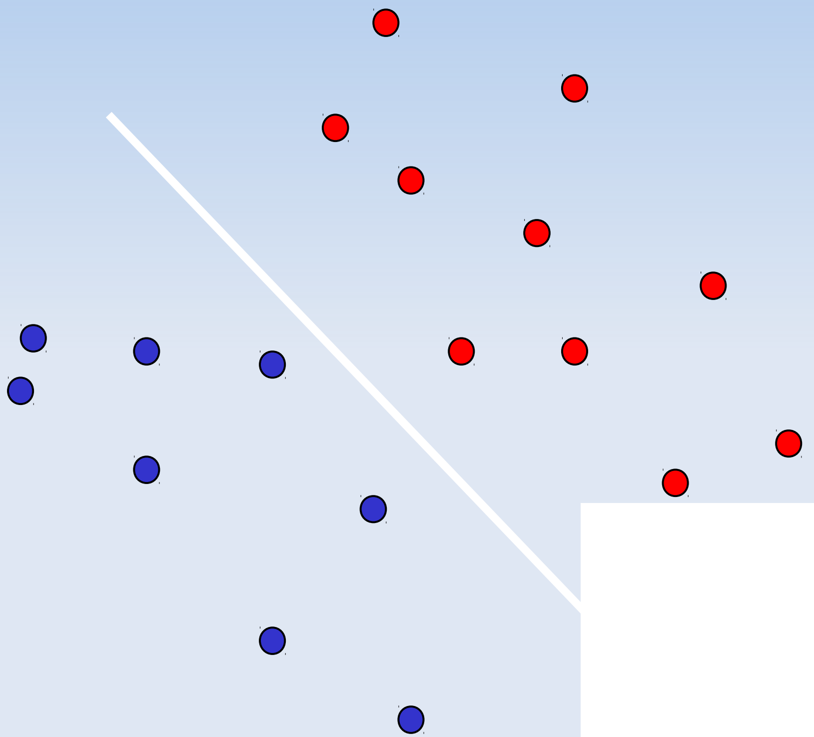
For support, vectors, $\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$

Distance between point and hyperplane: $\dfrac{|\mathbf{x}_i \cdot \mathbf{w} + b|}{||\mathbf{w}||}$

Therefore, the margin is $2 / \|\mathbf{w}\|$

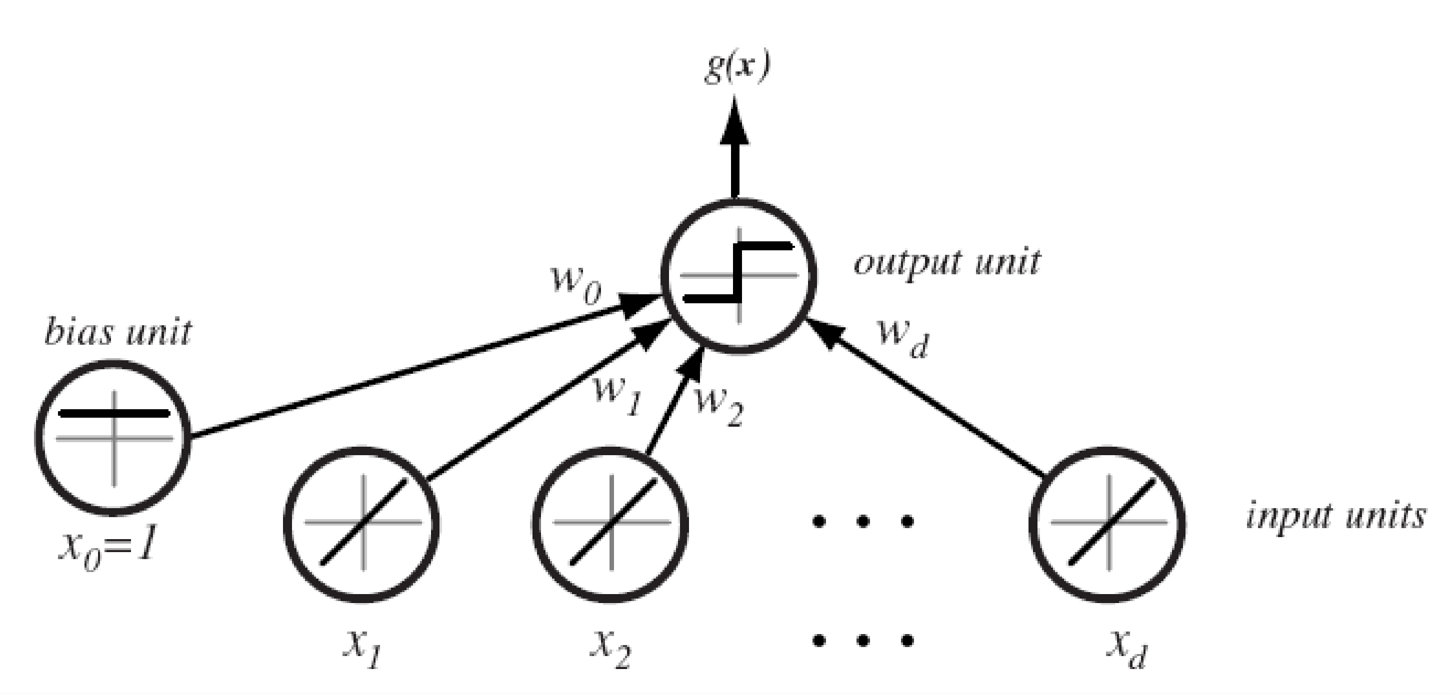
C. Burges, A Tutorial on Support Vector Machines for Pattern Recognition, Data Mining and Knowledge Discovery, 1998

Linear classifiers



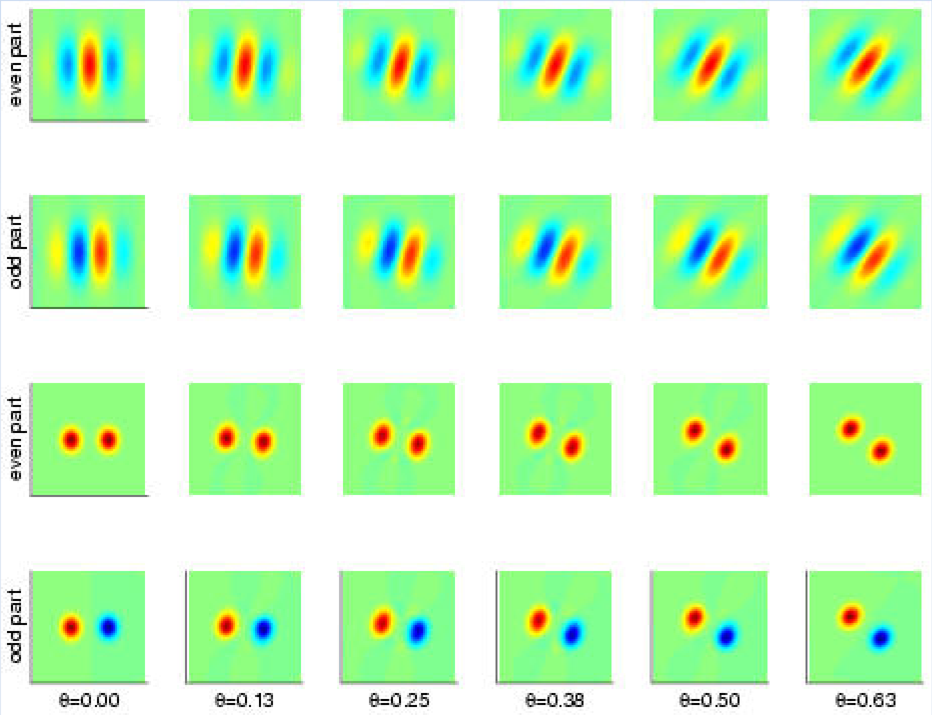
The perceptron (Rosenblatt'57)

What the perceptron can learn, it will learn using a simple weight update rule.



What does w look like?

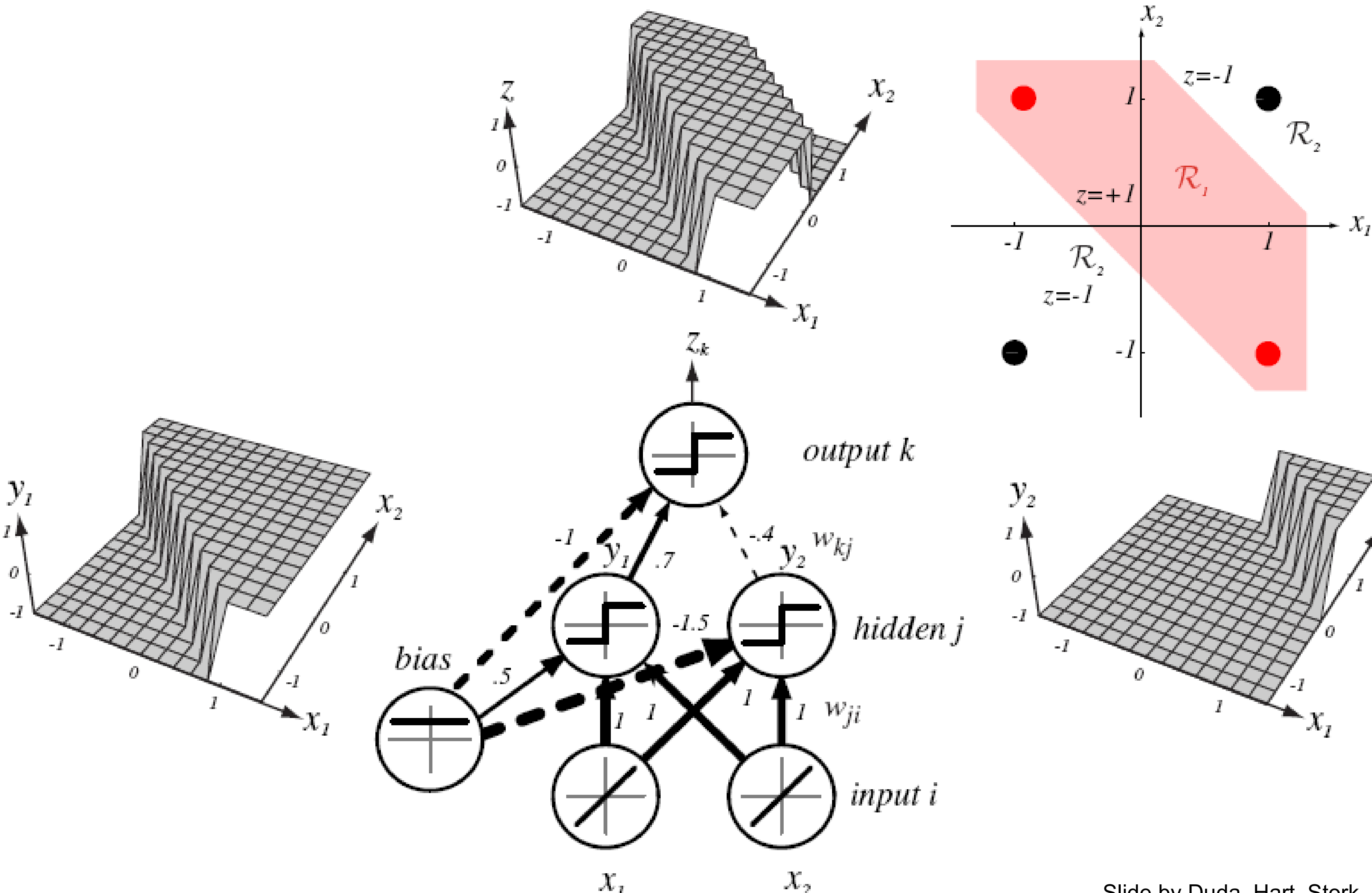
- Gabor filters
  - Edge detectors
  - Various angles
  - Various frequencies



Some problems are not linear

- Can we learn them using a combination of linear filters?
- ”Features” are more and more complex!

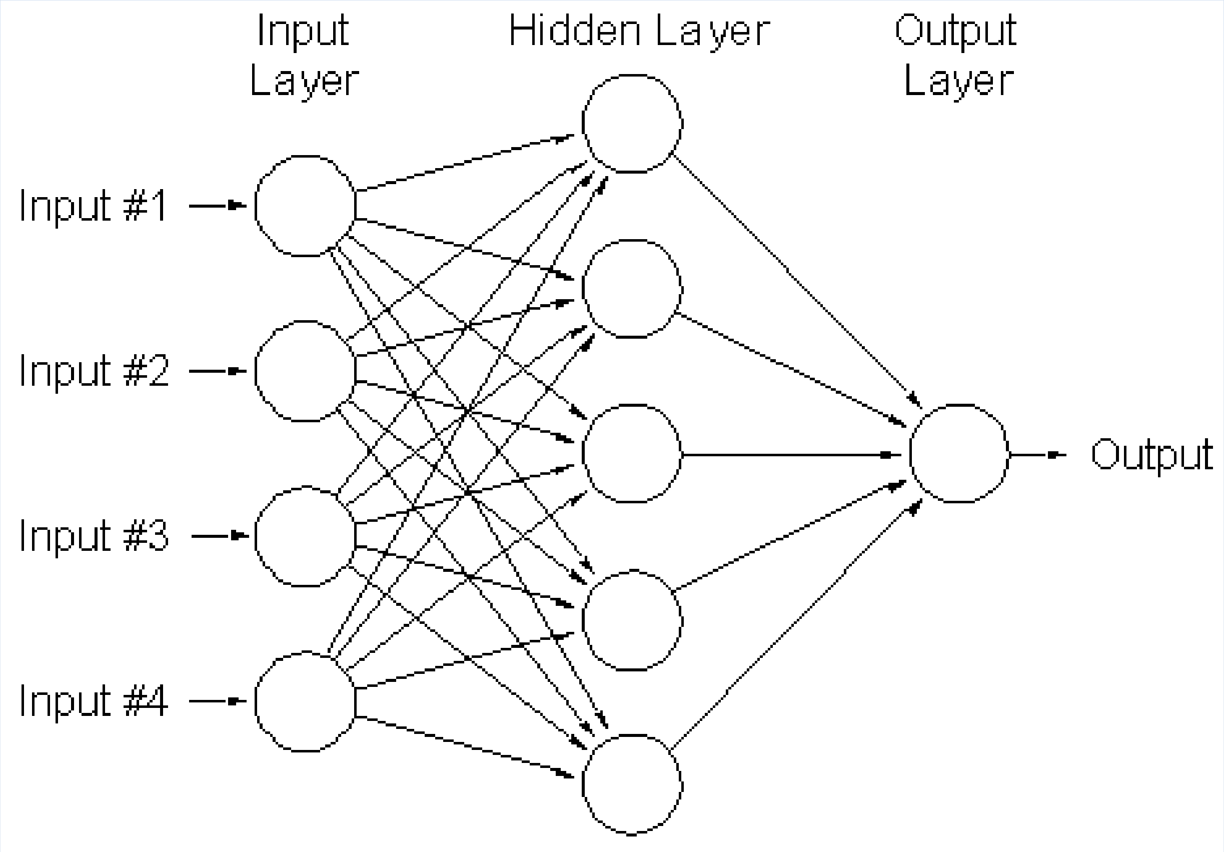
Modeling non-linear functions



Slide by Duda, Hart, Stork

A multilayer neural network

- Linear classifier at the end!
- Unrestricted hidden layer



Link between NNs and SVMs

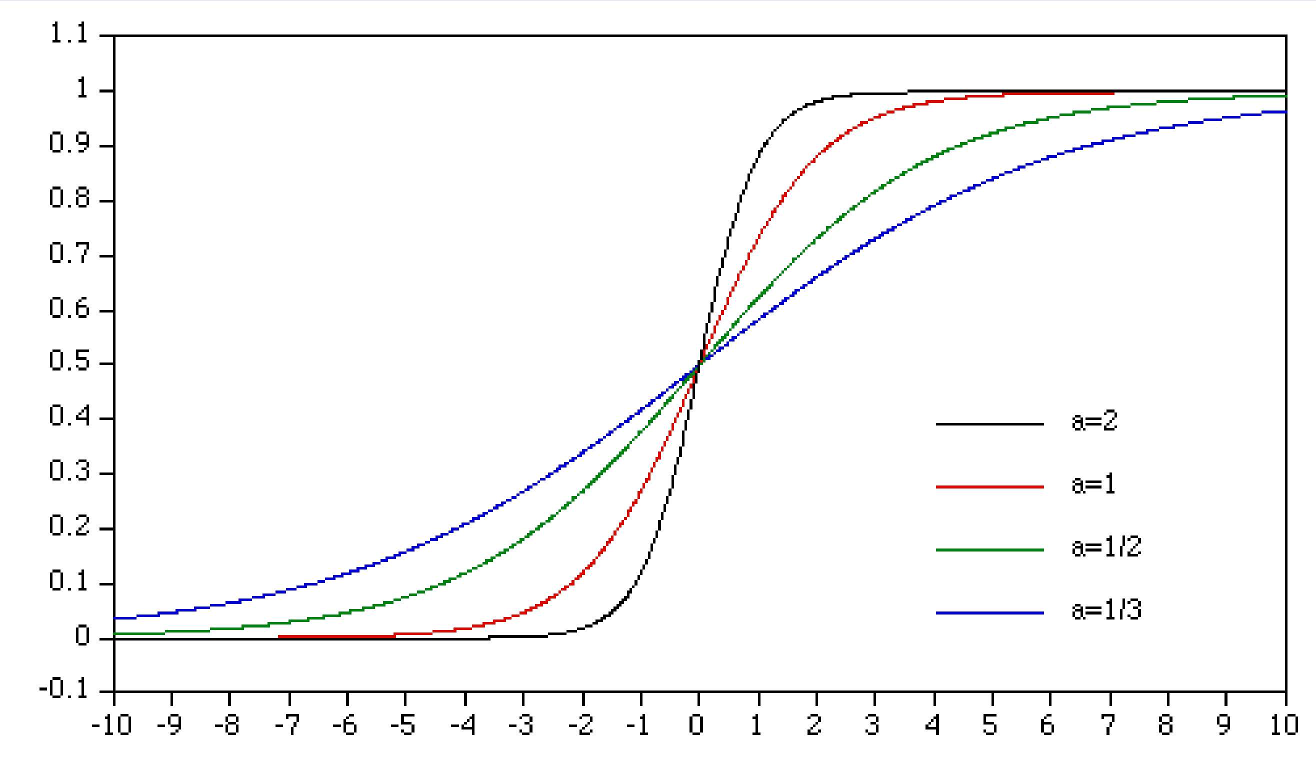
- A neural network is a linear classifier in a new space.
- It is a universal approximator.
- It is an SVM whose kernel can be (sometimes badly) learnt!
- Wait... Learnt?

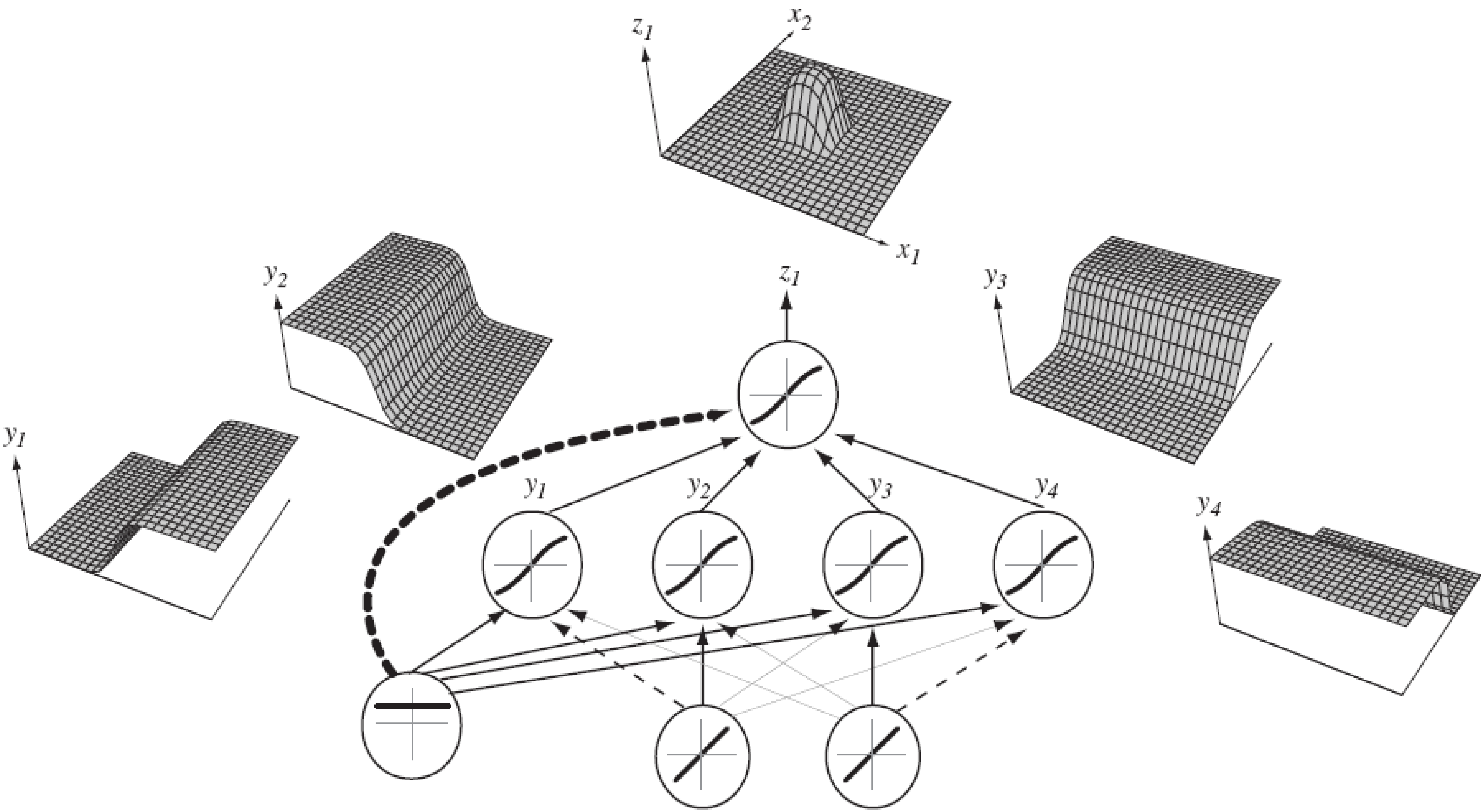
Learning a neural network

- A neural network is just a function (which one?)
- If it has a gradient, we can do gradient descent.
- Does it?

Changing the activation

- A sigmoid is a smoothed version of the threshold





Any function can be learned by a 3-layer network with enough hidden units

Gradient-based supervised learning

- Parametric prediction function: f (x, w) →ℑ y

- Learning: Minimize

$$E = \sum i\ L\ (yi\ , f\ (xi,\ w))$$

- Recognition: y = f(x, w)

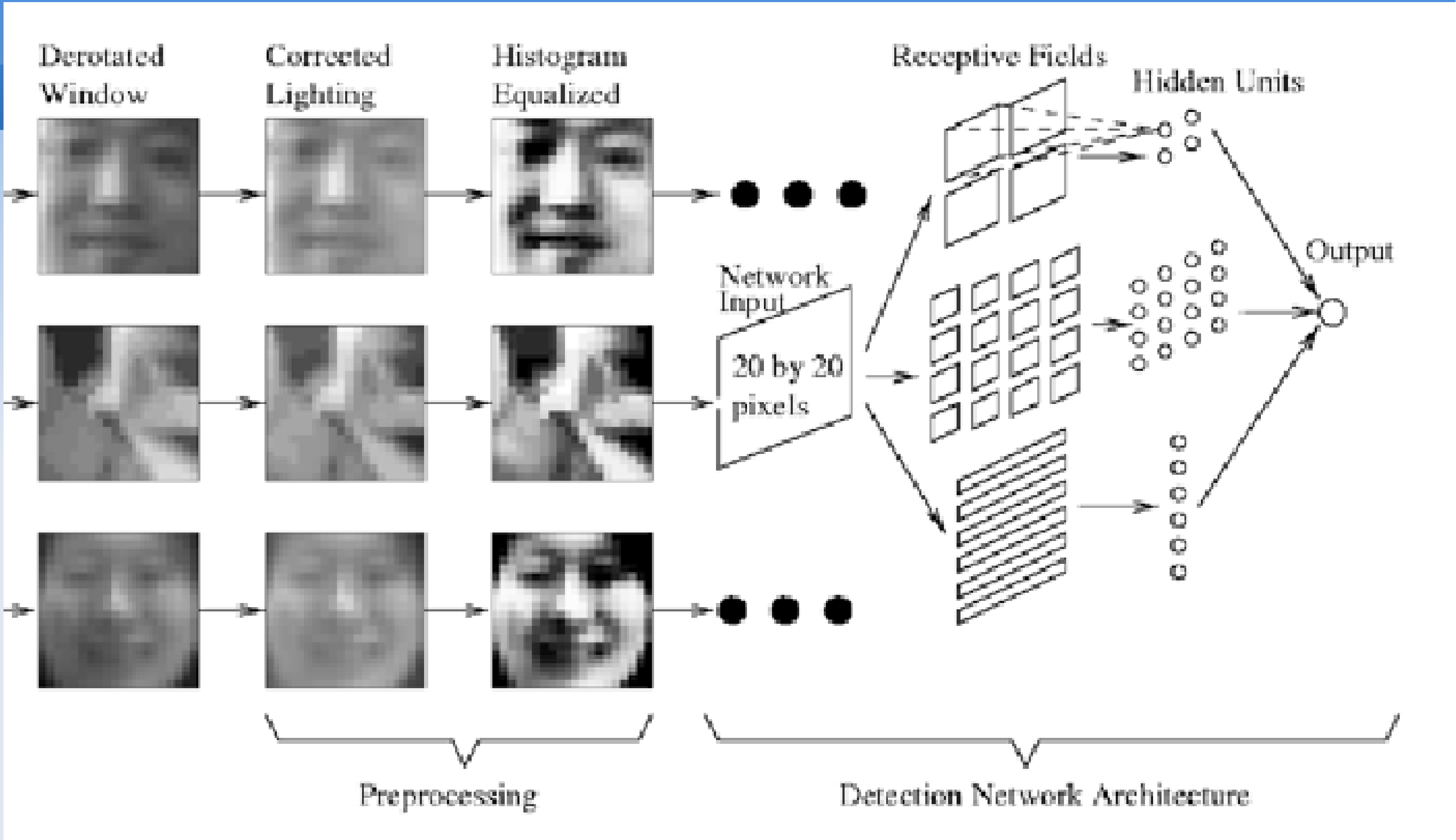
How can we minimize E? ..Gradient descent..

Gradient-based supervised learning II

- Difference between stochastic and batch.

Backpropagation

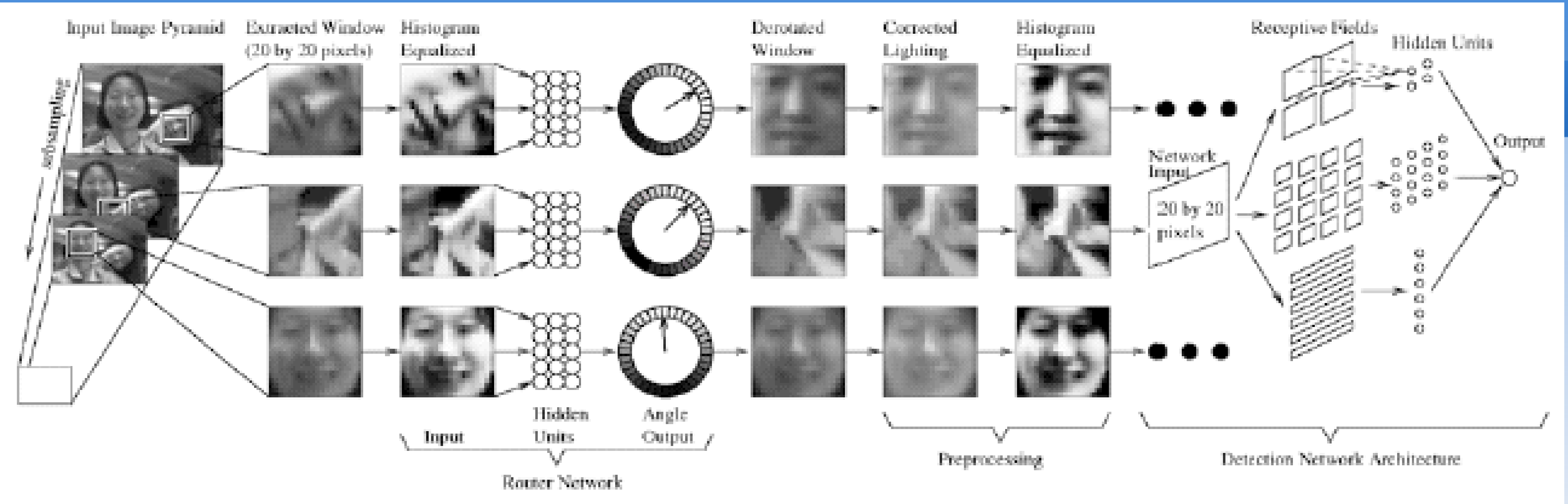
- The gradient with respect to one layer depends on the gradient with respect to the layer above.
- We can ”backpropagate” the gradient to the layers below.



The vertical face-finding part of Rowley, Baluja and Kanade's system

Figure from "Rotation invariant neural-network based face detection," H.A. Rowley, S. Baluja and T. Kanade, Proc. Computer Vision and Pattern Recognition, 1998, copyright 1998, IEEE

Slide by D.A. Forsyth



Architecture of the complete system: they use another neural net to estimate orientation of the face, then rectify it. They search over scales to find bigger/smaller faces.

Figure from "Rotation invariant neural-network based face detection," H.A. Rowley, S. Baluja and T. Kanade, Proc. Computer Vision and Pattern Recognition, 1998, copyright 1998, IEEE

Figure from “Rotation invariant neural-network based face detection,” H.A. Rowley, S. Baluja and T. Kanade, Proc. Computer Vision and Pattern Recognition, 1998, copyright 1998, IEEE

Slide by D.A. Forsyth

Advantages of MLP

- Can learn anything
- Extremely fast at test time (computing the answer for a new datapoint)
- Complete control over the power of the network (by controlling the hidden layers sizes).

Problems with MLP

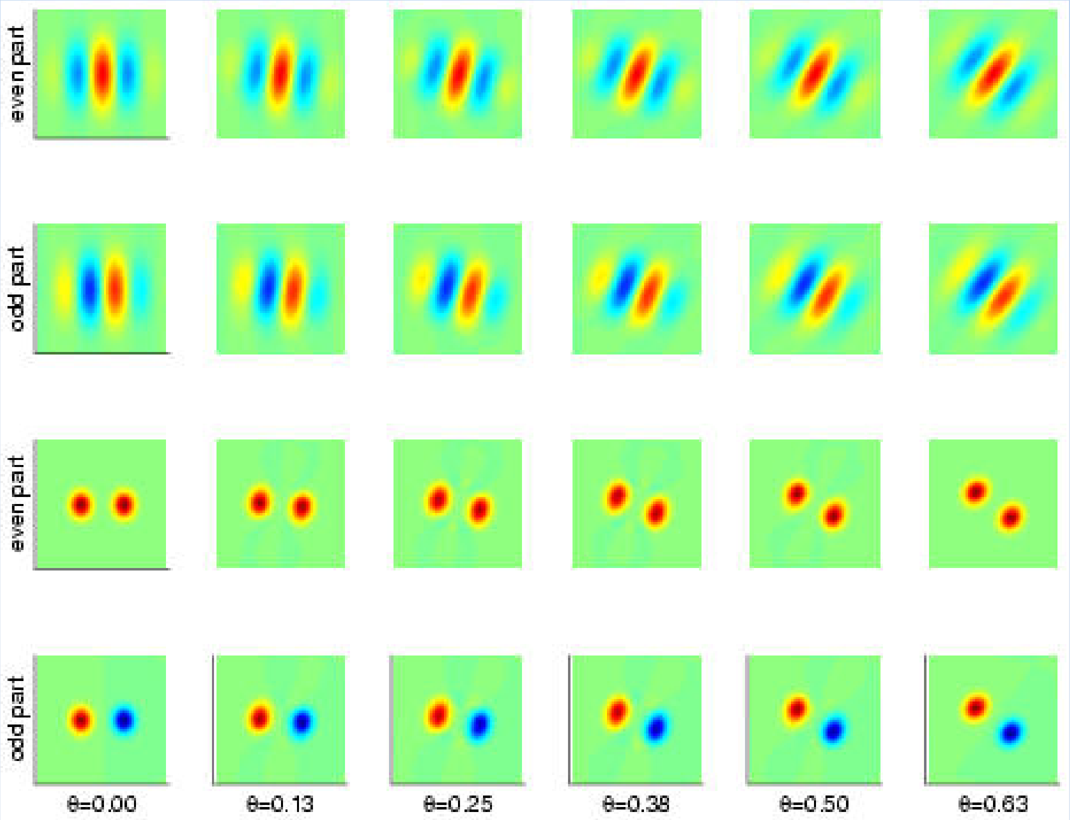
- Highly non-convex → many local minima
- Upper layers much harder to train than lower layers
- Can learn anything → needs tons of examples to be good (make some awesome Tennis analogy here).

Take-home messages

- Neural networks can learn anything
- But it is HARD!
- If you wish to use them, be smart (or ask someone who knows)!
- If you have a huge dataset, they CAN be awesome!

Convolutional NNets

- An image was just a huge vector
- Can we make more assumptions?
- Filters are mostly LOCAL!

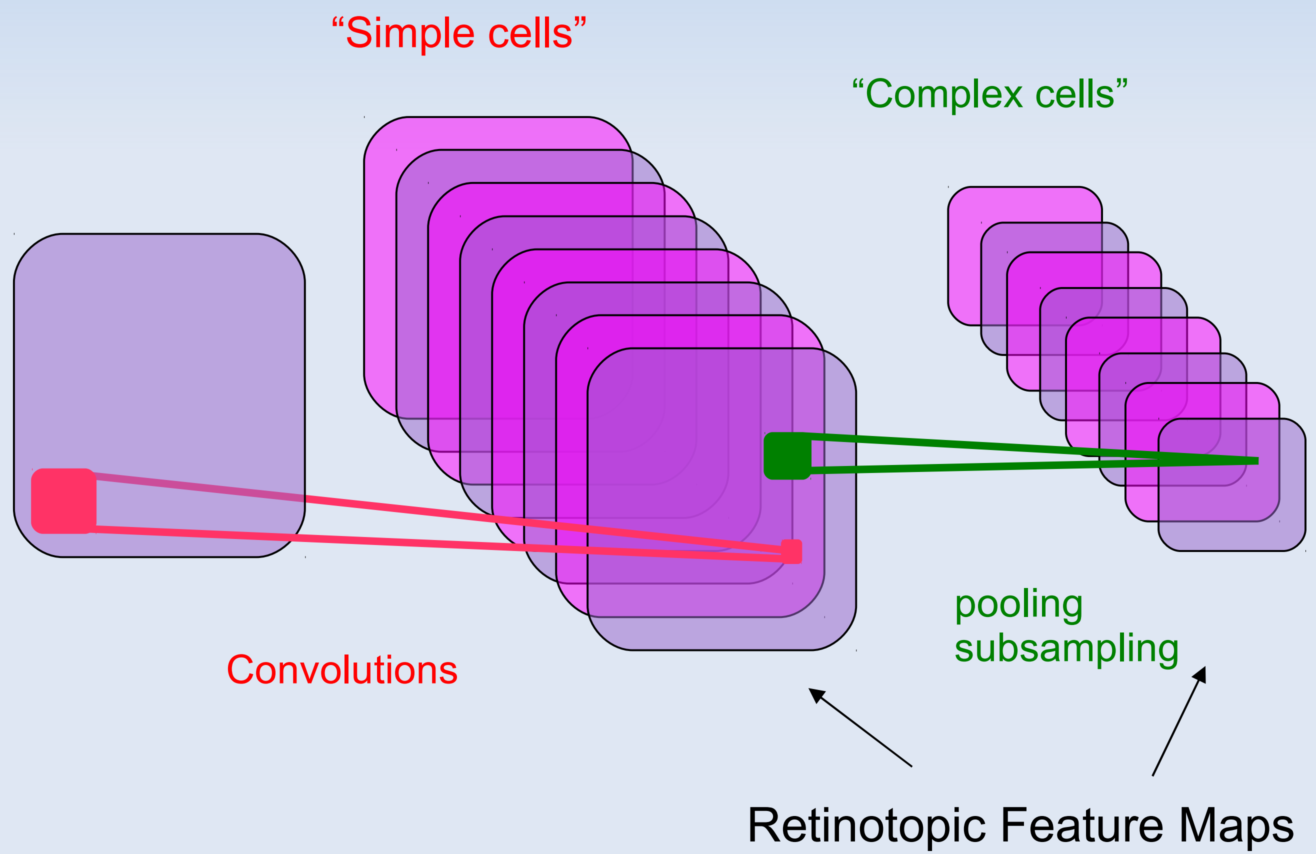


Basic idea

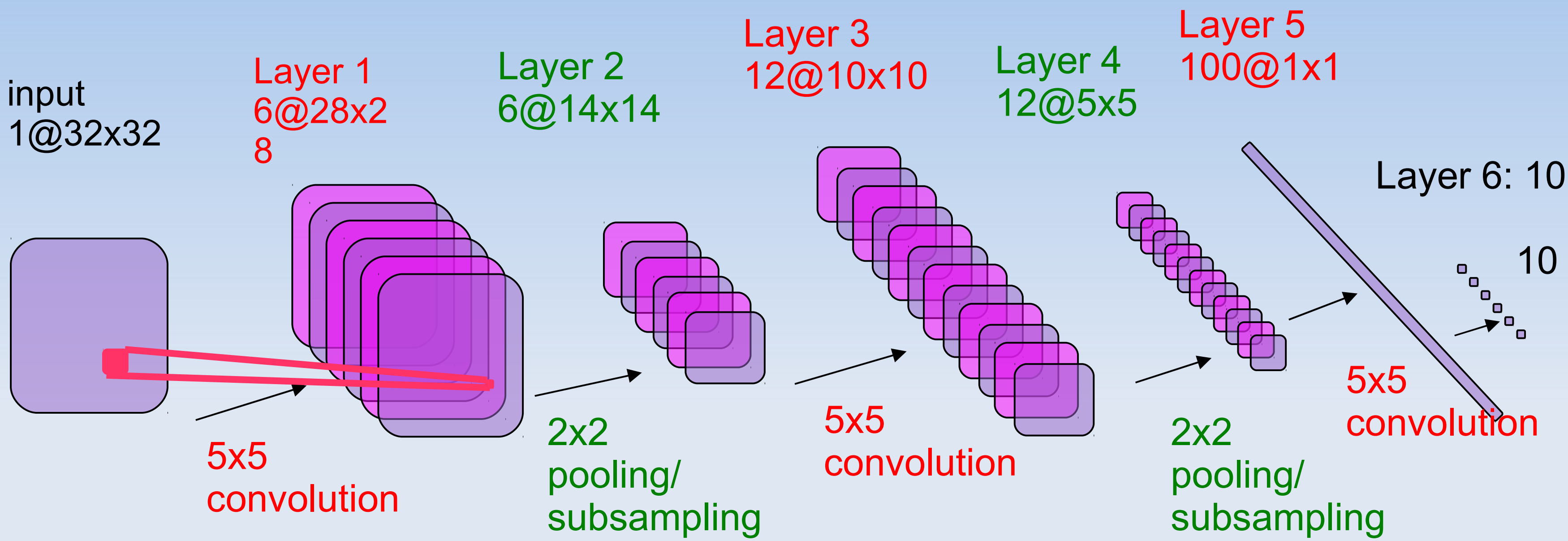
- Insted of computing features over the entire image, compute it over small patches.
- Repeat for every patch.
- ”Pool” features to get local invariance.

An Old Idea for Local Shift Invariance

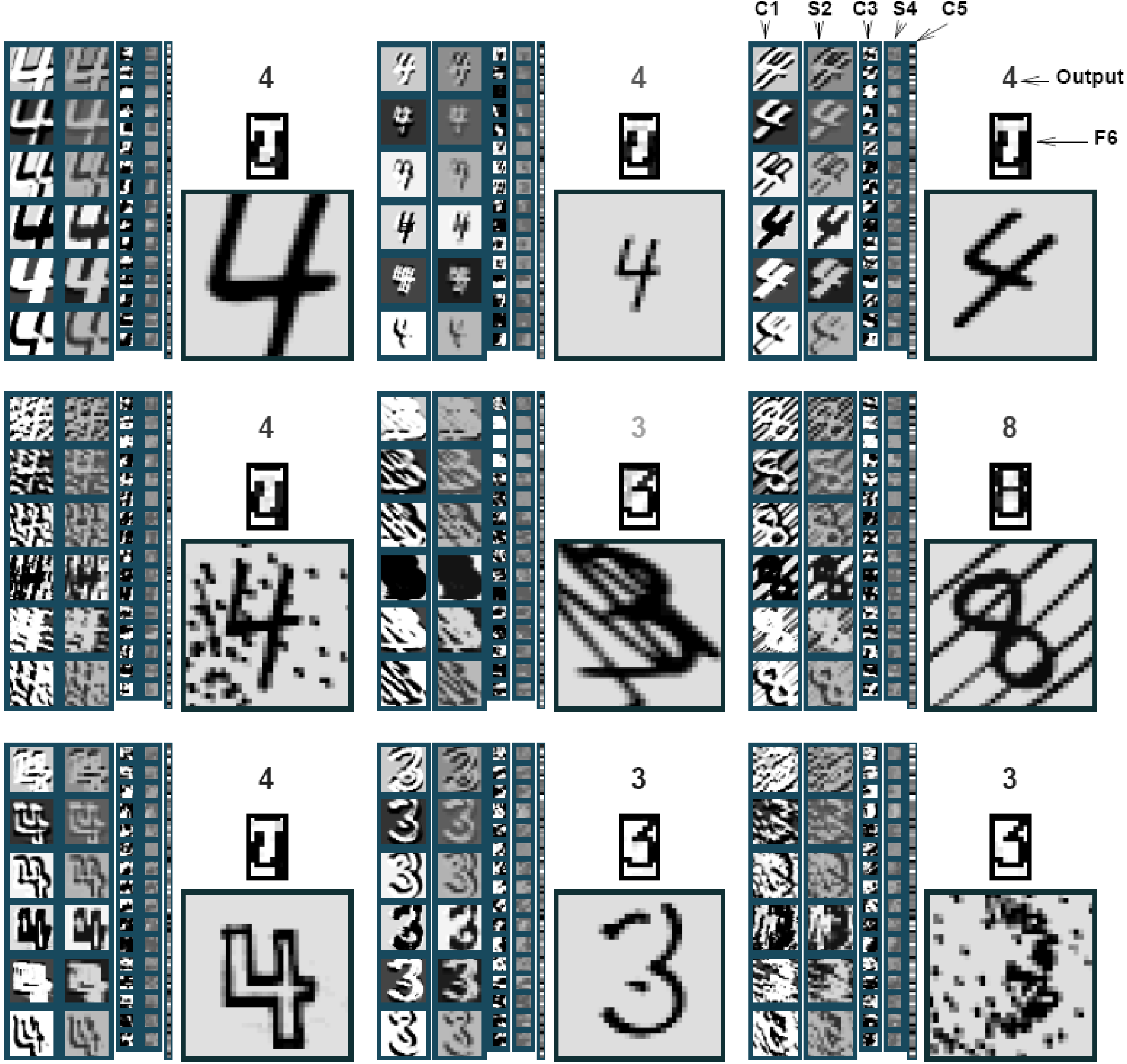
- **[Hubel & Wiesel 1962]:**
  - simple cells detect local features
  - complex cells "pool" the outputs of simple cells within a retinotopic neighborhood.



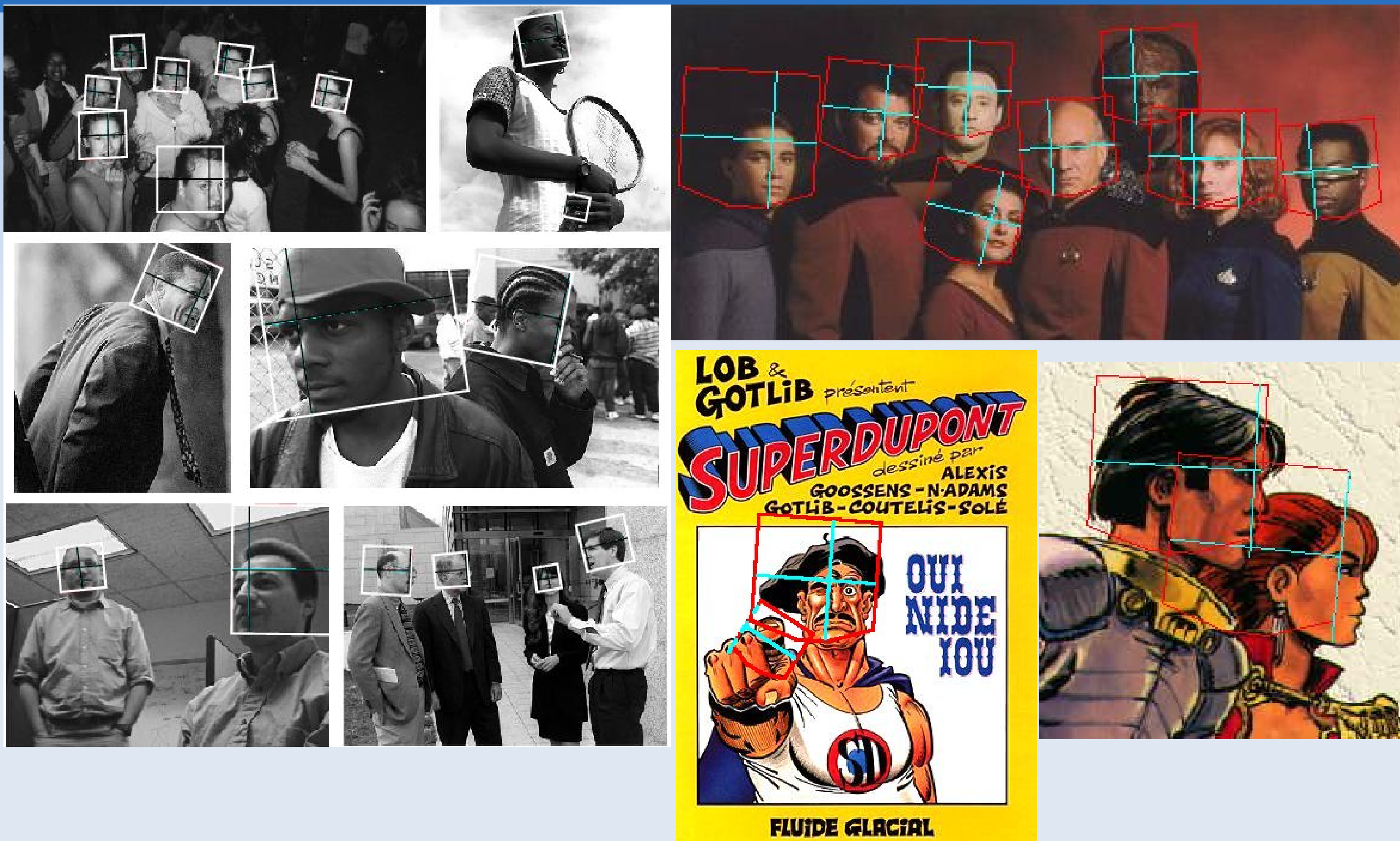
CNN architecture



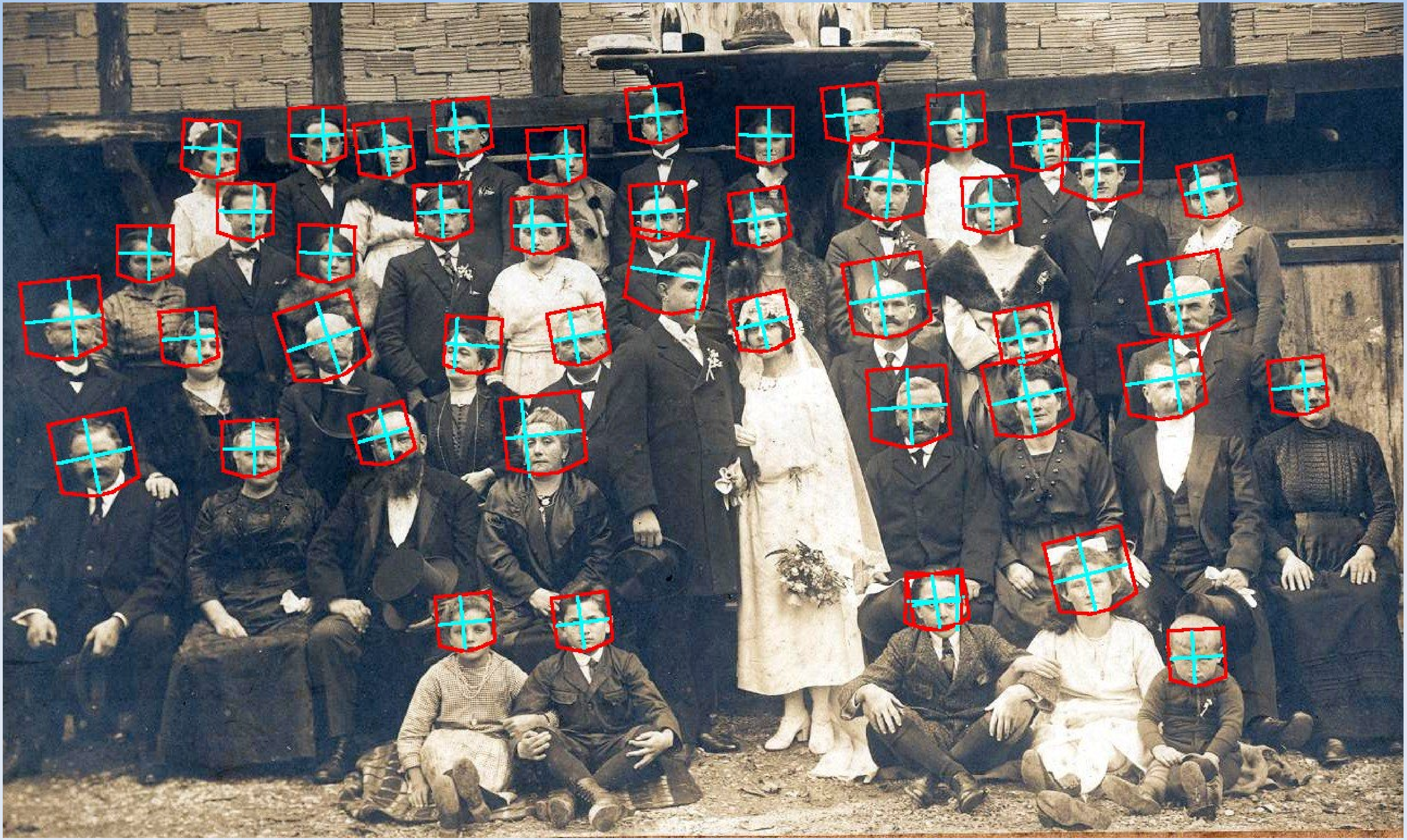
- **Convolutional net for handwriting recognition** (400,000 synapses)
- Convolutional layers (simple cells): all units in a feature plane share the same weights
- Pooling/subsampling layers (complex cells): for invariance to small distortions.
- **Supervised gradient-descent learning using back-propagation**
- **The entire network is trained end-to-end. All the layers are trained simultaneously**.


C1
S2
C3
S4
C5
4
4
4
Output
F6
4
3
8
4
3
3

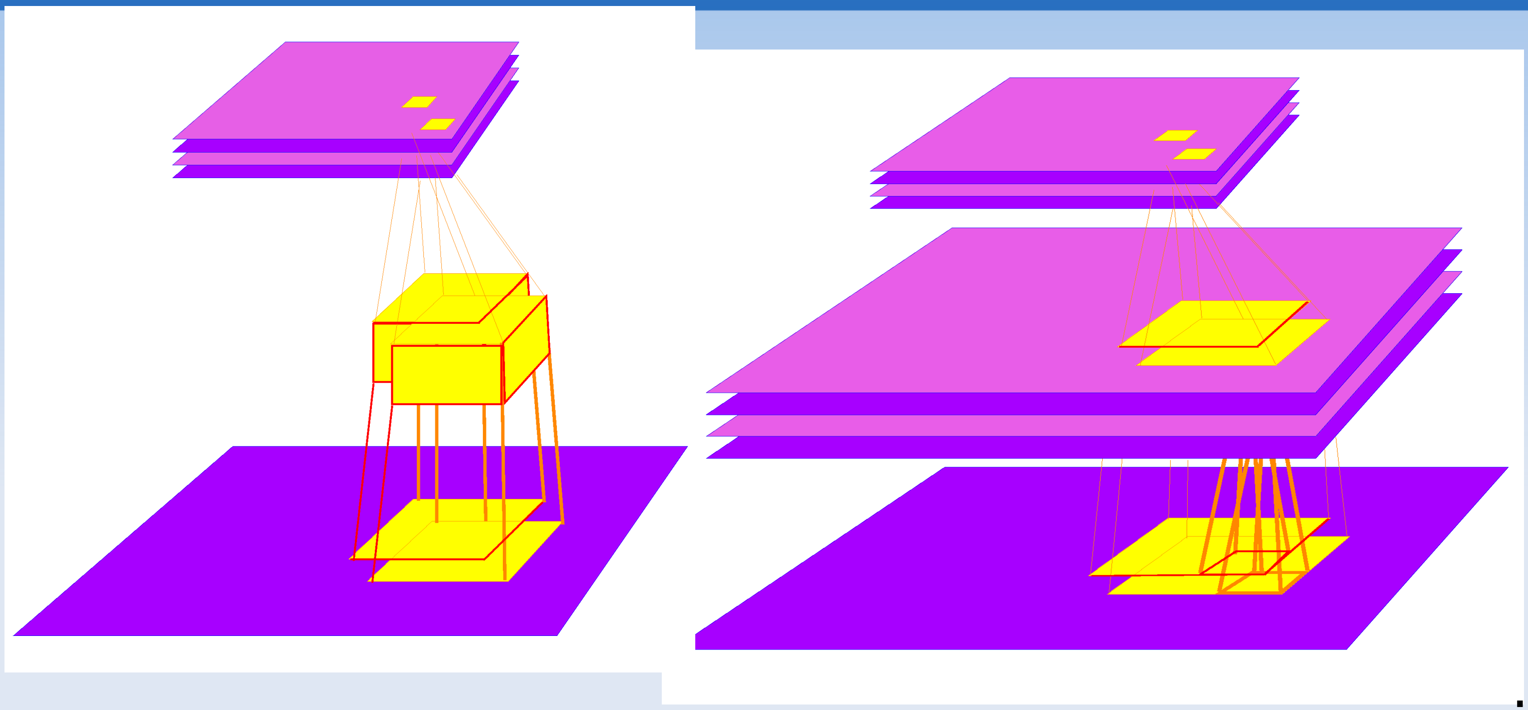
Face detection - pose estimation

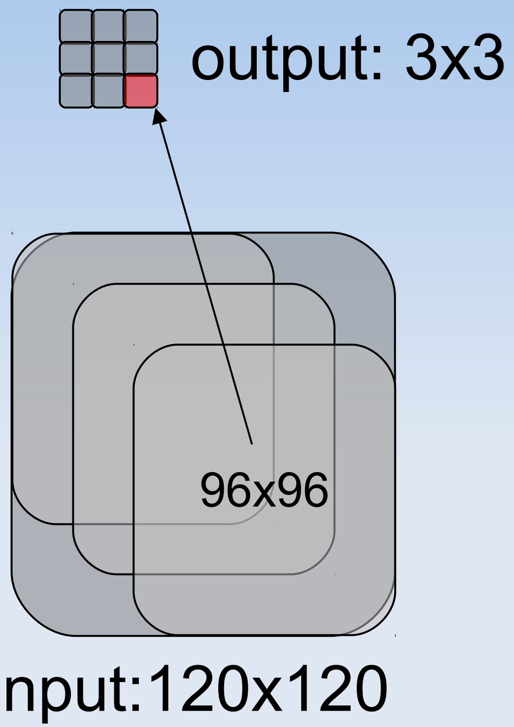


Face detection



Applying a ConvNet on Sliding Windows is Very Cheap!

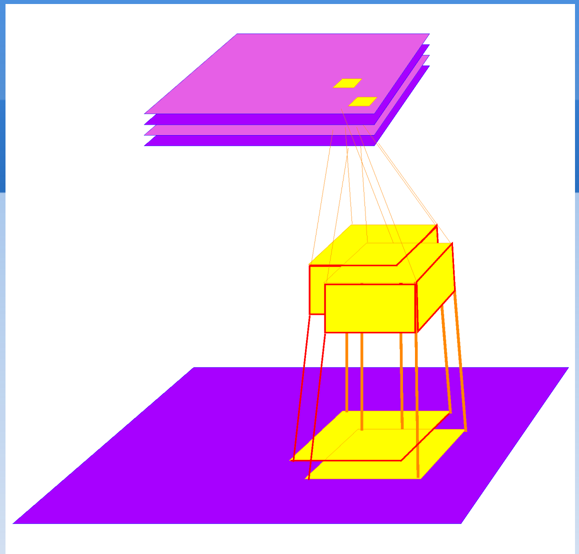


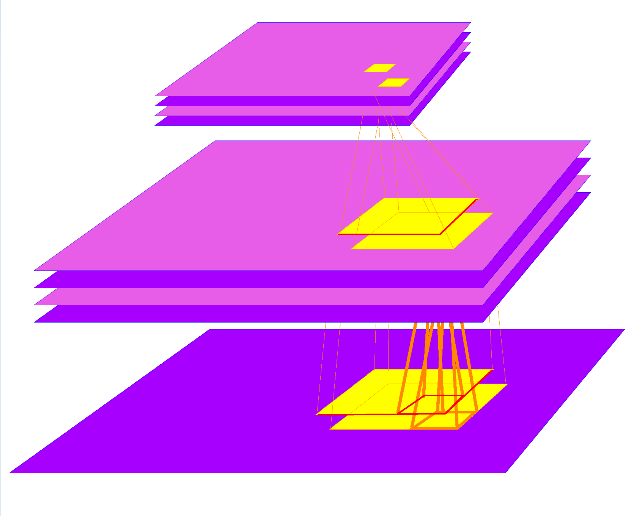


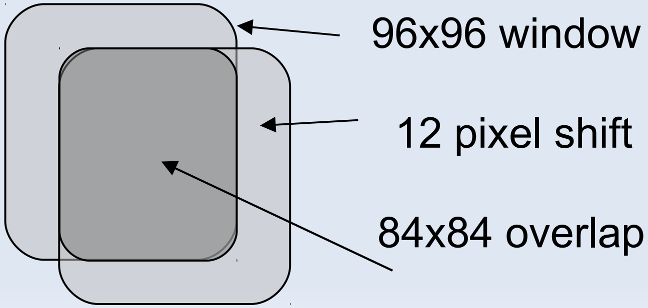
- Traditional Detectors/Classifiers must be applied to every location on a large input image, at multiple scales.
- Convolutional nets can replicated over large images very cheaply.
- The network is applied to multiple scales spaced by 1.5.

Building a Detector/Recognizer: Replicated Convolutional Nets

- Computational cost for replicated convolutional net:
  - 96x96 -> 4.6 million multiply-accumulate operations
  - 120x120 -> 8.3 million multiply-accumulate operations
  - 240x240 -> 47.5 million multiply-accumulate operations
  - 480x480 -> 232 million multiply-accumulate operations
- Computational cost for a non-convolutional detector of the same size, applied every 12 pixels:
  - 96x96 -> 4.6 million multiply-accumulate operations
  - 120x120 -> 42.0 million multiply-accumulate operations
  - 240x240 -> 788.0 million multiply-accumulate operations
  - 480x480 -> 5,083 million multiply-accumulate operations



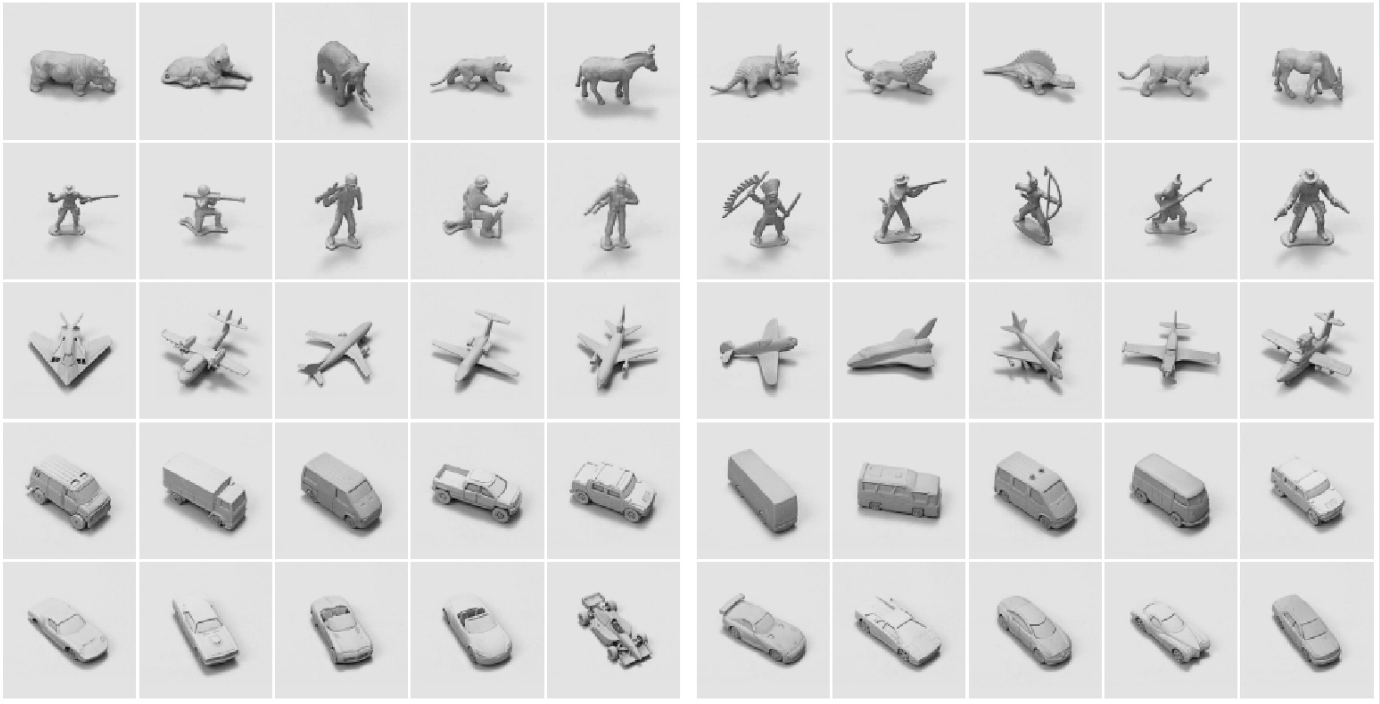




Generic Object Detection and Recognition with Invariance to Pose and Illumination

- **50** toys belonging to 5 categories: **animal, human figure, airplane, truck, car**
- **10** instance per category: 5 instances used for training, 5 instances for testing
- **Raw dataset: 972** stereo pair of each object instance. **48,600** image pairs total.

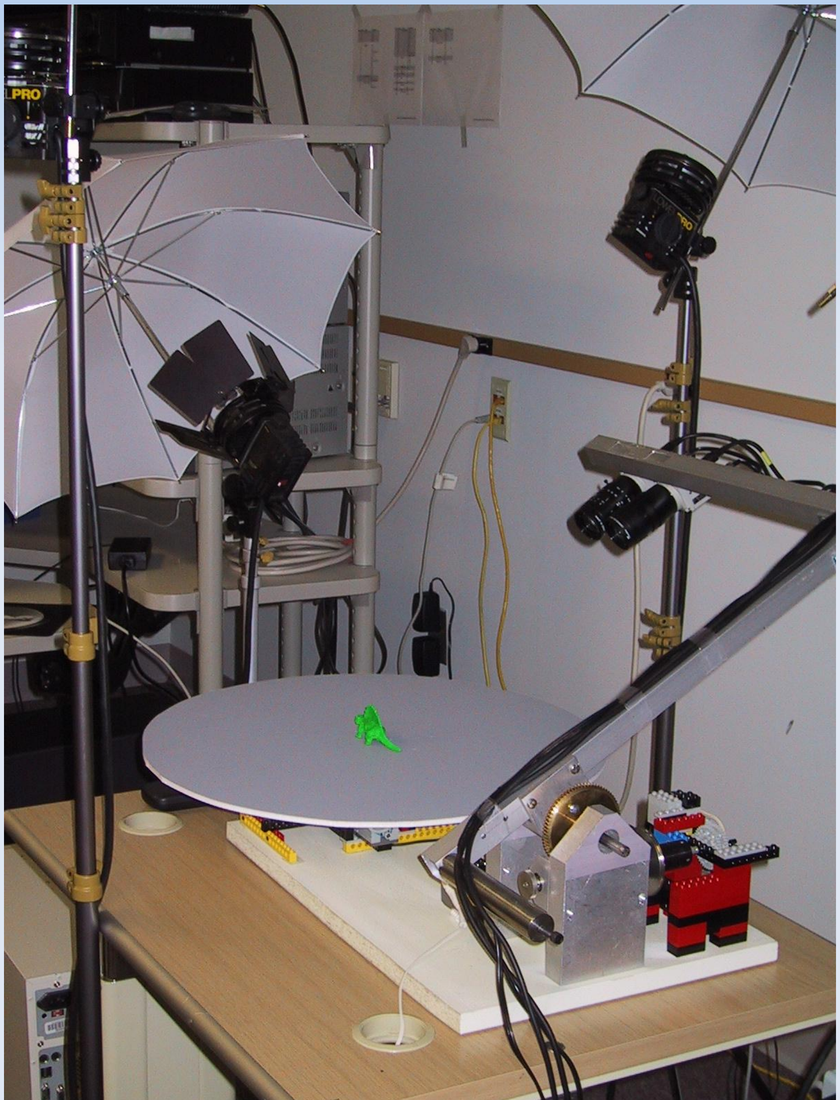
- **For each instance:**
- **18 azimuths**
  - 0 to 350 degrees every 20 degrees
- **9 elevations**
  - 30 to 70 degrees from horizontal every 5 degrees
- **6 illuminations**
  - on/off combinations of 4 lights
  - **2 cameras (stereo)**
- 7.5 cm apart
  - 40 cm from the object



Training instances **Test instances**

Data Collection, Sample Generation

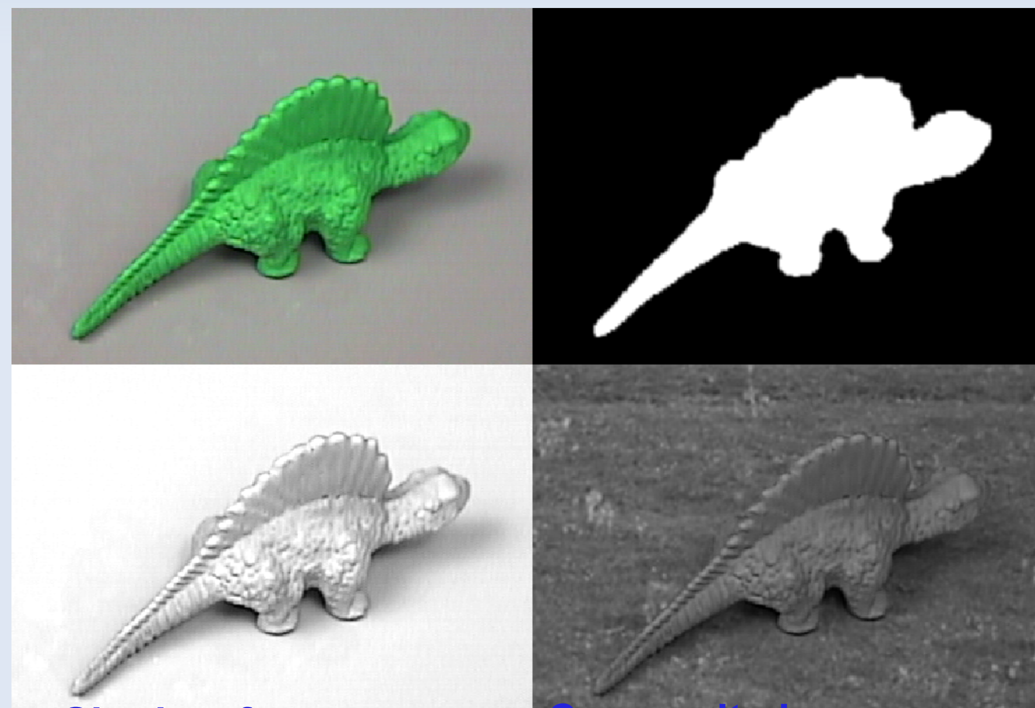
Image capture setup



Objects are painted green so that:
- all features other than shape are removed
- objects can be segmented, transformed,
  and composited onto various backgrounds

Original image

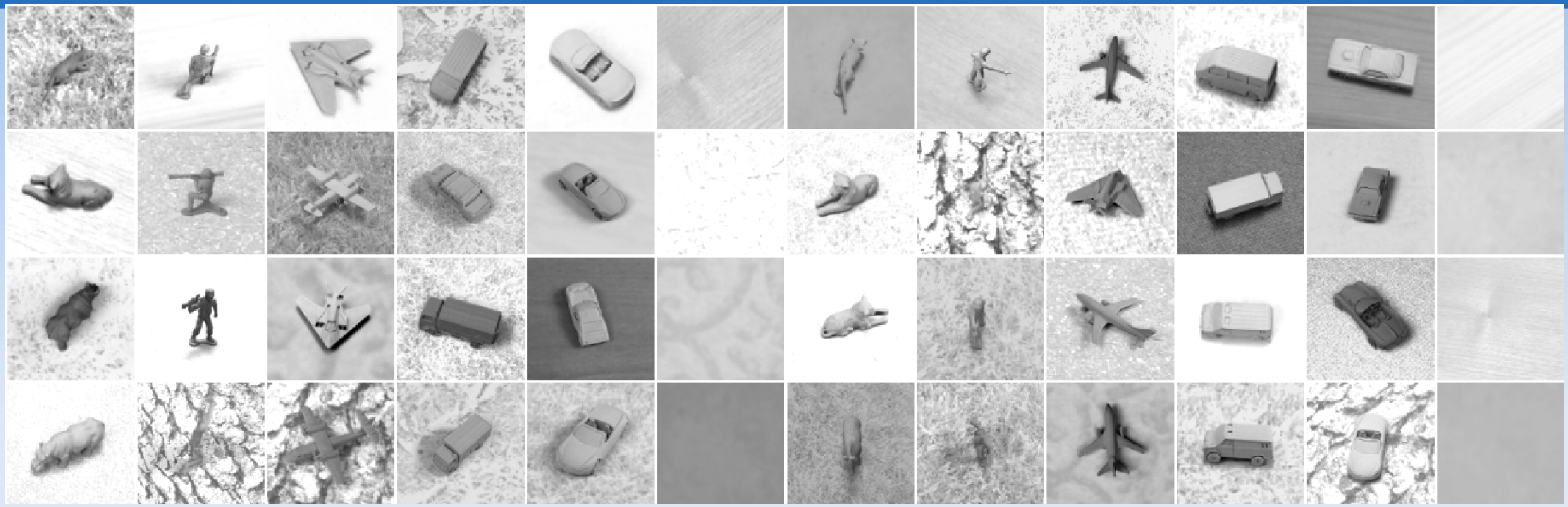
Object mask



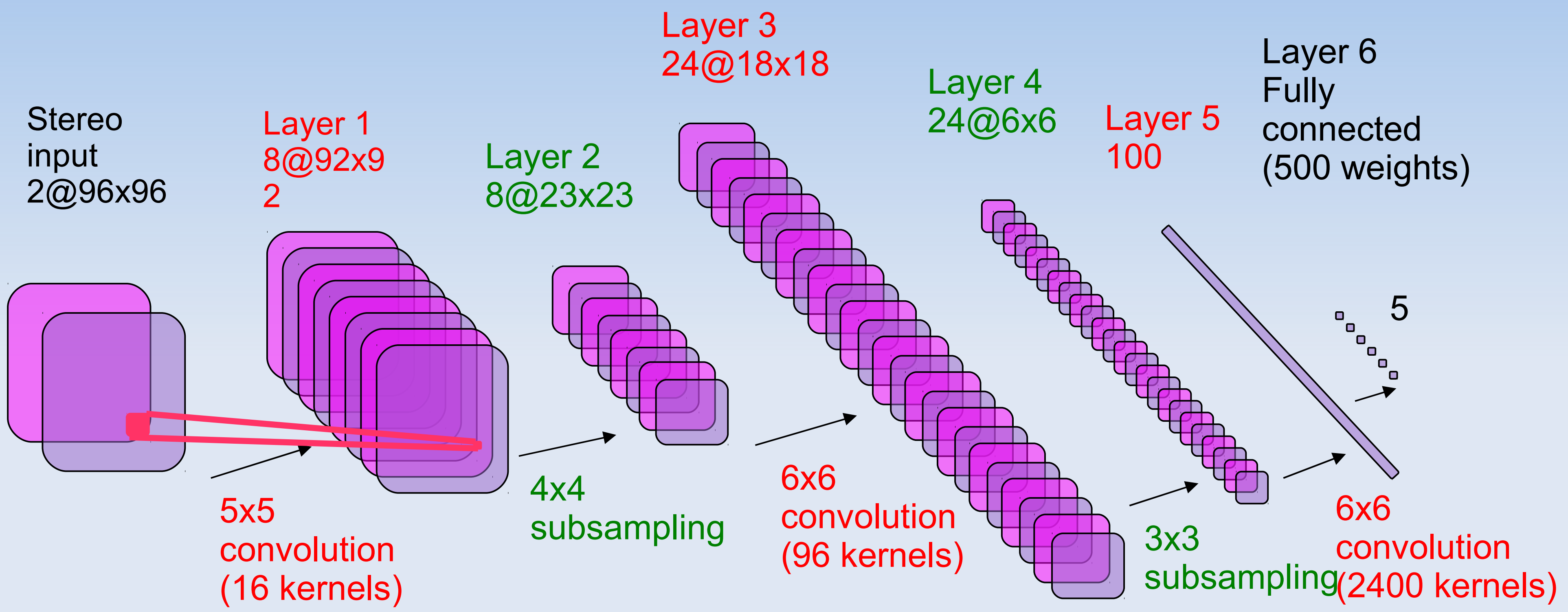
Shadow factor

Composite image

Textured and Cluttered Datasets

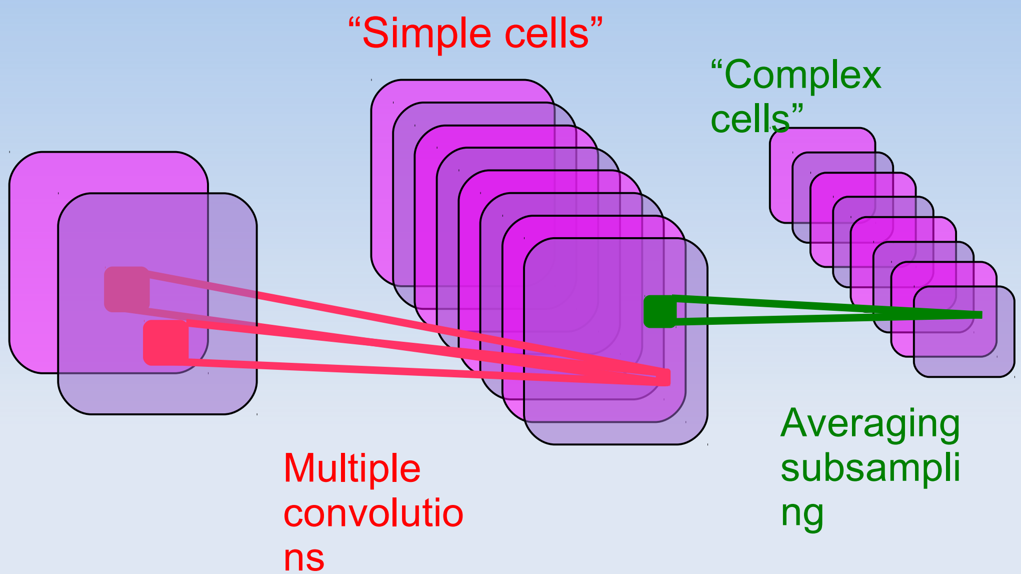


Convolutional Network

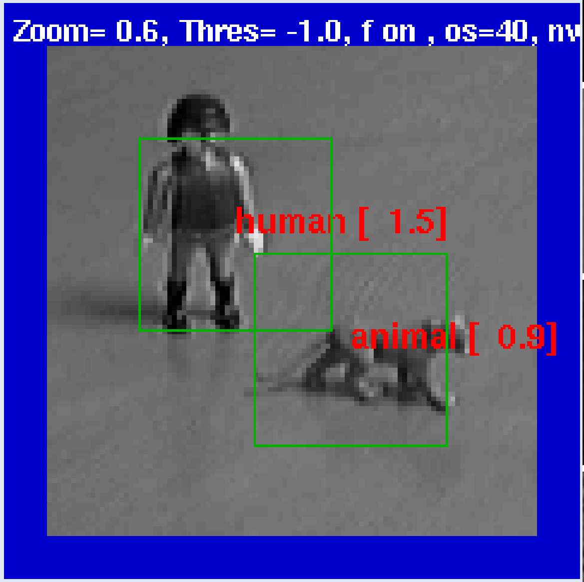


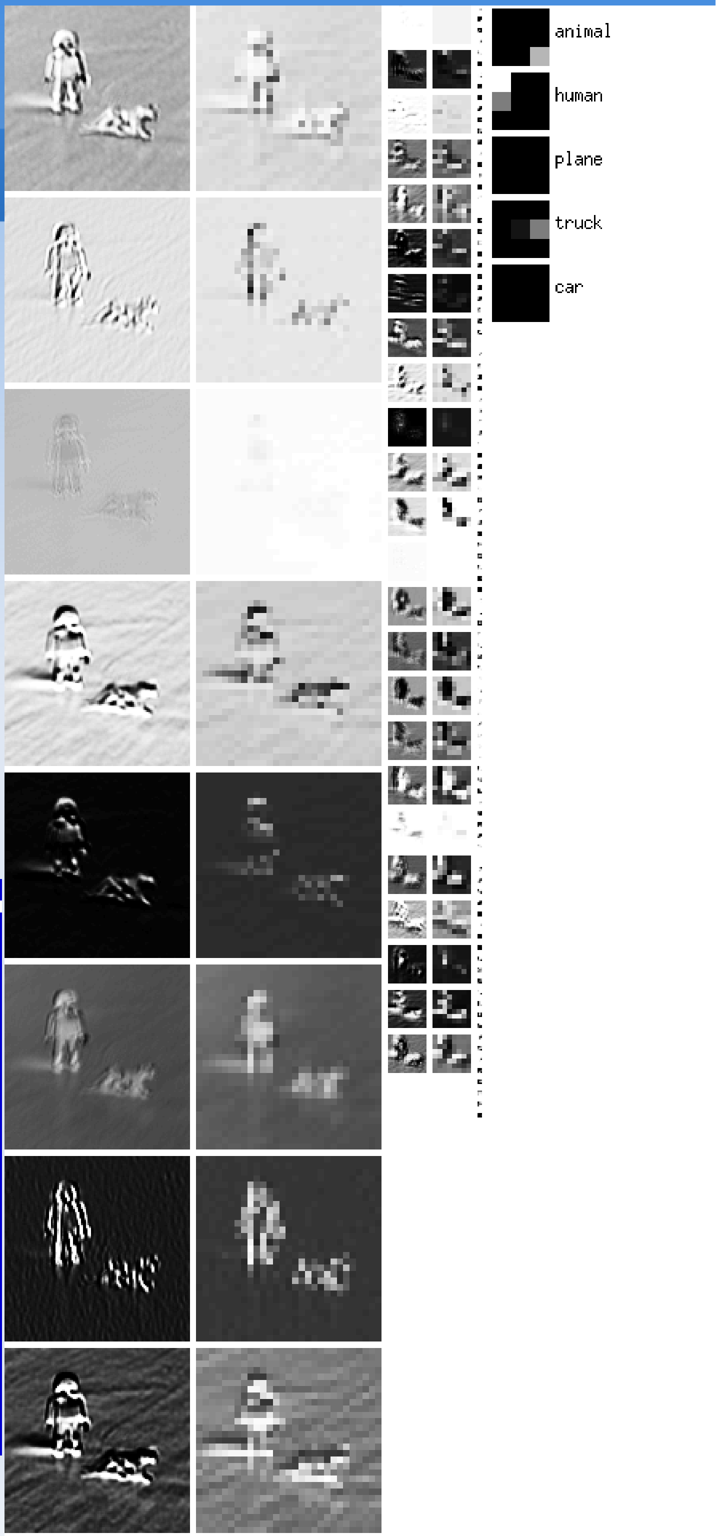
- **90,857 free parameters, 3,901,162 connections.**
- The architecture alternates convolutional layers (feature detectors) and subsampling layers (local feature pooling for invariance to small distortions).
- **The entire network is trained end-to-end** (all the layers are trained simultaneously).
- A gradient-based algorithm is used to minimize a supervised loss function.

Alternated Convolutions and Subsampling



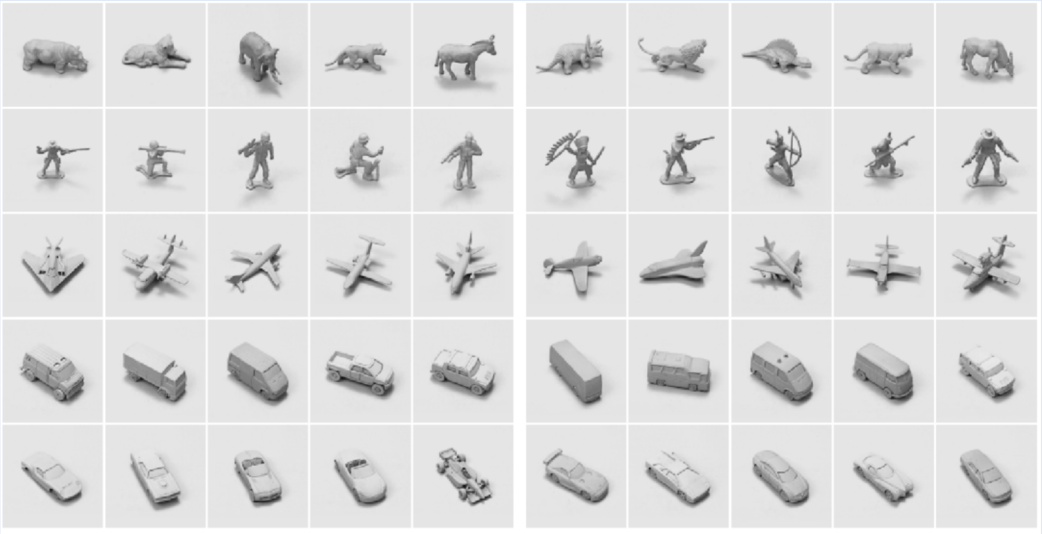
- Local features are extracted everywhere.
- averaging/subsampling layer builds robustness to variations in feature locations.
- Hubel/Wiesel'62, Fukushima'71, LeCun'89, Riesenhuber & Poggio'02, Ullman'02,....





Normalized-Uniform Set: Error Rates

- Linear Classifier on raw stereo images: 30.2% error.
- K-Nearest-Neighbors on raw stereo images: 18.4% error.
- K-Nearest-Neighbors on PCA-95: 16.6% error.
- Pairwise SVM on 96x96 stereo images: 11.6% error
- Pairwise SVM on 95 Principal Components: 13.3% error.
- Convolutional Net on 96x96 stereo images: 5.8% error.



Training instances Test instances

Jittered-Cluttered Dataset

- **Jittered-Cluttered Dataset:**
- **291,600** stereo pairs for training, **58,320** for testing
- Objects are jittered: position, scale, in-plane rotation, contrast, brightness, backgrounds, distractor objects,...
- Input dimension: 98x98x2 (approx 18,000)

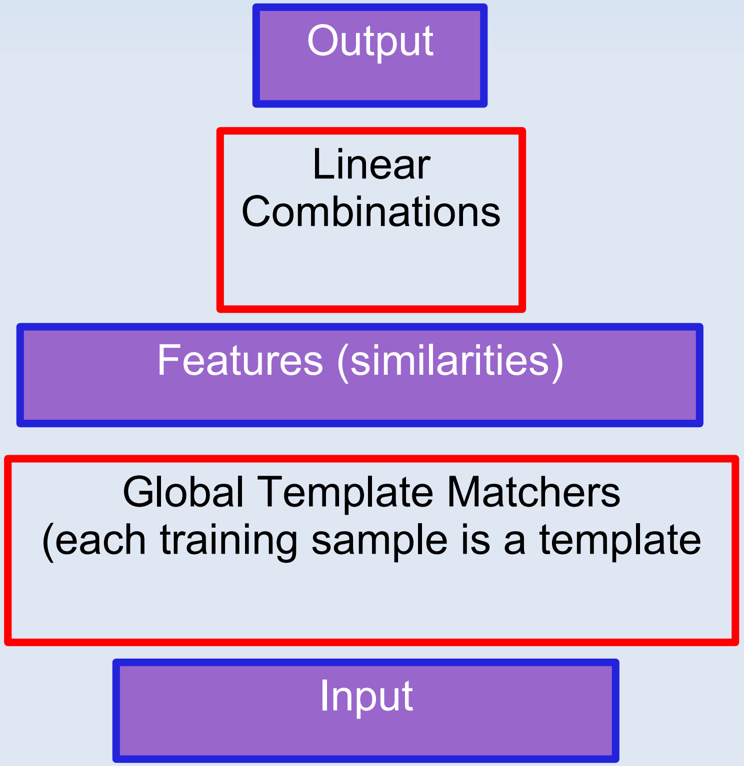
Experiment 2: Jittered-Cluttered Dataset

- **291,600** training samples, **58,320** test samples
- SVM with Gaussian kernel — 43.3% error
- Convolutional Net with binocular input: — 7.8% error
- Convolutional Net + SVM on top: — 5.9% error
- Convolutional Net with monocular input: — 20.8% error
- Smaller mono net (DEMO): — 26.0% error
- Dataset available from http://www.cs.nyu.edu/~yann

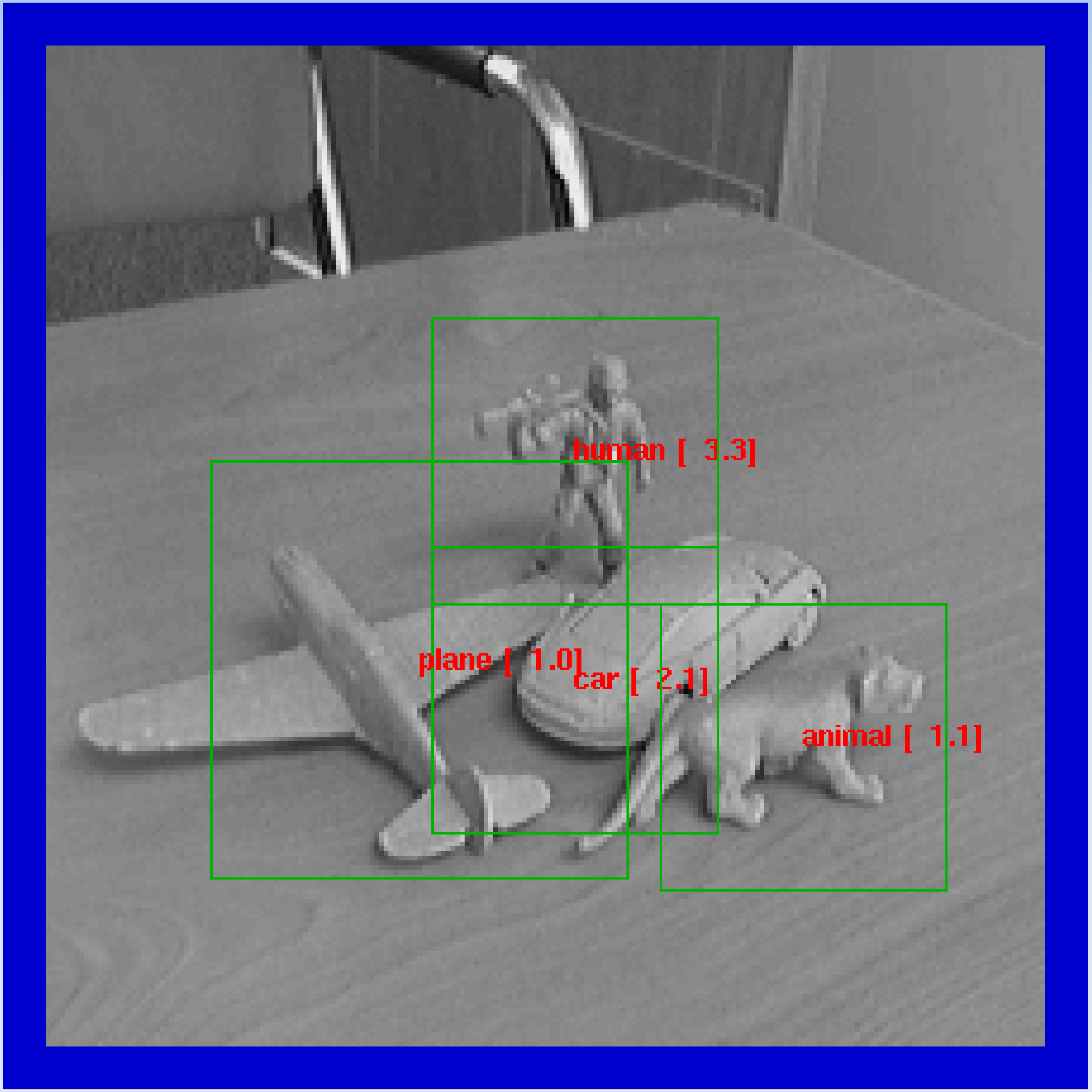
What's wrong with K-NN and SVMs?

- K-NN and SVM with Gaussian kernels are based on matching global templates
- Both are "shallow" architectures
- There is now way to learn invariant recognition tasks with such naïve architectures (unless we use an impractically large number of templates).

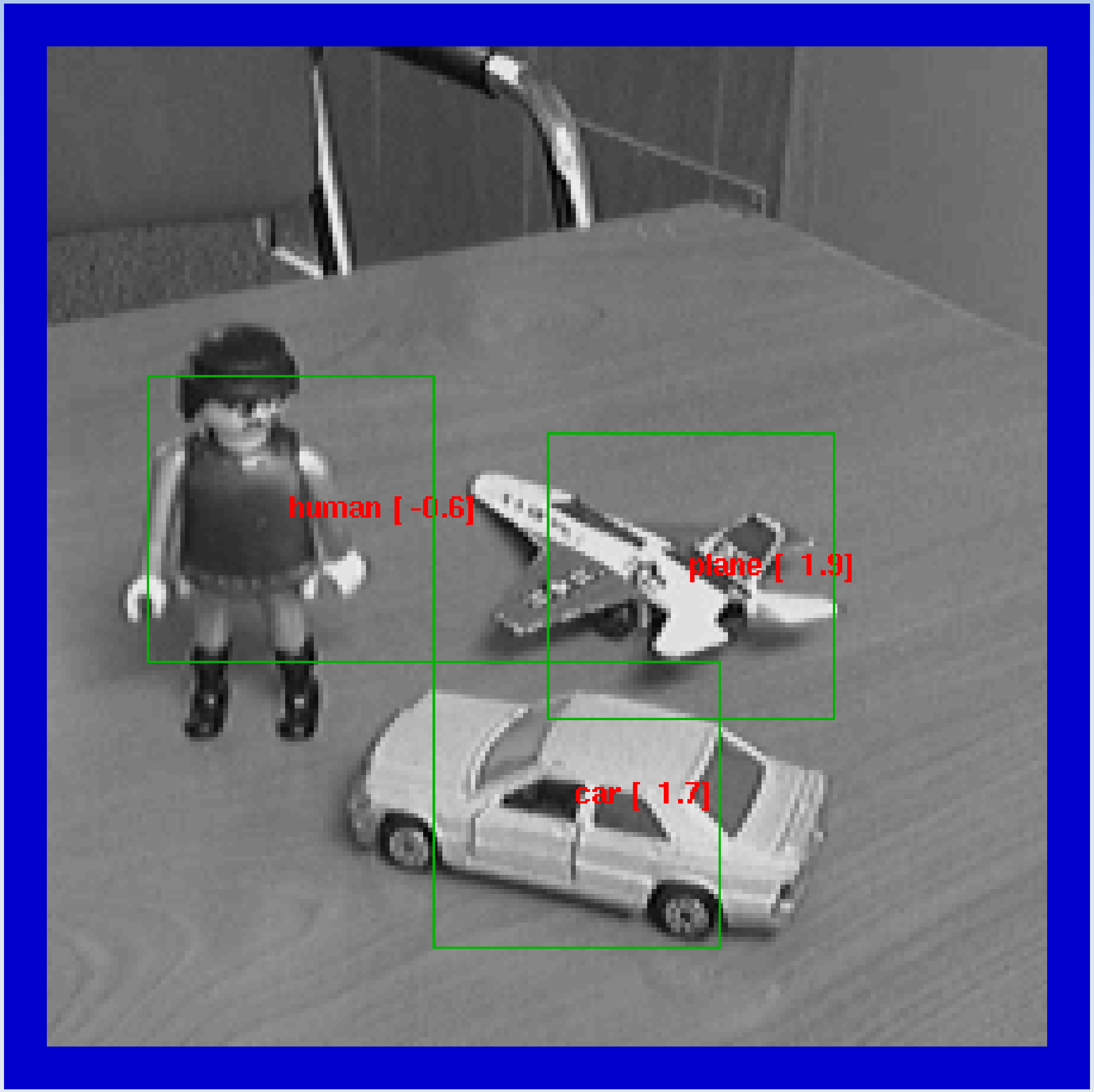
- The number of necessary templates grows exponentially with the number of dimensions of variations.
- Global templates are in trouble when the variations include: category, instance shape, configuration (for articulated object), position, azimuth, elevation, scale, illumination, texture, albedo, in-plane rotation, background luminance, background texture, background clutter, ...



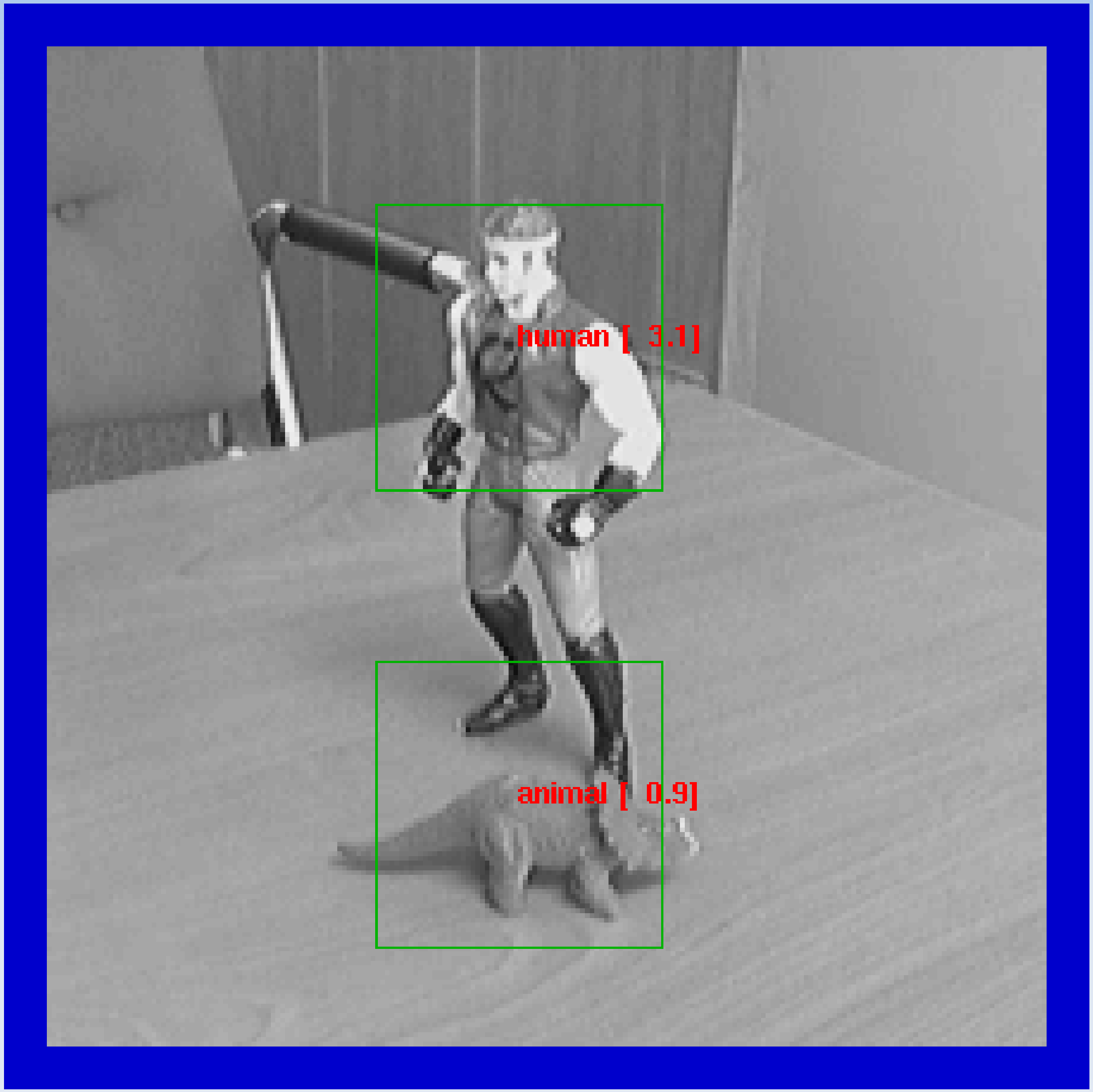
Examples (Monocular Mode)



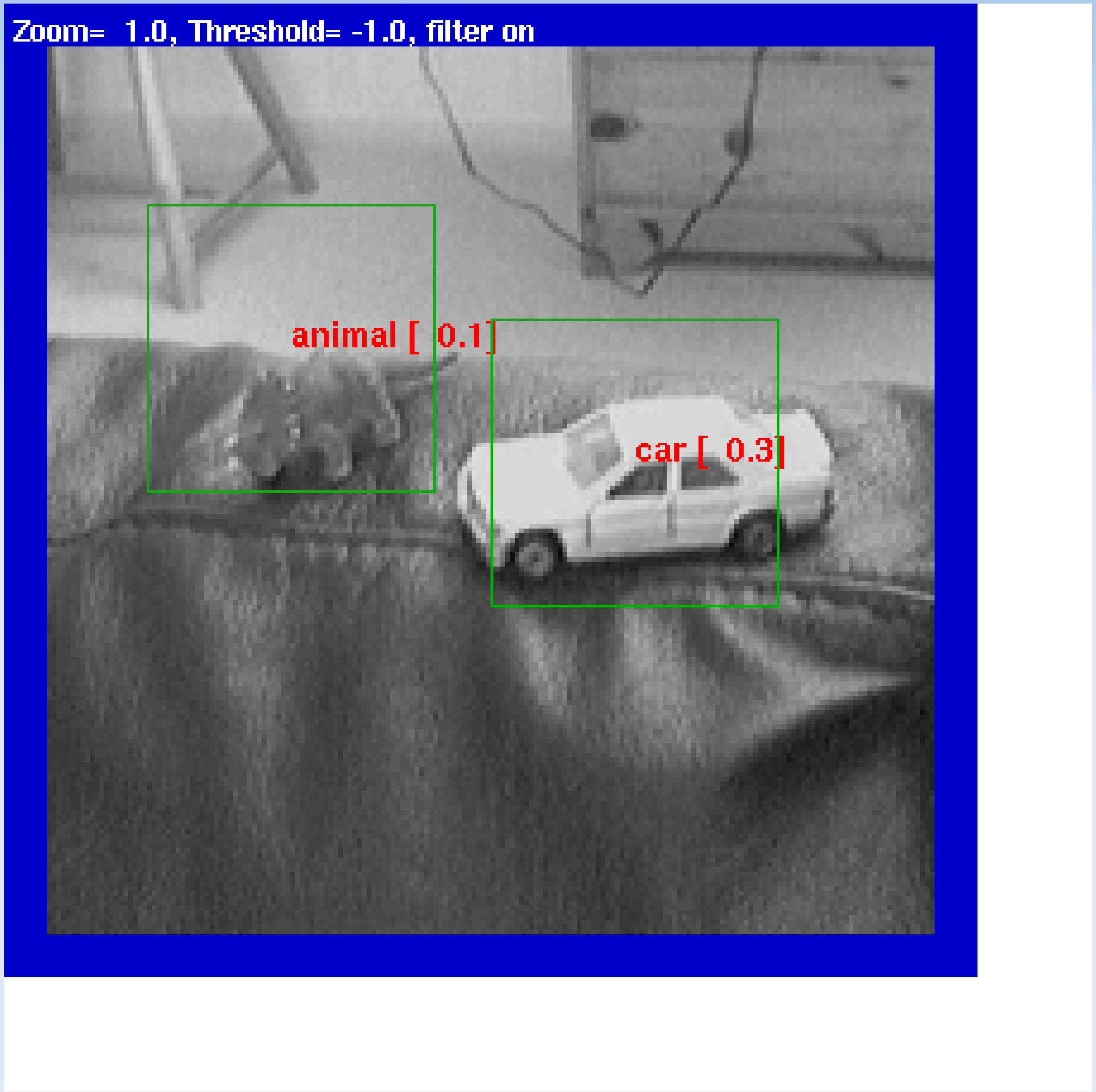
Examples (Monocular Mode)



Examples (Monocular Mode)



Examples (Monocular Mode)



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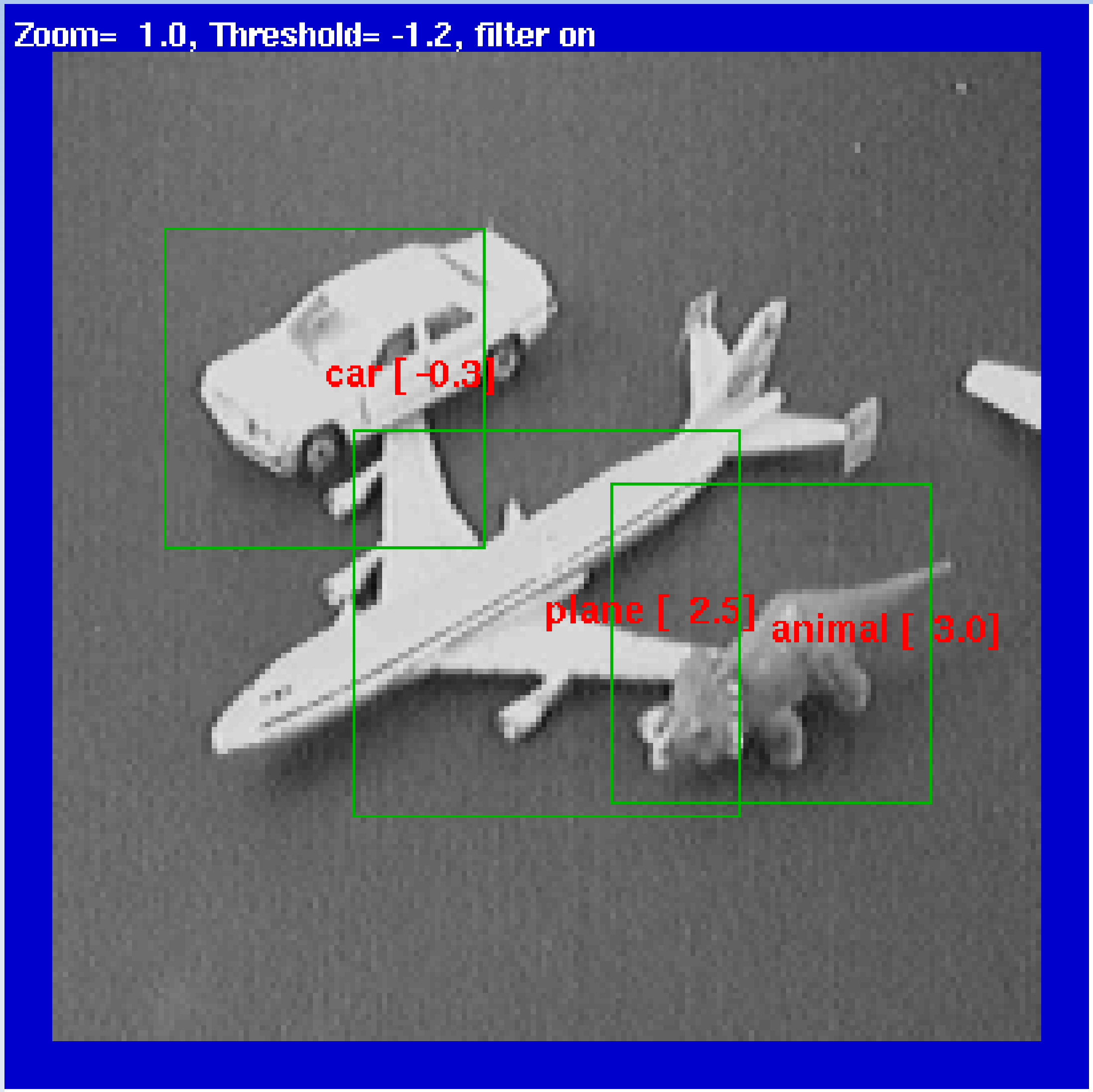
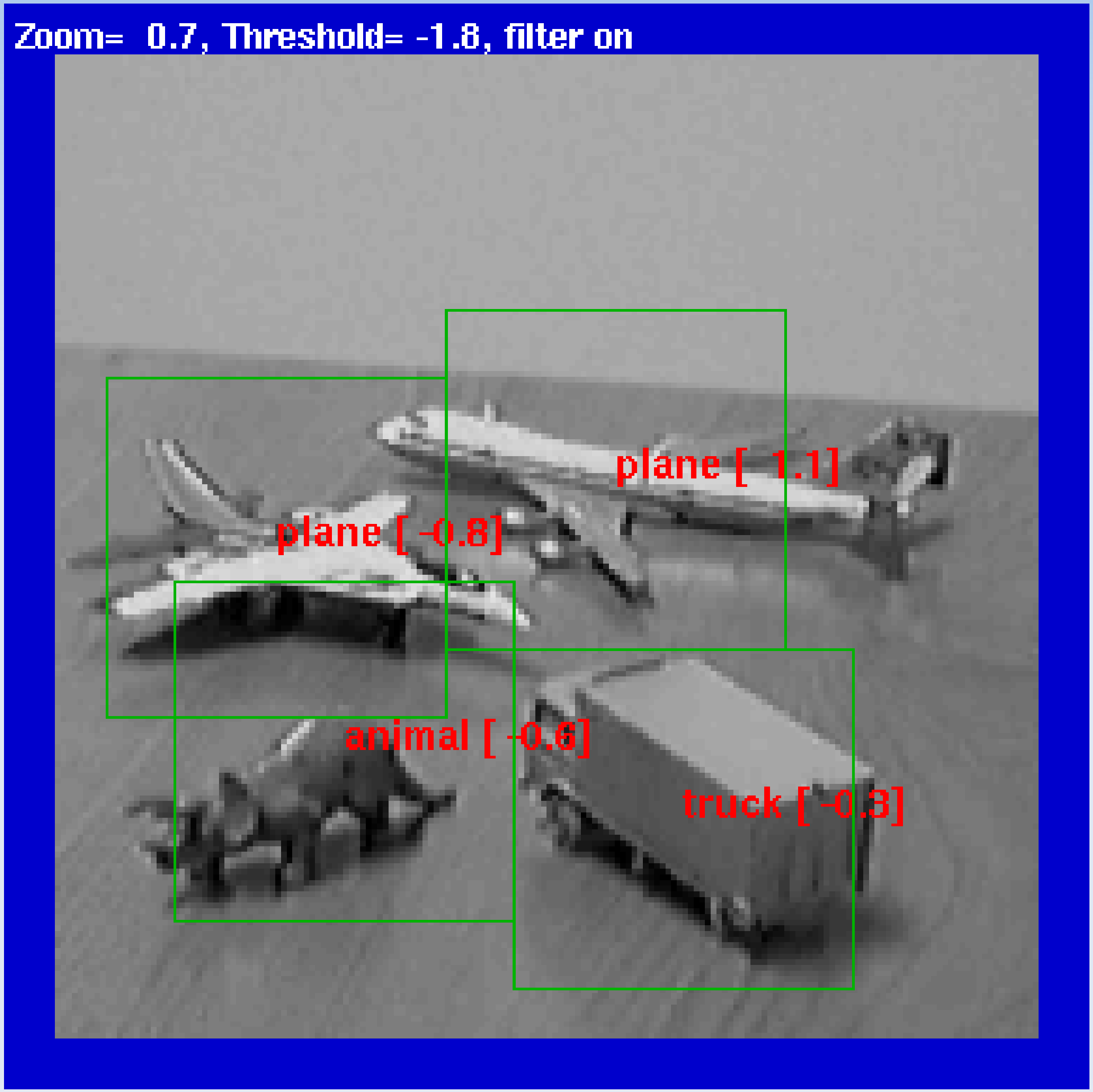


Examples (Monocular Mode)



Supervised Convolutional Nets: Pros and Cons

- **Convolutional nets can be trained to perform a wide variety of visual tasks.**
  - Global supervised gradient descent can produce parsimonious architectures
- **BUT: they require lots of labeled training samples**
  - 60,000 samples for handwriting
  - 120,000 samples for face detection
  - 25,000 to 350,000 for object recognition
- **Since low-level features tend to be non task specific, we should be able to learn them unsupervised.**
- **Hinton has shown that layer-by-layer unsupervised "pre-training" can be used to initialize "deep" architectures**
  - [Hinton & Shalakhutdinov, Science 2006]
- **Can we use this idea to reduce the number of necessary labeled examples.**

Learning fast

- Common point of neural networks: need many examples
- → we need to be able to use these examples fast

Objectives and Essential Remarks

- Baseline large-scale learning algorithm

Randomly discarding data is the simplest way to handle large datasets.

  - What are the statistical benefits of processing more data?
  - What is the computational cost of processing more data?

- We need a theory that joins Statistics and Computation!

– 1967: Vapnik's theory does not discuss computation.

– 1981: Valiant's learnability excludes exponential time algorithms, but (i) polynomial time can be too slow, (ii) few actual results.

– We propose a simple analysis of approximate optimization. . .

Learning Algorithms: Standard Framework

- Assumption: examples are drawn independently from an unknown probability distribution $P(x, y)$ that represents the rules of Nature.
- Expected Risk: $E(f) = \int \ell(f(x), y)\, dP(x, y)$.
- Empirical Risk: $E_n(f) = \frac{1}{n} \sum \ell(f(x_i), y_i)$.
- We would like f^* that minimizes $E(f)$ among all functions.
- In general $f^* \notin \mathcal{F}$.
- The best we can have is $f^*_{\mathcal{F}} \in \mathcal{F}$ that minimizes $E(f)$ inside $\mathcal{F}$.
- But $P(x, y)$ is unknown by definition.
- Instead we compute $f_n \in \mathcal{F}$ that minimizes $E_n(f)$.
  Vapnik-Chervonenkis theory tells us when this can work.

Learning with Approximate Optimization

Computing $f_n = \arg\min_{f \in \mathcal{F}} E_n(f)$ is often costly.

Since we already make lots of approximations,

why should we compute f_n exactly?

Let's assume our optimizer returns $\tilde{f}_n$

such that $E_n(\tilde{f}_n) < E_n(f_n) + \rho$.

For instance, one could stop an iterative
optimization algorithm long before its convergence.

Decomposition of the Error (i)

$$
\begin{aligned}
E(\tilde{f}_n) - E(f^*) &= E(f^*_{\mathcal{F}}) - E(f^*) && \text{Approximation error} \\
&+ E(f_n) - E(f^*_{\mathcal{F}}) && \text{Estimation error} \\
&+ E(\tilde{f}_n) - E(f_n) && \text{Optimization error}
\end{aligned}
$$

Problem:
Choose $\mathcal{F}$, n, and ρ to make this as small as possible,

$$
\text{subject to budget constraints} \begin{cases} \text{maximal number of examples } n \\ \text{maximal computing time } T \end{cases}
$$

Decomposition of the Error (ii)

Approximation error bound: (Approximation theory)
– decreases when $\mathcal{F}$ gets larger.

Estimation error bound: (Vapnik-Chervonenkis theory)
– decreases when n gets larger.
– increases when $\mathcal{F}$ gets larger.

Optimization error bound: (Vapnik-Chervonenkis theory plus tricks)
– increases with ρ.

Computing time T: (Algorithm dependent)
– decreases with ρ
– increases with n
– increases with $\mathcal{F}$

Small-scale vs. Large-scale Learning

We can give *rigorous definitions.*

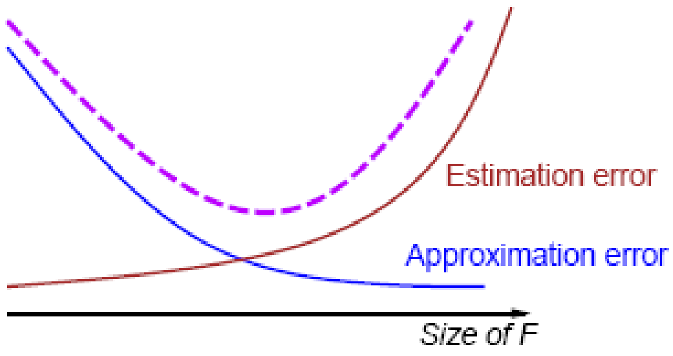
- **Definition 1:**
  We have a **small-scale learning** problem when the **active budget constraint is the number of examples n.**

- **Definition 2:**
  We have a **large-scale learning** problem when the **active budget constraint is the computing time T.**

Small-scale Learning

The active budget constraint is the number of examples.

- To reduce the estimation error, take n as large as the budget allows.
- To reduce the optimization error to zero, take $\rho = 0$.
- We need to adjust the size of $\mathcal{F}$.



See Structural Risk Minimization (Vapnik 74) and later works.

Large-scale Learning

The active budget constraint is the computing time.

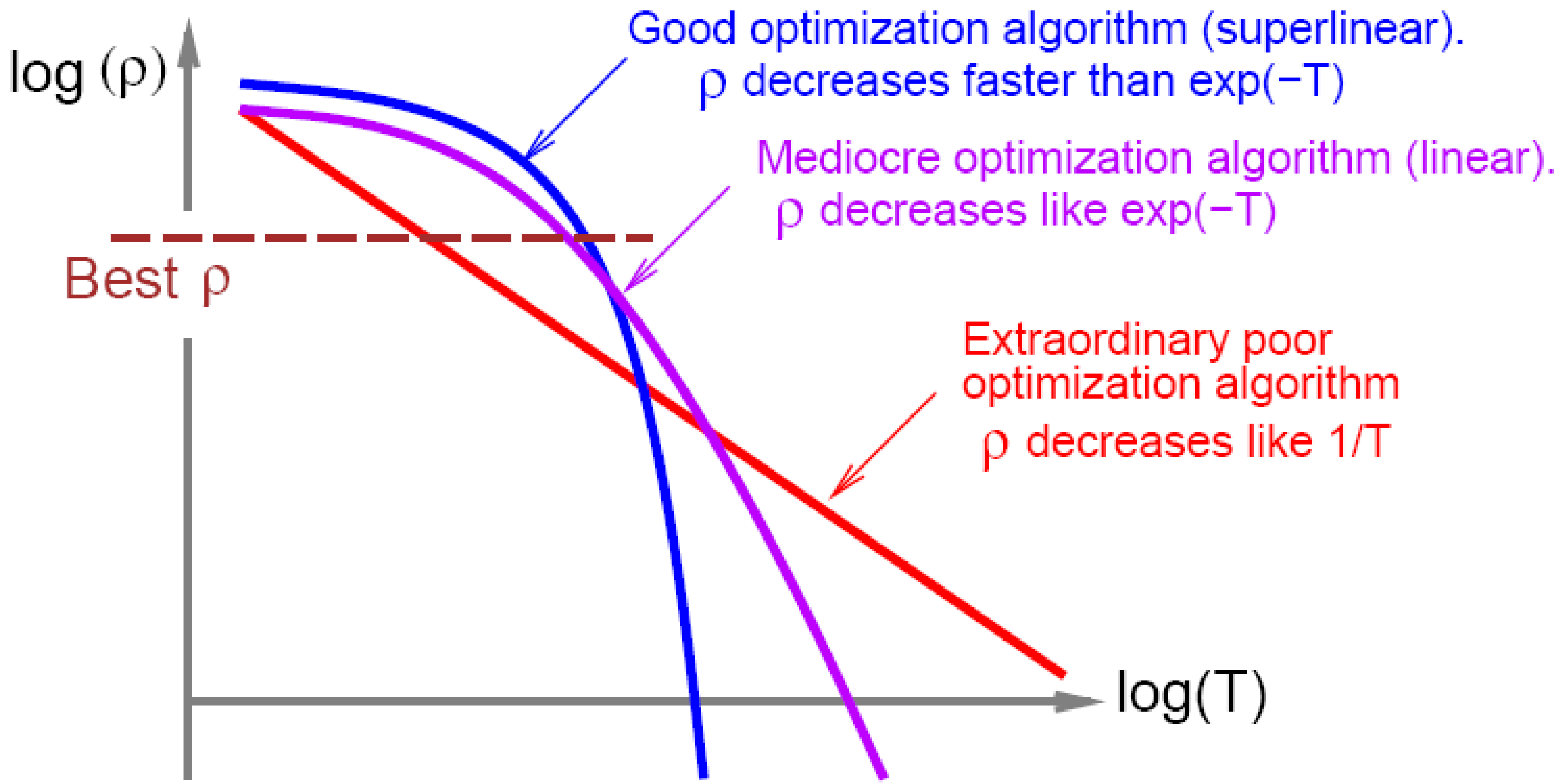
- More complicated tradeoffs.
  The computing time depends on the three variables: $\mathcal{F}$, n, and ρ.

- Example.
  If we choose ρ small, we decrease the optimization error. But we must also decrease $\mathcal{F}$ and/or n with adverse effects on the estimation and approximation errors.

- The exact tradeoff depends on the optimization algorithm.

- We can compare optimization algorithms rigorously.

Executive Summary



Case Study

Simple parametric setup

- $\mathcal{F}$ is fixed.
- Functions $f_w(x)$ linearly parametrized by $w \in \mathbb{R}^d$.

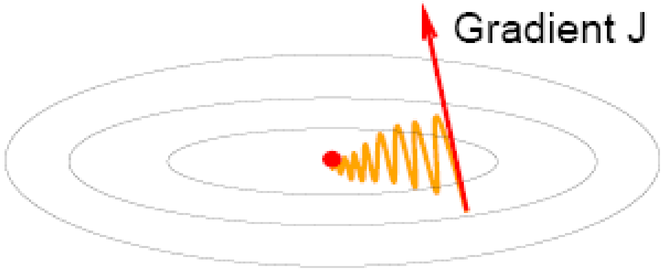
Comparing four iterative optimization algorithms for $E_n(f)$

1. Gradient descent.
2. Second order gradient descent (Newton).
3. Stochastic gradient descent.
4. Stochastic second order gradient descent.

Gradient Descent (GD)

Iterate

- $w_{t+1} \leftarrow w_t - \eta \dfrac{\partial E_n(f_{w_t})}{\partial w}$



Best speed achieved with fixed learning rate $\eta = \frac{1}{\lambda_{\max}}$.
(e.g., Dennis & Schnabel, 1983)

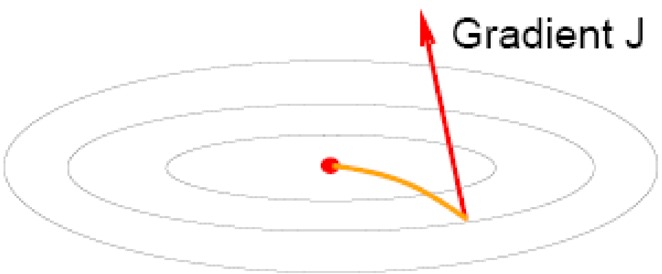
	Cost per iteration	Iterations to reach ρ	Time to reach accuracy ρ	Time to reach $E(\tilde{f}_n) - E(f^*_{\mathcal{F}}) < \varepsilon$
GD	$\mathcal{O}(nd)$	$\mathcal{O}\left(\kappa \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(nd\kappa \log \frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2\kappa}{\varepsilon^{1/\alpha}} \log^2 \frac{1}{\varepsilon}\right)$

– In the last column, n and ρ are chosen to reach ε as fast as possible.
– Solve for ε to find the best error rate achievable in a given time.
– Remark: abuses of the $\mathcal{O}()$ notation

Second Order Gradient Descent (2GD)

Iterate

- $w_{t+1} \leftarrow w_t - H^{-1} \dfrac{\partial E_n(f_{w_t})}{\partial w}$



We assume H^{-1} is known in advance.
Superlinear optimization speed (e.g., Dennis & Schnabel, 1983)

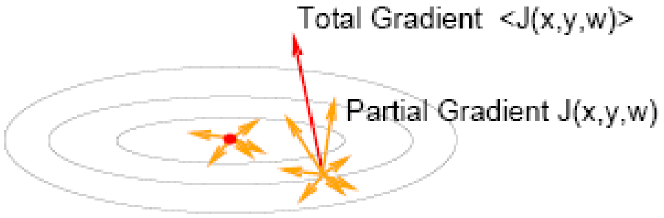
	Cost per iteration	Iterations to reach ρ	Time to reach accuracy ρ	Time to reach $E(\tilde{f}_n) - E(f^*_{\mathcal{F}}) < \varepsilon$
2GD	$\mathcal{O}\big(d(d+n)\big)$	$\mathcal{O}\left(\log\log\frac{1}{\rho}\right)$	$\mathcal{O}\left(d(d+n)\log\log\frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2}{\varepsilon^{1/\alpha}}\log\frac{1}{\varepsilon}\log\log\frac{1}{\varepsilon}\right)$

– Optimization speed is much faster.
– Learning speed only saves the condition number κ.

Stochastic Gradient Descent (SGD)

Iterate

- Draw random example (x_t, y_t).
- $w_{t+1} \leftarrow w_t - \dfrac{\eta}{t} \dfrac{\partial \ell(f_{w_t}(x_t), y_t)}{\partial w}$



Best decreasing gain schedule with $\eta = \frac{1}{\lambda_{\min}}$.
(see Murata, 1998; Bottou & LeCun, 2004)

	Cost per iteration	Iterations to reach ρ	Time to reach accuracy ρ	Time to reach $E(\tilde{f}_n) - E(f^*_{\mathcal{F}}) < \varepsilon$
SGD	$\mathcal{O}(d)$	$\frac{\nu k}{\rho} + o\left(\frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d\,\nu\,k}{\rho}\right)$	$\mathcal{O}\left(\frac{d\,\nu\,k}{\varepsilon}\right)$

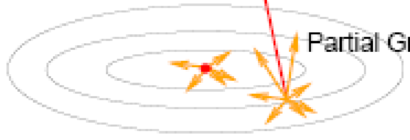
With $1 \leq k \leq \kappa^2$

– Optimization speed is *catastrophic*.
– Learning speed does not depend on the statistical estimation rate α.
– Learning speed depends on condition number κ but *scales very well*.

Second order Stochastic Descent (2SGD)

Iterate

- Draw random example (x_t, y_t).
- $w_{t+1} \leftarrow w_t - \frac{1}{t} H^{-1} \frac{\partial \ell(f_{w_t}(x_t), y_t)}{\partial w}$



Replace scalar gain $\frac{\eta}{t}$ by matrix $\frac{1}{t} H^{-1}$.

	Cost per iteration	Iterations to reach ρ	Time to reach accuracy ρ	Time to reach $E(\tilde{f}_n) - E(f^*_{\mathcal{F}}) < \varepsilon$
2SGD	$\mathcal{O}(d^2)$	$\frac{\nu}{\rho} + o\left(\frac{1}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2 \nu}{\rho}\right)$	$\mathcal{O}\left(\frac{d^2 \nu}{\varepsilon}\right)$

– Each iteration is d times more expensive.
– The number of iterations is reduced by κ^2 (or less.)
– Second order only changes the constant factors.

Benchmarking SGD in Simple Problems

- The theory suggests that SGD is very competitive.
  - Many people associate SGD with trouble.

- SGD historically associated with back-propagation.
  - Multilayer networks are very hard problems (nonlinear, nonconvex)
  - What is difficult, SGD or MLP?

- **Try PLAIN SGD on simple learning problems.**
  - Support Vector Machines
  - Conditional Random Fields

Download from `http://leon.bottou.org/projects/sgd`.
These simple programs are very short.

See also (Shalev-Schwartz et al., 2007; Vishwanathan et al., 2006)

Text Categorization with SVMs

- **Dataset**
  - Reuters RCV1 document corpus.
  - 781,265 training examples, 23,149 testing examples.
  - 47,152 TF-IDF features.
- **Task**
  - Recognizing documents of category `CCAT`.
  - Minimize $E_n = \frac{1}{n} \sum_i \left(\frac{\lambda}{2} w^2 + \ell(w\, x_i + b,\, y_i) \right)$.
  - Update $w \leftarrow w - \eta_t \nabla(w_t, x_t, y_t) \quad = \quad w - \eta_t \left(\lambda w + \frac{\partial \ell(w\, x_t + b,\, y_t)}{\partial w} \right)$

Same setup as (Shalev-Schwartz et al., 2007) but plain SGD.

Text Categorization with SVMs

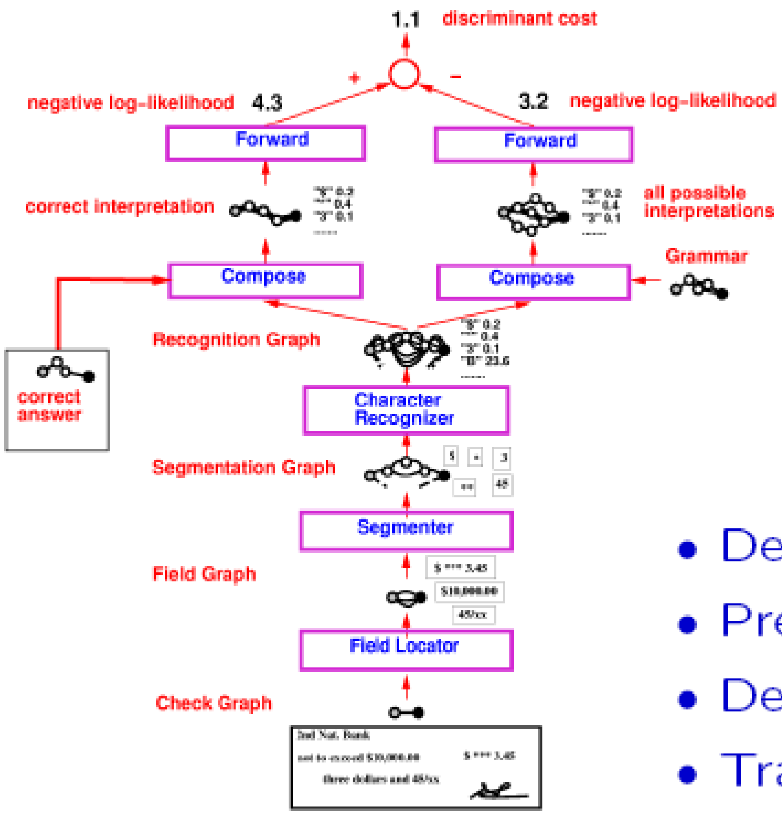
- **Results: Linear SVM**
  $\ell(\hat{y}, y) = \max\{0, 1 - y\hat{y}\} \quad \lambda = 0.0001$

	Training Time	Primal cost	Test Error
SVMLight	23,642 secs	*0.2275*	*6.02%*
SVMPerf	66 secs	0.2278	6.03%
SGD	1.4 secs	*0.2275*	*6.02%*

- **Results: Log-Loss Classifier**
  $\ell(\hat{y}, y) = log(1 + exp(-y\hat{y})) \quad \lambda = 0.00001$

	Training Time	Primal cost	Test Error
LibLinear ($\varepsilon = 0.01$)	30 secs	0.18907	*5.68%*
LibLinear ($\varepsilon = 0.001$)	44 secs	*0.18890*	5.70%
SGD	2.3 secs	0.18893	*5.66%*

SGD for Real Life Applications



A Check Reader

Examples are pairs (image,amount).

Problem with strong structure:

– Field segmentation

– Character segmentation

– Character recognition

– Syntactical interpretation.

- Define differentiable modules.
- Pretrain modules with hand-labelled data.
- Define global cost function (e.g., CRF).
- Train with SGD for a few weeks.

Industrially deployed in 1996. Ran billions of checks over 10 years.

Credits: Bengio, Bottou, Burges, Haffner, LeCun, Nohl, Simard, et al.