

# Reconnaissance d'objets et vision artificielle

<http://www.di.ens.fr/willow/teaching/recvis09>

## Lecture 3 bis

Fitting and the Hough transform

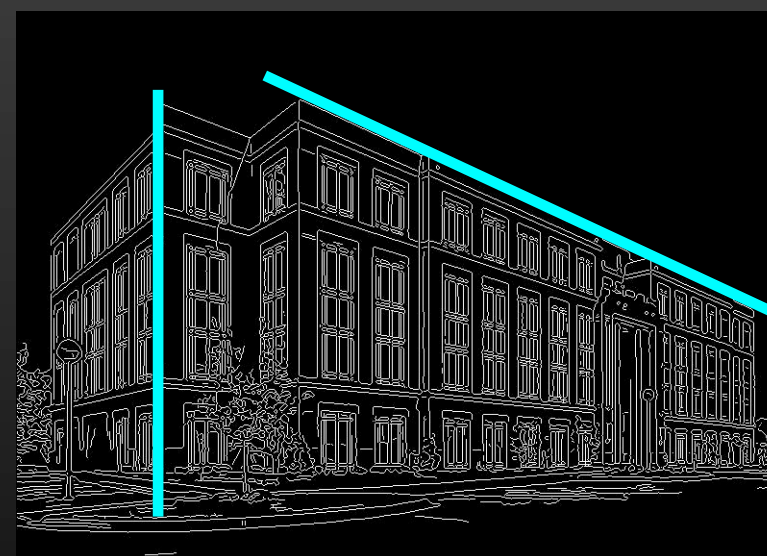
# N'oubliez pas!

Premier exercice de programmation du  
maintenant le 3 novembre!!

<http://www.di.ens.fr/willow/teaching/recvis09/assignment1/>

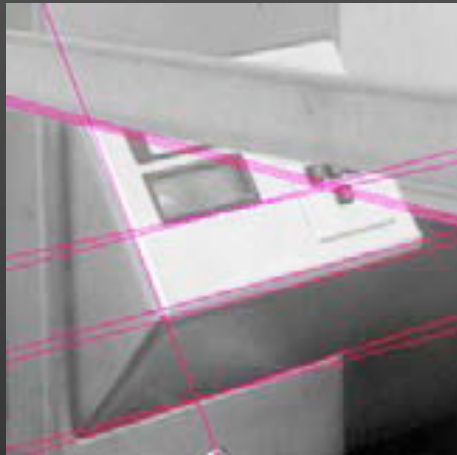
# Fitting: Motivation

- We've learned how to detect edges, corners, blobs. Now what?
- We would like to form a higher-level, more compact representation of the features in the image by grouping multiple features according to a simple model



# Fitting

- Choose a parametric model to represent a set of features



simple model: lines



simple model: circles



complicated model: car

# Fitting

- Choose a parametric model to represent a set of features
- Membership criterion is not local
  - Can't tell whether a point belongs to a given model just by looking at that point
- Three main questions:
  - What model represents this set of features best?
  - Which of several model instances gets which feature?
  - How many model instances are there?
- Computational complexity is important
  - It is infeasible to examine every possible set of parameters and every possible combination of features

# Fitting: Issues

- **Noise** in the measured feature locations
- **Extraneous data:** clutter (outliers), multiple lines
- **Missing data:** occlusions

Case study: Line detection



# Voting schemes

- Let each feature vote for all the models that are compatible with it
- Hopefully the noise features will not vote consistently for any single model
- Missing data doesn't matter as long as there are enough features remaining to agree on a good model

# Hough transform

- An early type of voting scheme
- General outline:
  - Discretize parameter space into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes

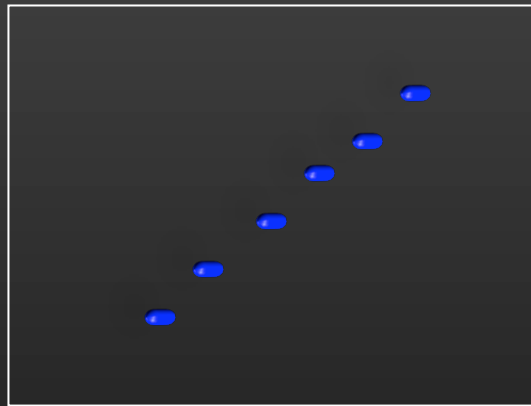
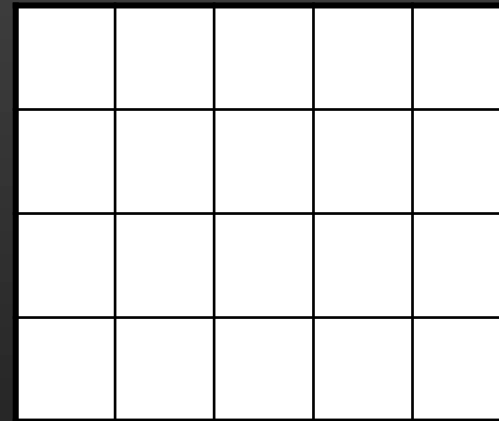


Image space



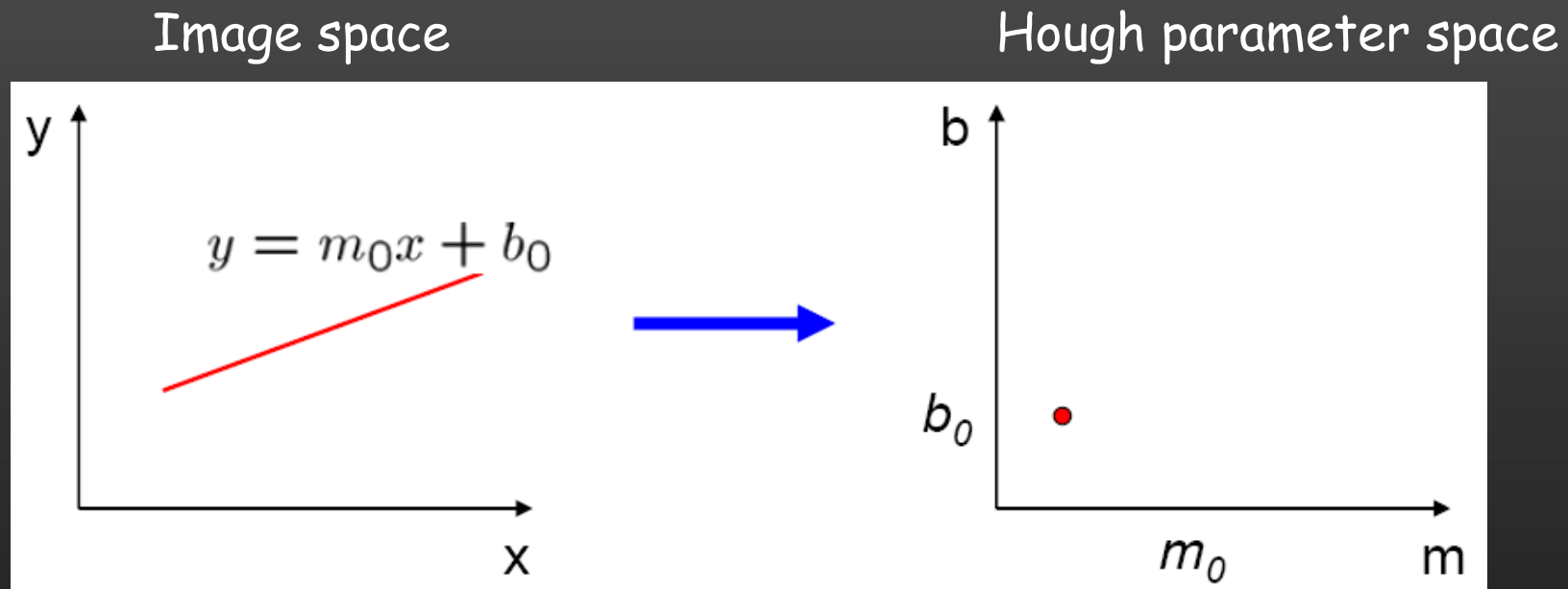
Hough parameter space

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures*, Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959



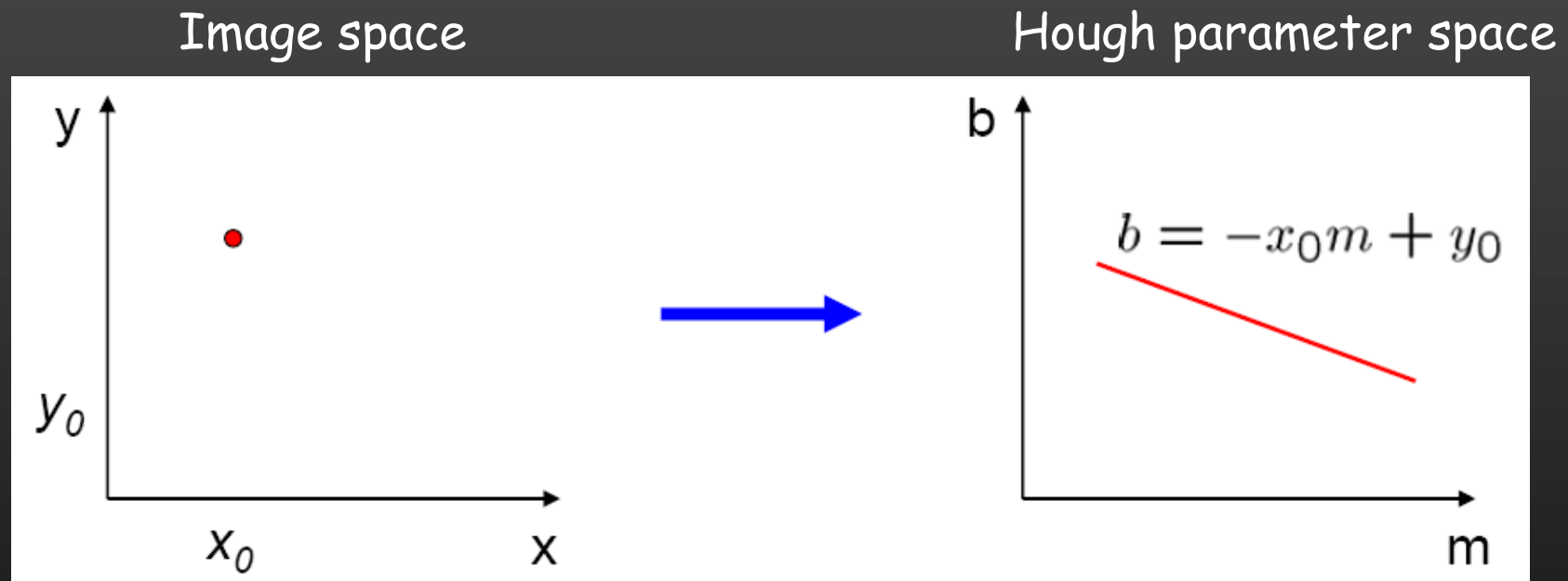
# Parameter space representation

- A line in the image corresponds to a point in Hough space



# Parameter space representation

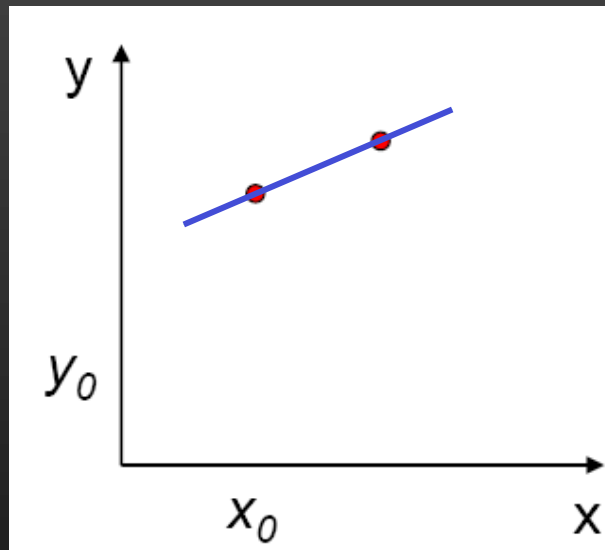
- What does a point  $(x_0, y_0)$  in the image space map to in the Hough space?
  - Answer: the solutions of  $b = -x_0m + y_0$
  - This is a line in Hough space



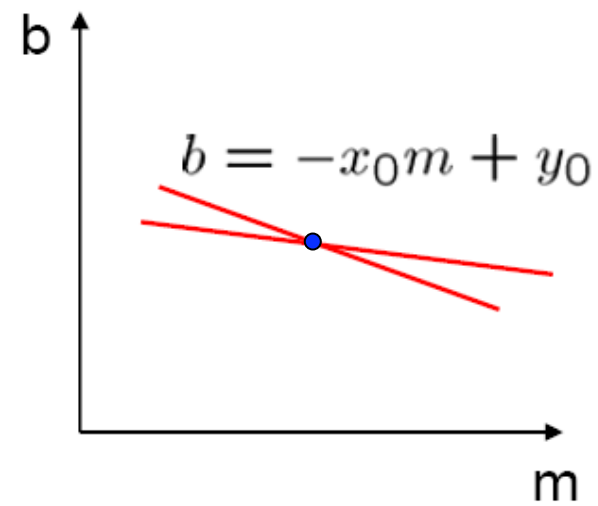
# Parameter space representation

- Where is the line that contains both  $(x_0, y_0)$  and  $(x_1, y_1)$ ?
  - It is the intersection of the lines  $b = -x_0m + y_0$  and  $b = -x_1m + y_1$

Image space



Hough parameter space

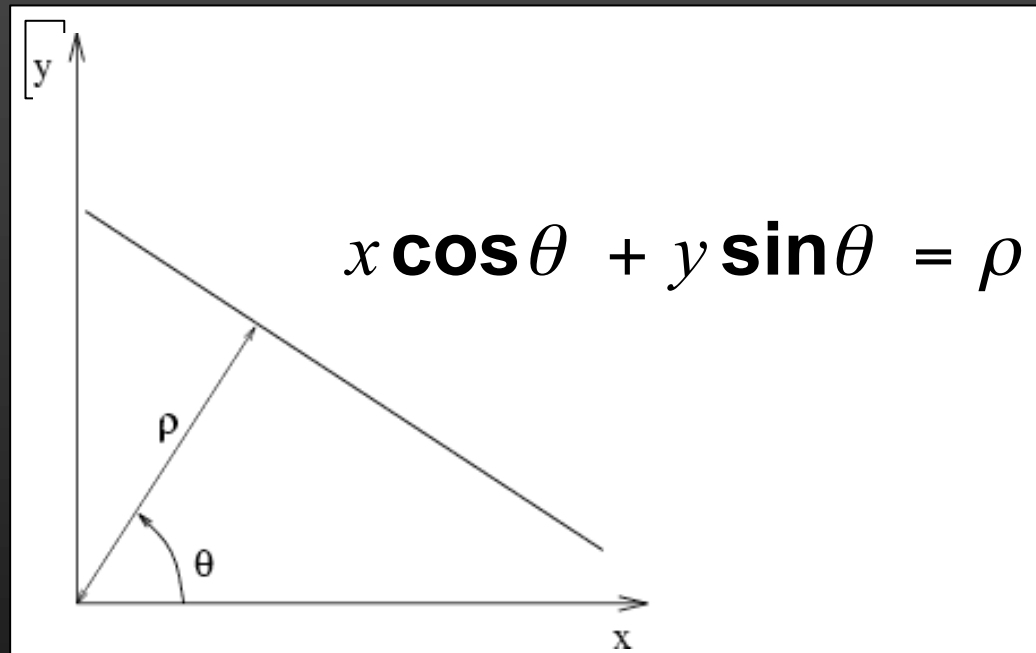


# Polar representation for lines

- Problems with the  $(m,b)$  space:
  - Unbounded parameter domain
  - Vertical lines require infinite  $m$

# Polar representation for lines

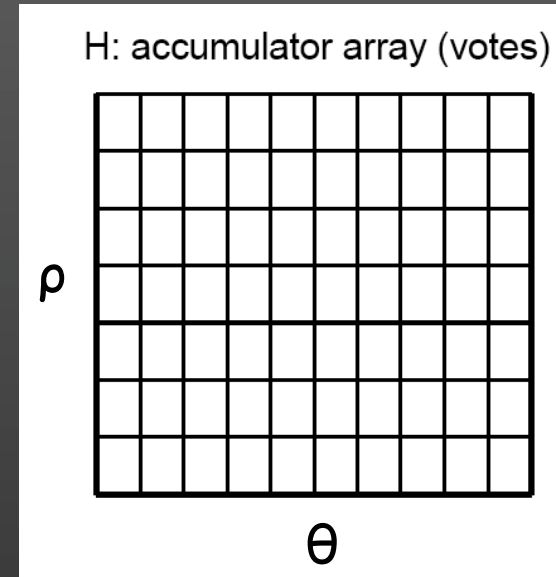
- Problems with the (m,b) space:
  - Unbounded parameter domain
  - Vertical lines require infinite m
- Alternative: polar representation



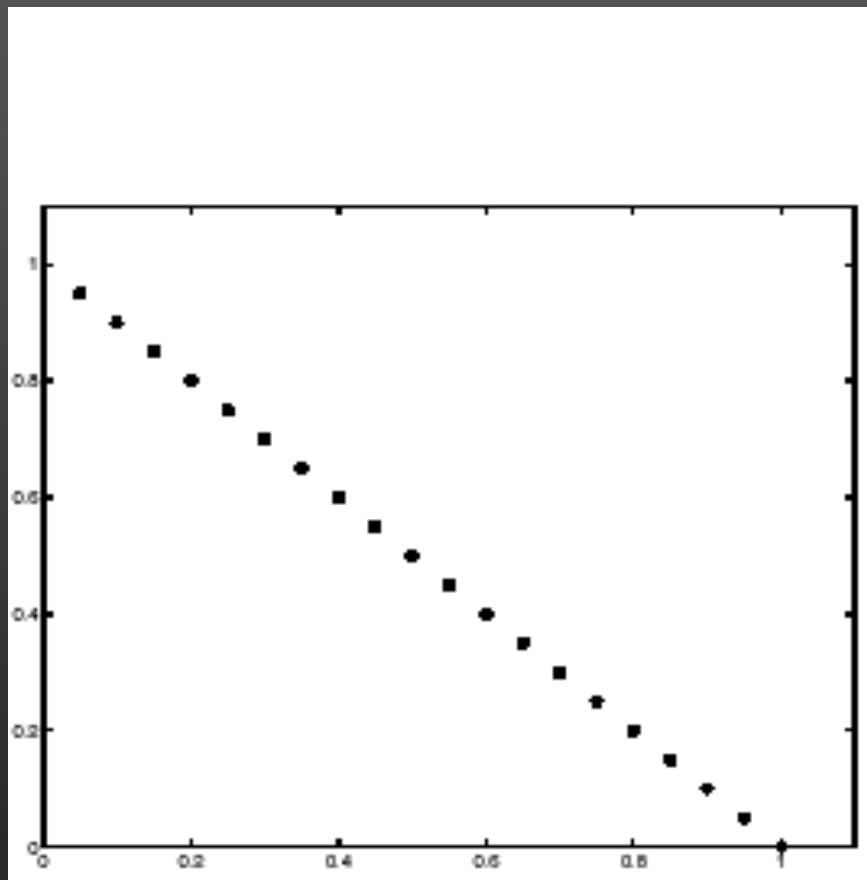
Each point will add a sinusoid in the  $(\theta, \rho)$  parameter space

# Algorithm outline

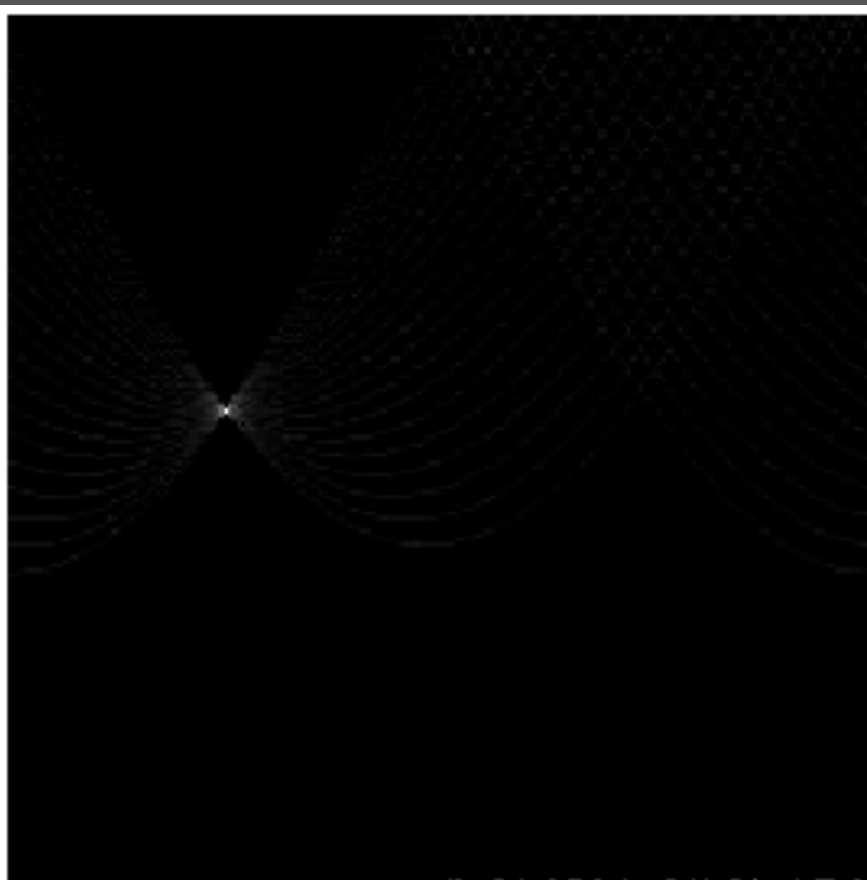
- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image
  - For  $\theta = 0$  to 180
    - $\rho = x \cos \theta + y \sin \theta$
    - $H(\theta, \rho) = H(\theta, \rho) + 1$
  - end
- end
- Find the value(s) of  $(\theta, \rho)$  where  $H(\theta, \rho)$  is a local maximum
  - The detected line in the image is given by  $\rho = x \cos \theta + y \sin \theta$



# Basic illustration



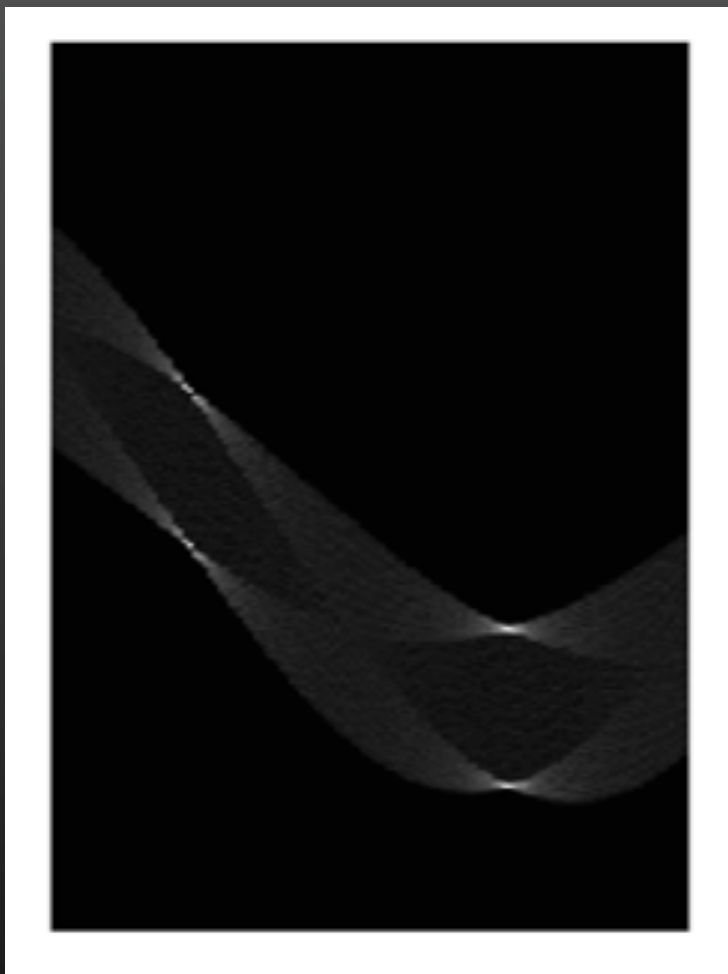
features



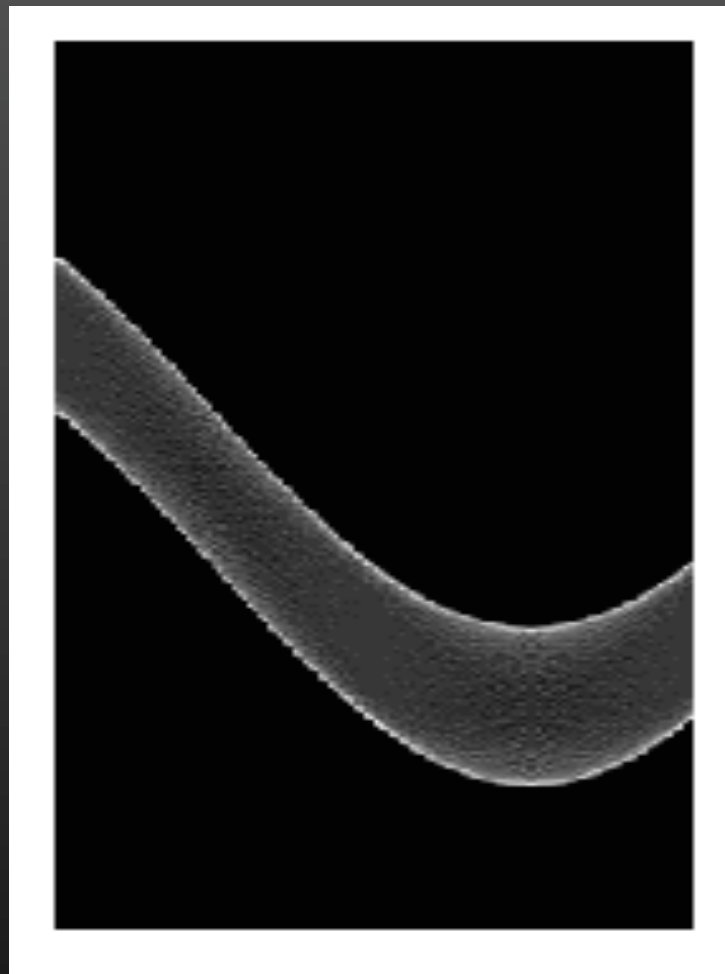
votes

# Other shapes

Square

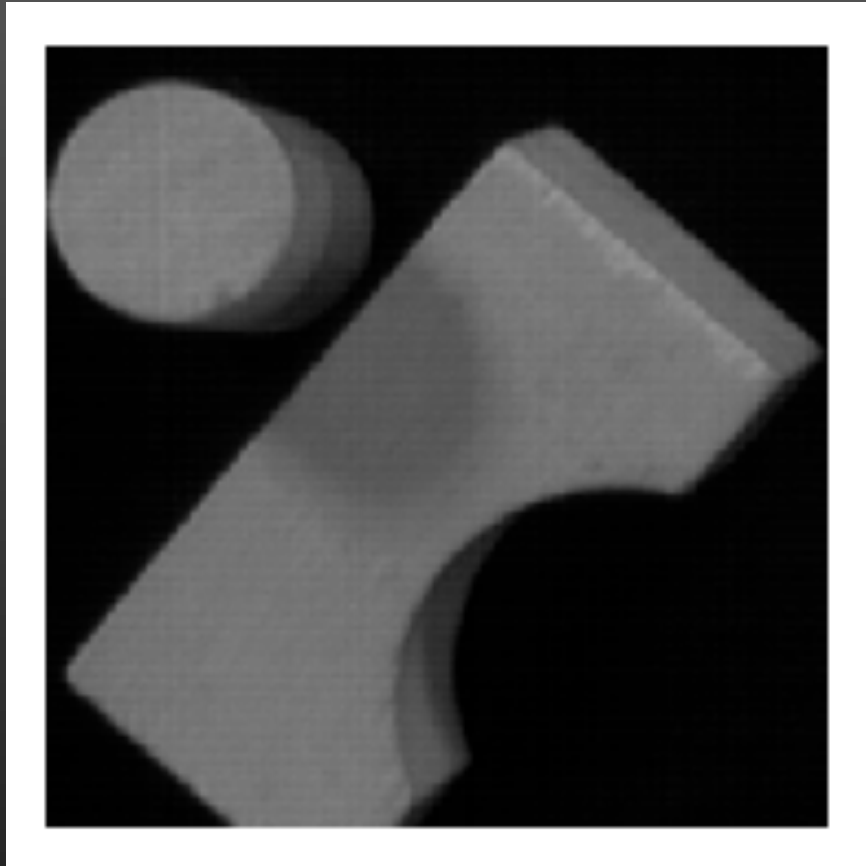


Circle

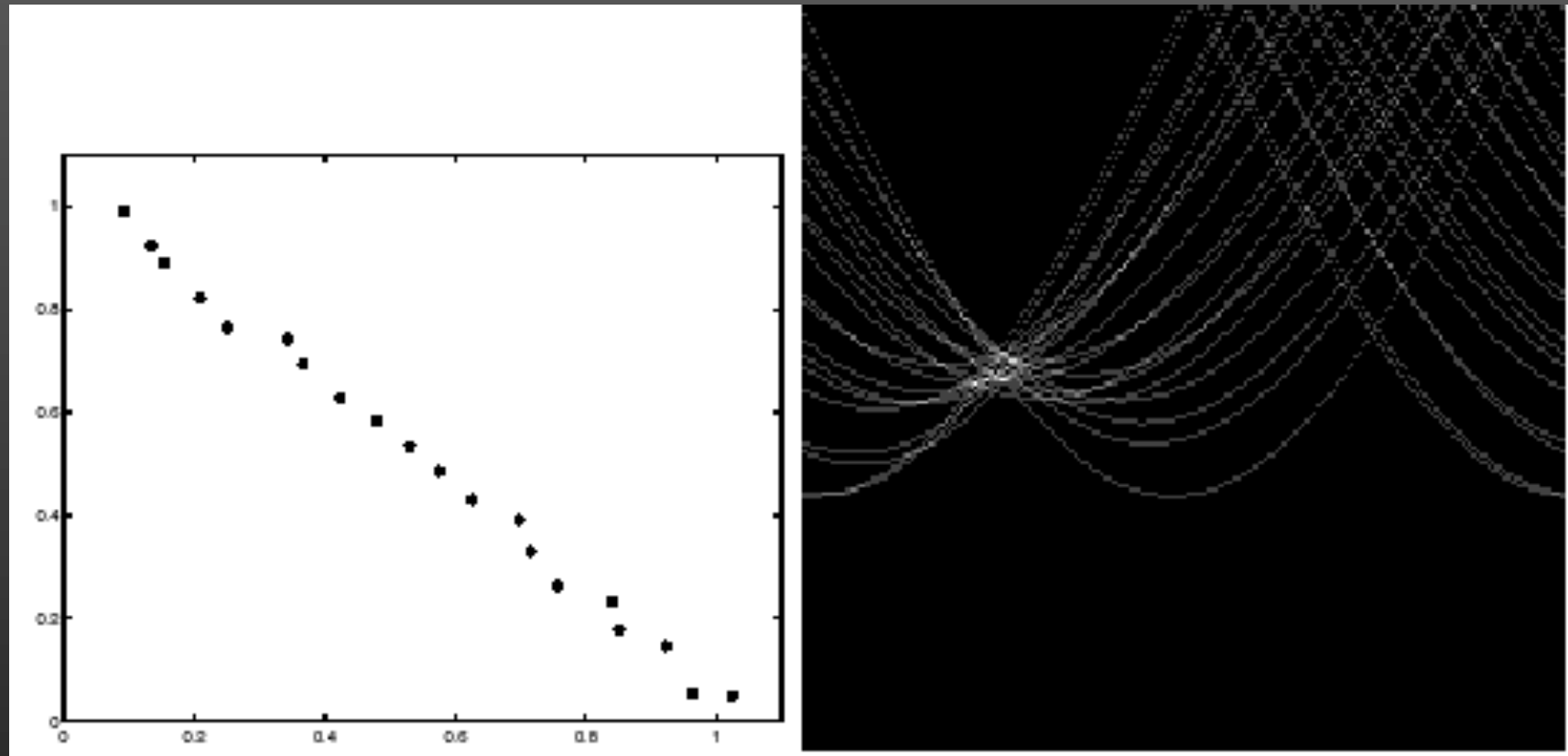




# Several lines



# Effect of noise



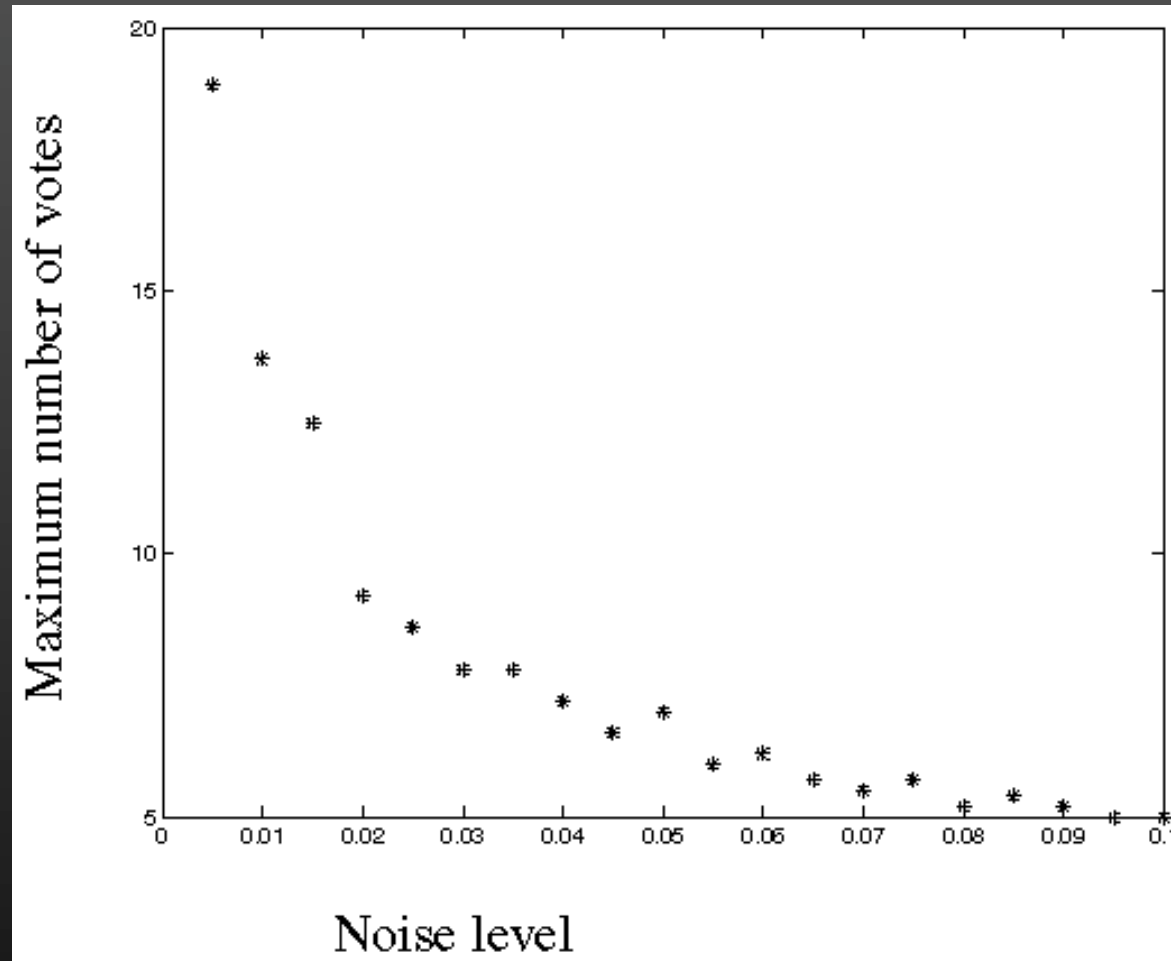
features

votes

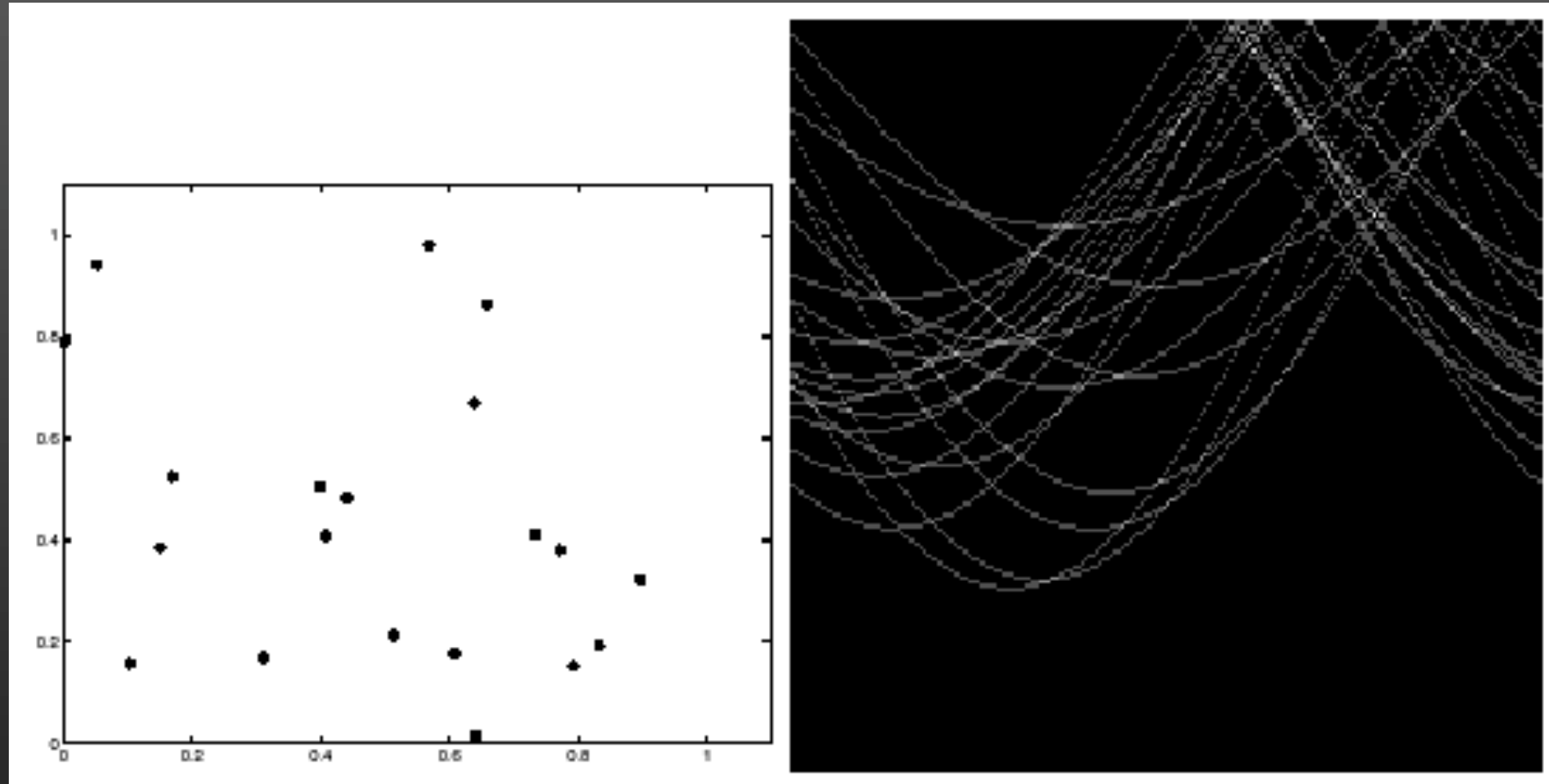
Peak gets fuzzy and hard to locate

# Effect of noise

- Number of votes for a line of 20 points with increasing noise:



# Random points



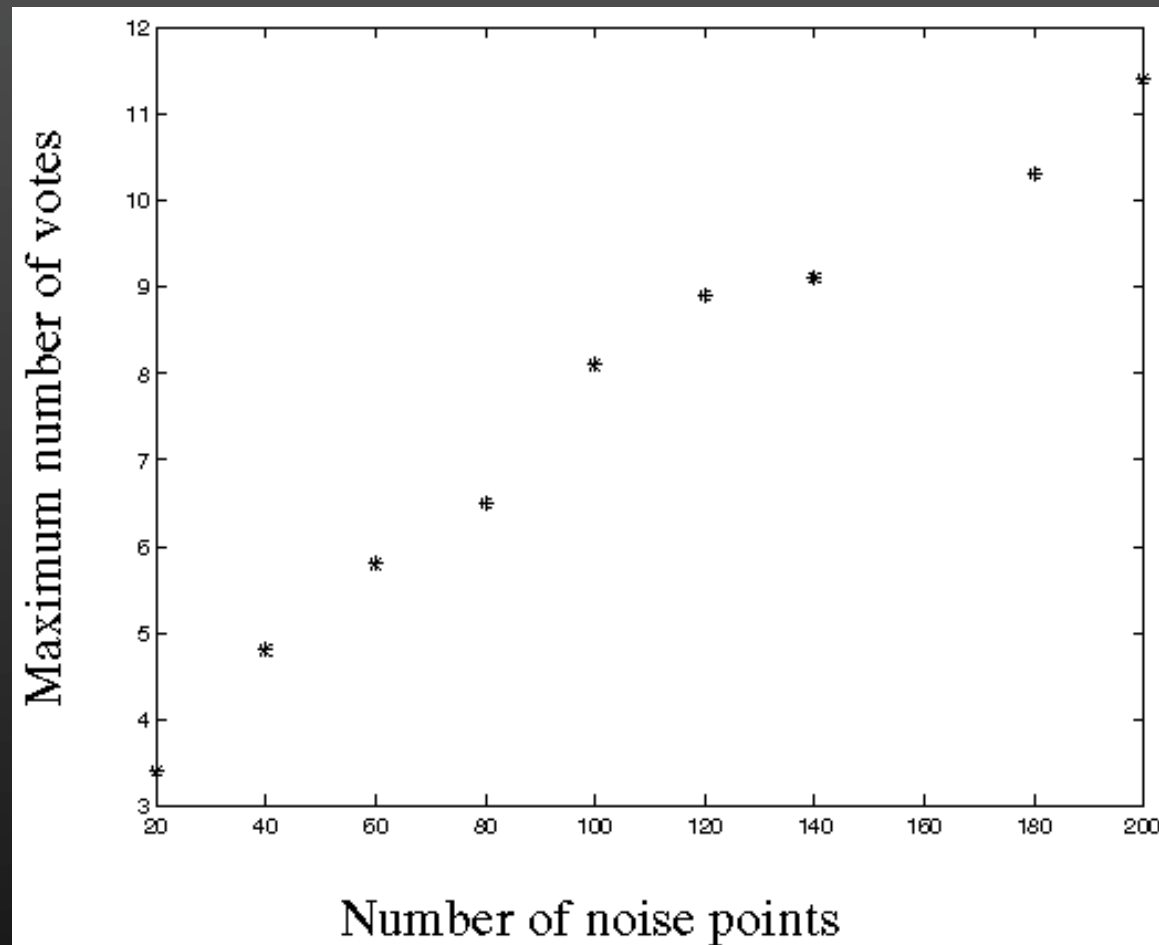
features

votes

Uniform noise can lead to spurious peaks in the array

# Random points

- As the level of uniform noise increases, the maximum number of votes increases too:



# Practical details

- Try to get rid of irrelevant features
  - Take edge points with significant gradient magnitude
- Choose a good grid / discretization
  - Too coarse: large votes obtained when too many different lines correspond to a single bucket
  - Too fine: miss lines because some points that are not exactly collinear cast votes for different buckets
- Increment neighboring bins (smoothing in accumulator array)
- Who belongs to which line?
  - Tag the votes

# Hough transform: Pros

- All points are processed independently, so can cope with occlusion
- Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Can deal with non-locality and occlusion
- Can detect multiple instances of a model in a single pass

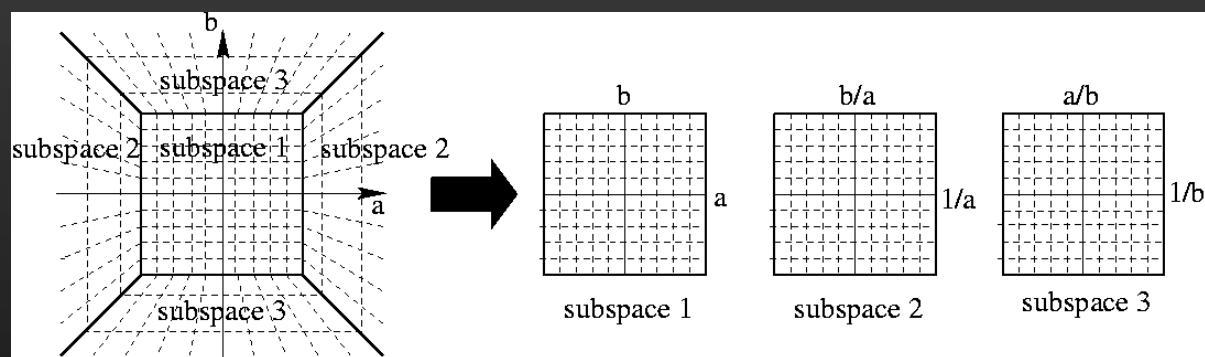
# Hough transform: Cons

- Complexity of search time increases exponentially with the number of model parameters
- Non-target shapes can produce spurious peaks in parameter space
- It's hard to pick a good grid size



# Extension: Cascaded Hough transform

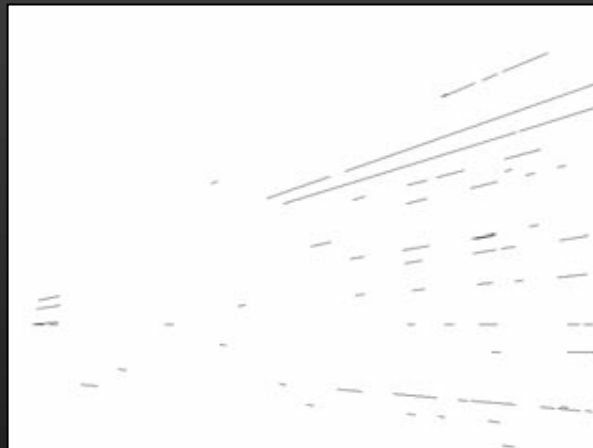
- Let's go back to the original  $(m,b)$  parametrization
- A line in the image maps to a pencil of lines in the Hough space
- What do we get with parallel lines or a pencil of lines?
  - Collinear peaks in the Hough space!
- So we can apply a Hough transform to the output of the first Hough transform to find vanishing points
- Issue: dealing with unbounded parameter space



T. Tuytelaars, M. Proesmans, L. Van Gool

["The cascaded Hough transform,"](#) *ICIP, vol. II, pp. 736-739, 1997.*

# Cascaded Hough transform

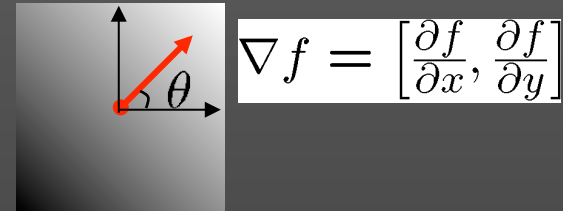


T. Tuytelaars, M. Proesmans, L. Van Gool

["The cascaded Hough transform,"](#) *ICIP*, vol. II, pp. 736-739, 1997.

# Extension: Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!
- Modified Hough transform:



$$\theta = \tan^{-1} \left( \frac{\partial f / \partial y}{\partial f / \partial x} \right)$$

For each edge point (x,y)

$\theta$  = gradient orientation at (x,y)

$\rho = x \cos \theta + y \sin \theta$

$H(\theta, \rho) = H(\theta, \rho) + 1$

end

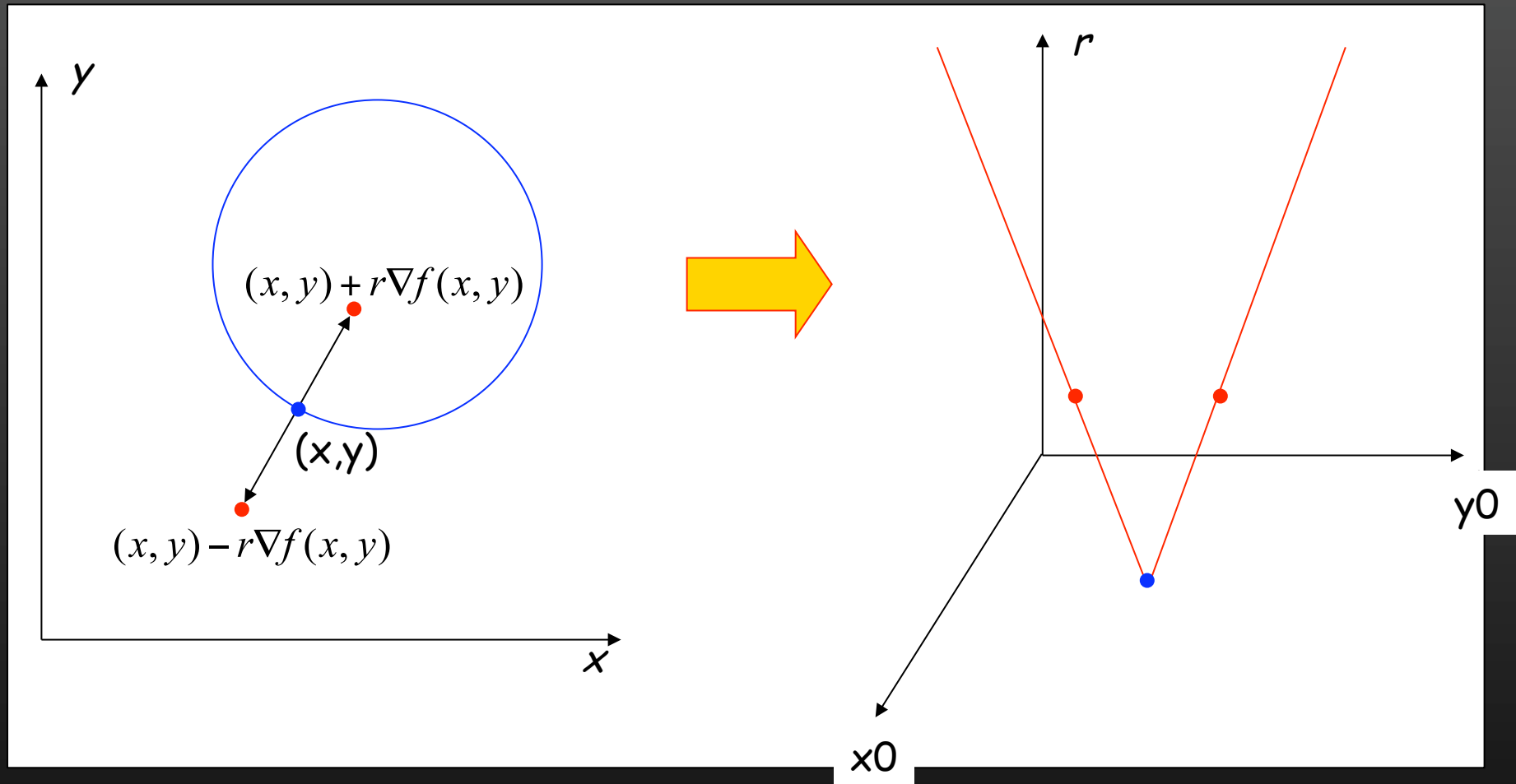
# Hough transform for circles

- How many dimensions will the parameter space have?
- Given an oriented edge point, what are all possible bins that it can vote for?

# Hough transform for circles

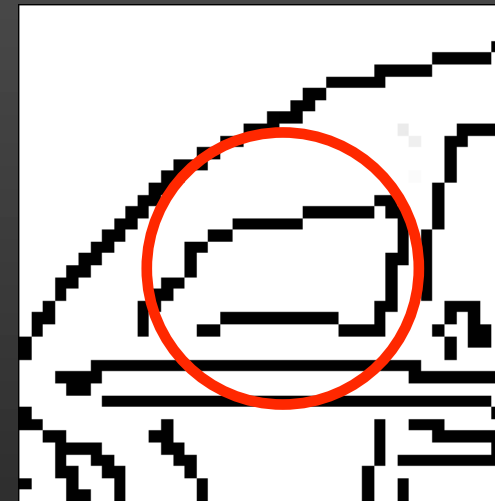
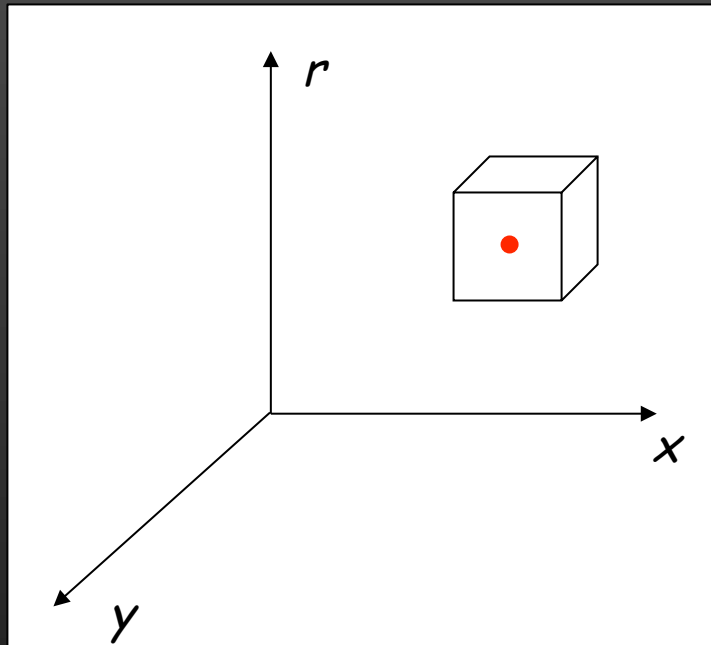
image space

Hough parameter space



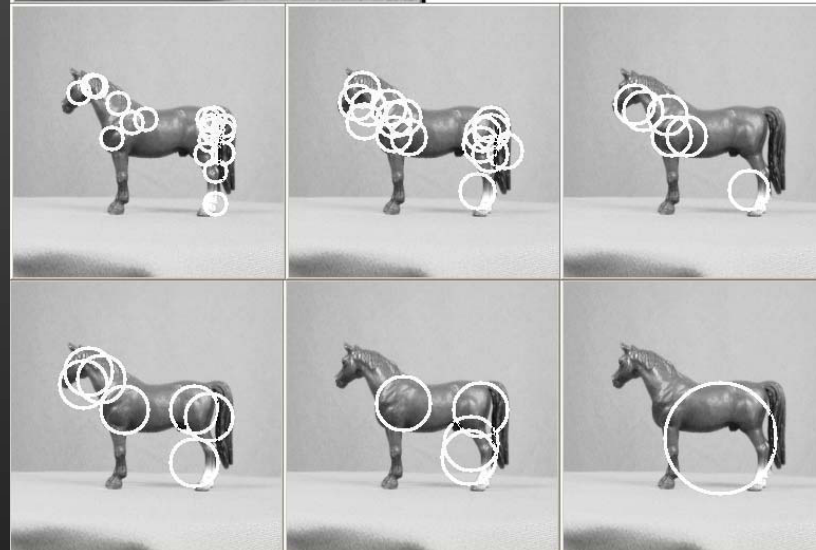
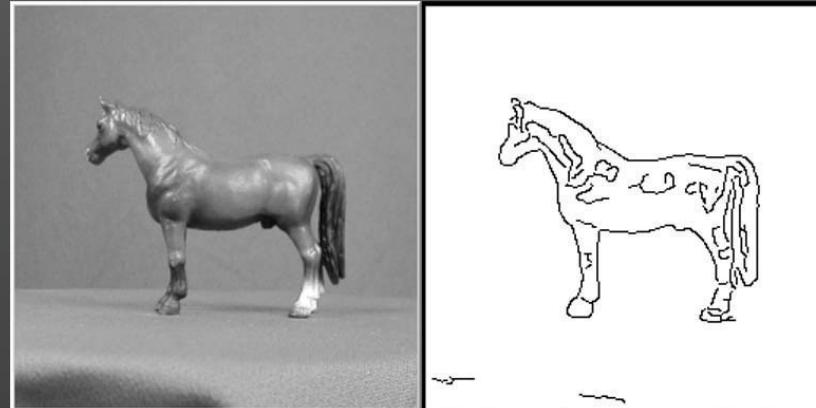
# Hough transform for circles

- Conceptually equivalent procedure: for each  $(x,y,r)$ , draw the corresponding circle in the image and compute its “support”



What is more efficient: going from the image space to the parameter space or vice versa?

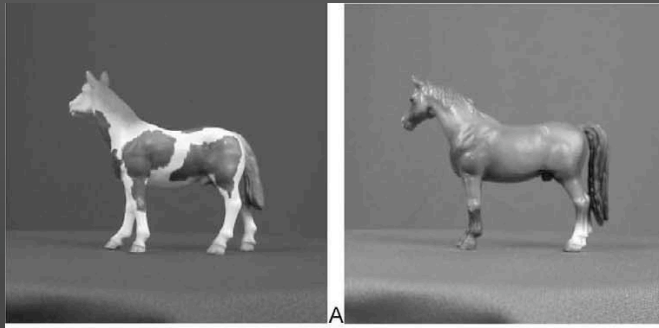
# Application in recognition



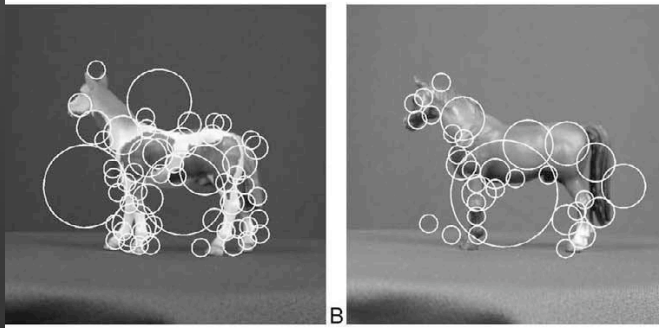
F. Jurie and C. Schmid,  
[Scale-invariant shape features for recognition of object categories](#),  
CVPR 2004

# Hough circles vs. Laplacian blobs

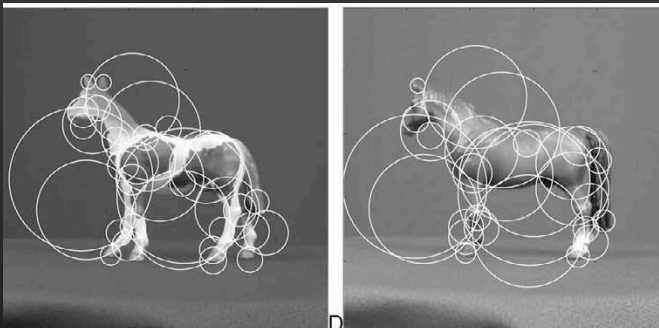
Original images



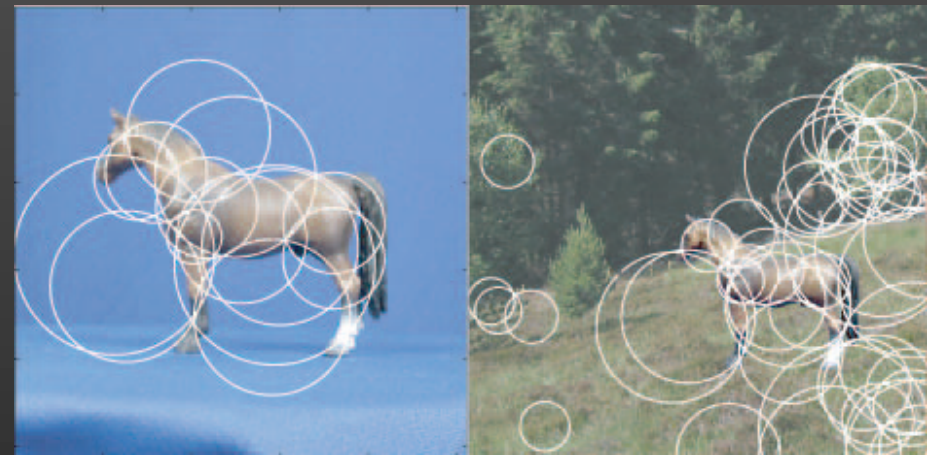
Laplacian circles



Hough-like circles



Robustness to scale and clutter

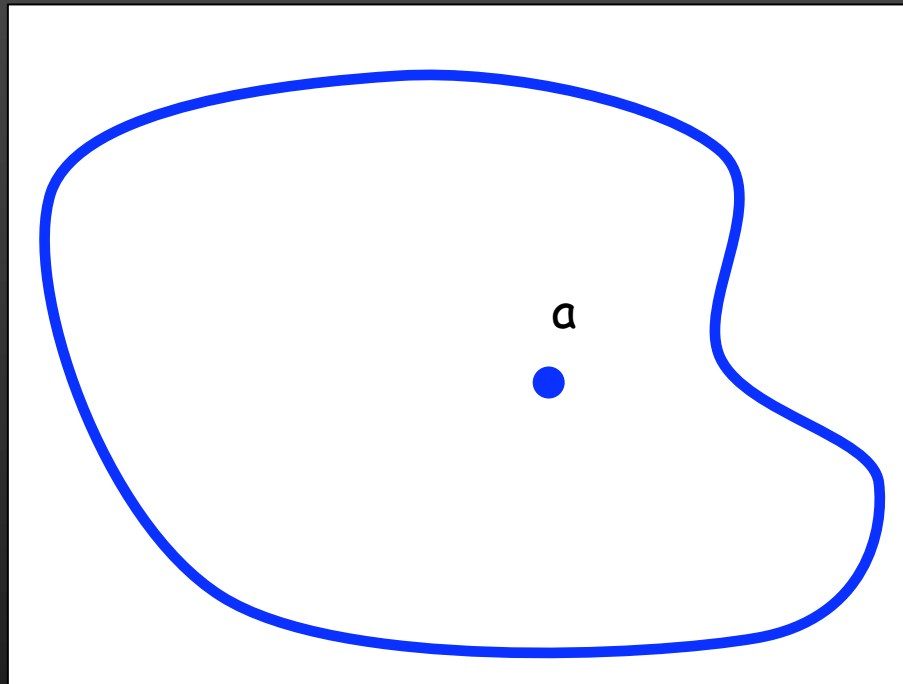


F. Jurie and C. Schmid,  
[\*Scale-invariant shape features for recognition of object categories\*](#),  
CVPR 2004



# Generalized Hough transform

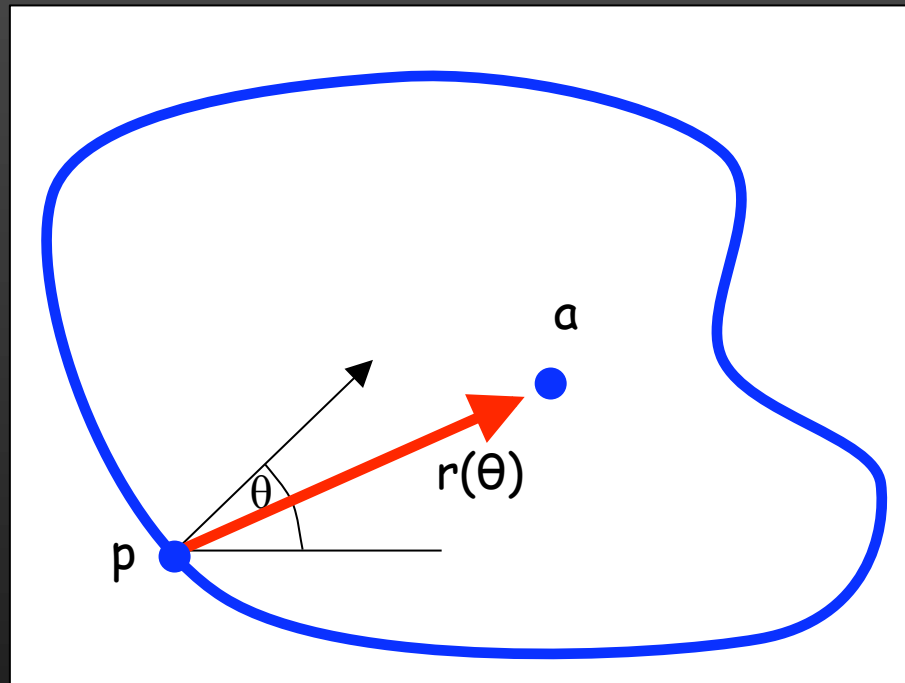
- We want to find a shape defined by its boundary points and a reference point



D. Ballard,  
[Generalizing the Hough Transform to Detect Arbitrary Shapes](#), Pattern  
Recognition 13(2), 1981, pp. 111-122.

# Generalized Hough transform

- We want to find a shape defined by its boundary points and a reference point
- For every boundary point  $p$ , we can compute the displacement vector  $r = a - p$  as a function of gradient orientation  $\theta$

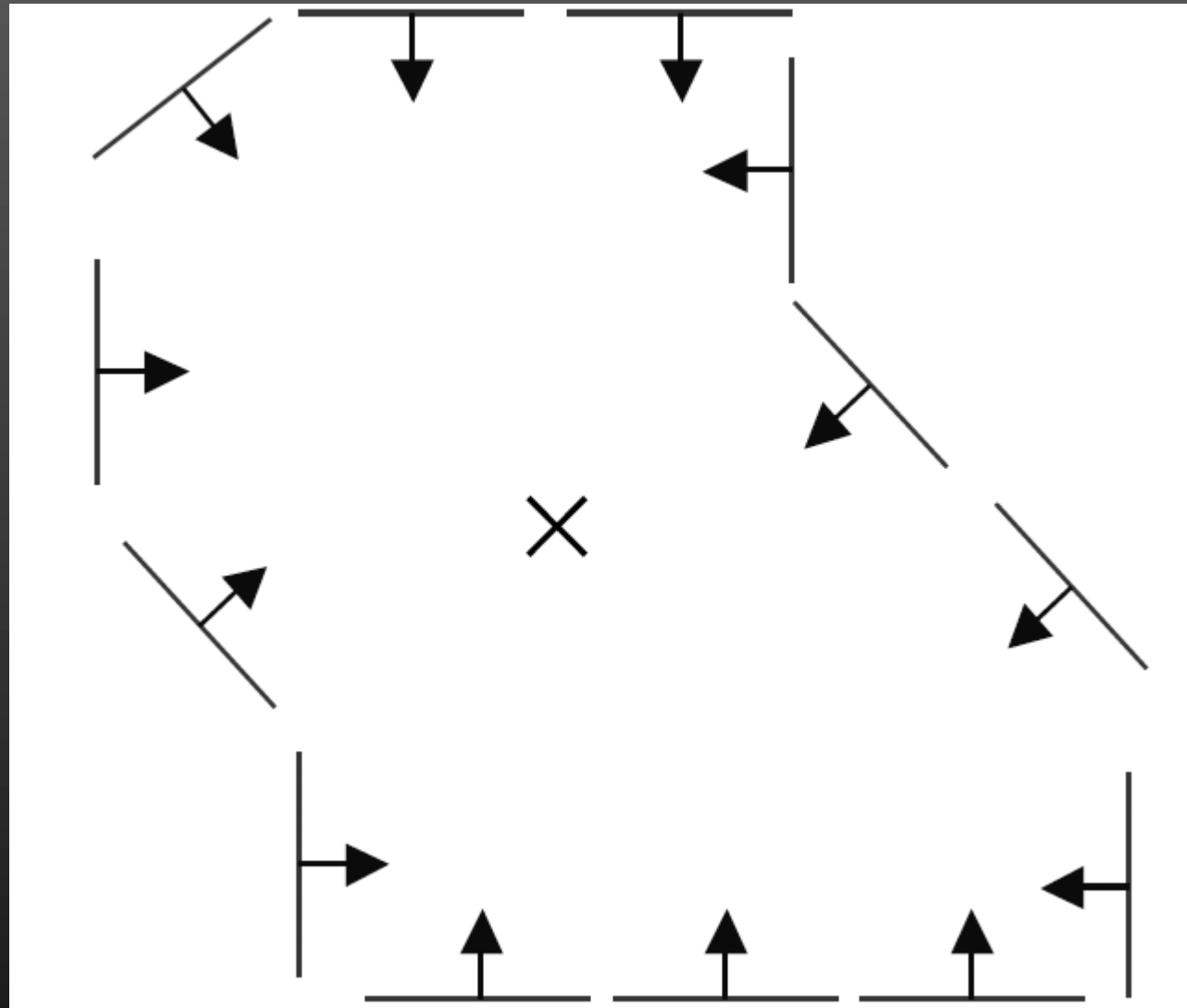


D. Ballard,  
[Generalizing the Hough Transform to Detect Arbitrary Shapes](#), Pattern  
Recognition 13(2), 1981, pp. 111-122.

# Generalized Hough transform

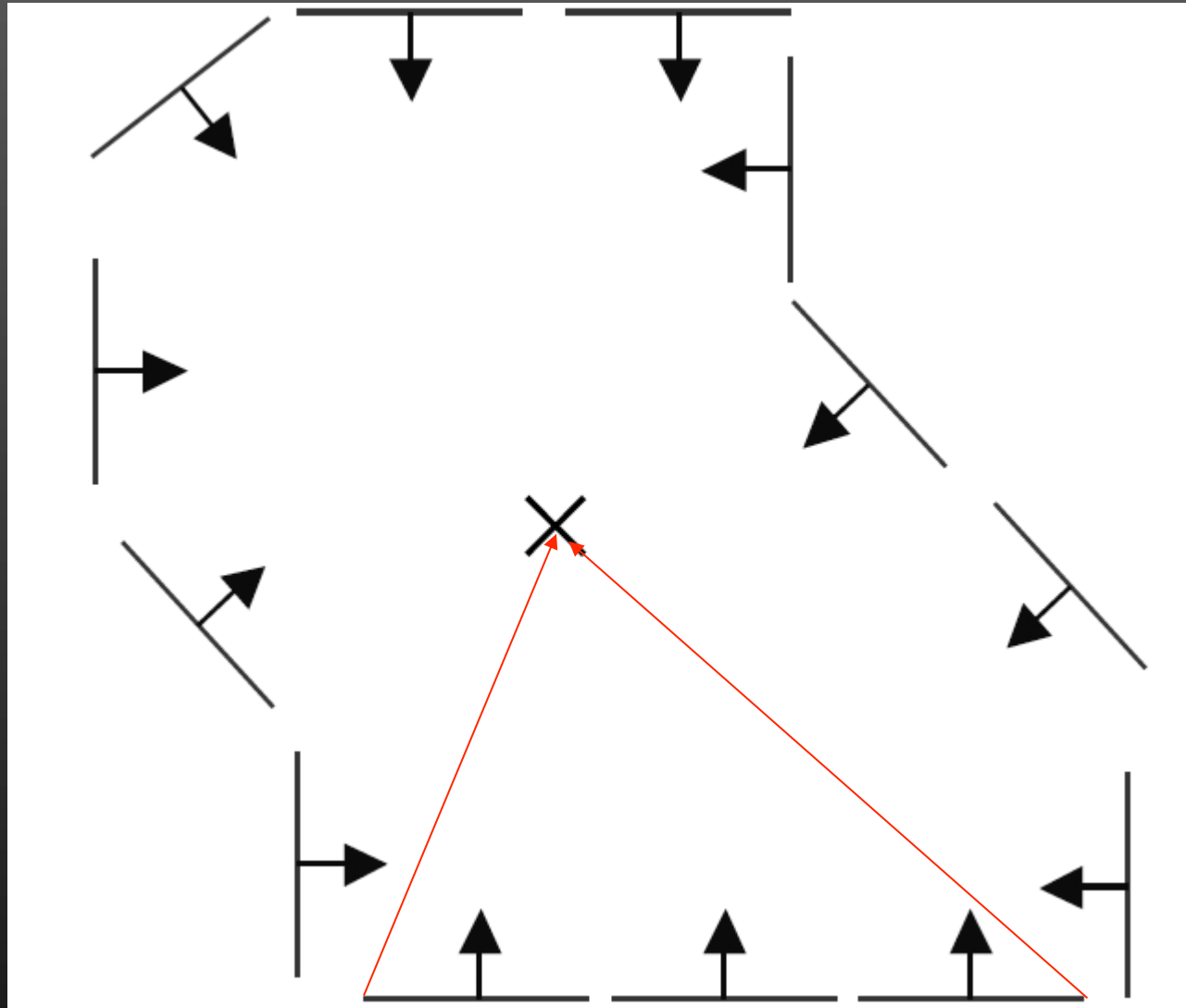
- For model shape: construct a table storing displacement vectors  $r$  as function of gradient direction
- Detection: For each edge point  $p$  with gradient orientation  $\theta$ :
  - Retrieve all  $r$  indexed with  $\theta$
  - For each  $r(\theta)$ , put a vote in the Hough space at  $p + r(\theta)$
- Peak in this Hough space is reference point with most supporting edges
- *Assumption: translation is the only transformation here, i.e., orientation and scale are fixed*

# Example



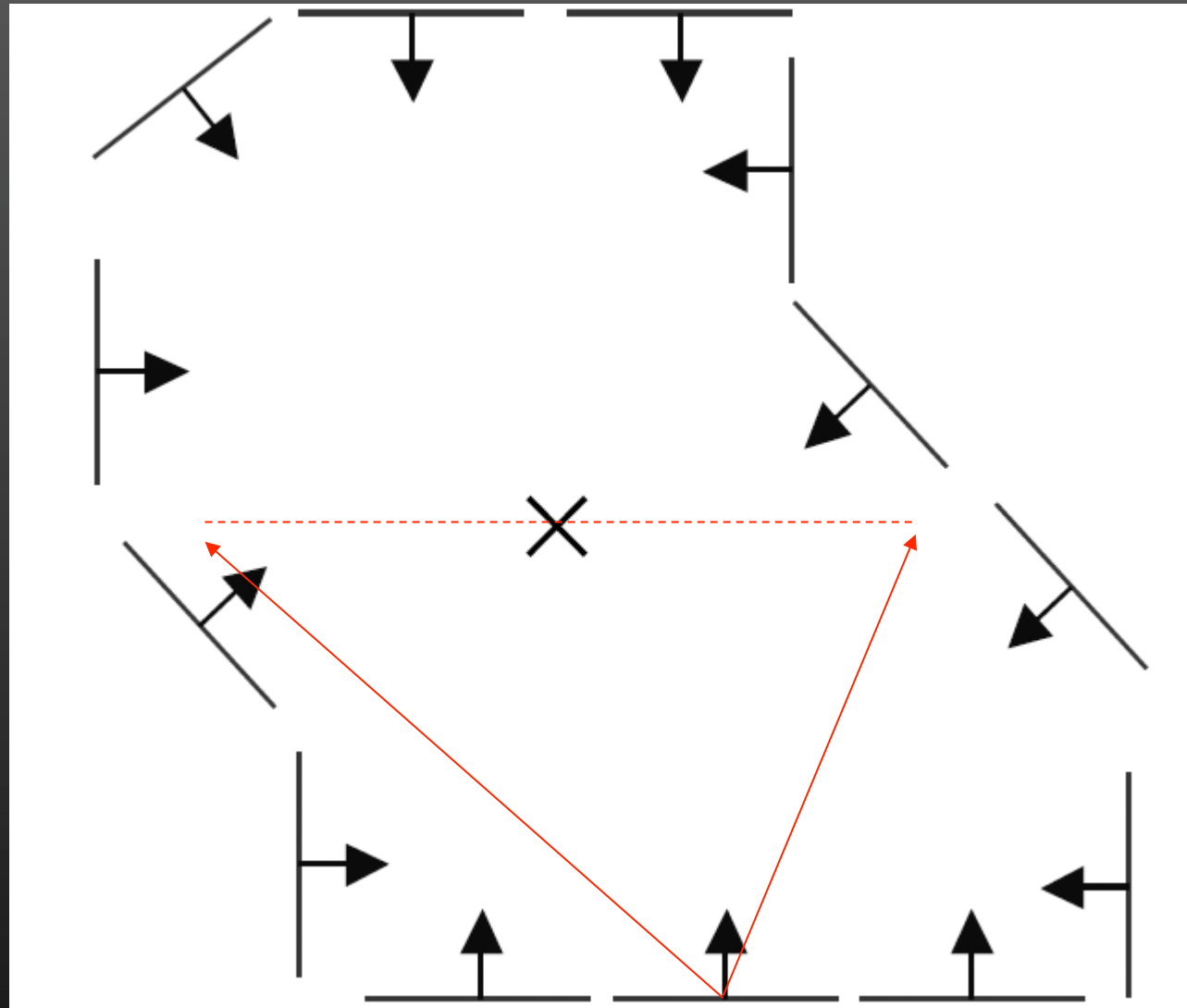
model shape

# Example



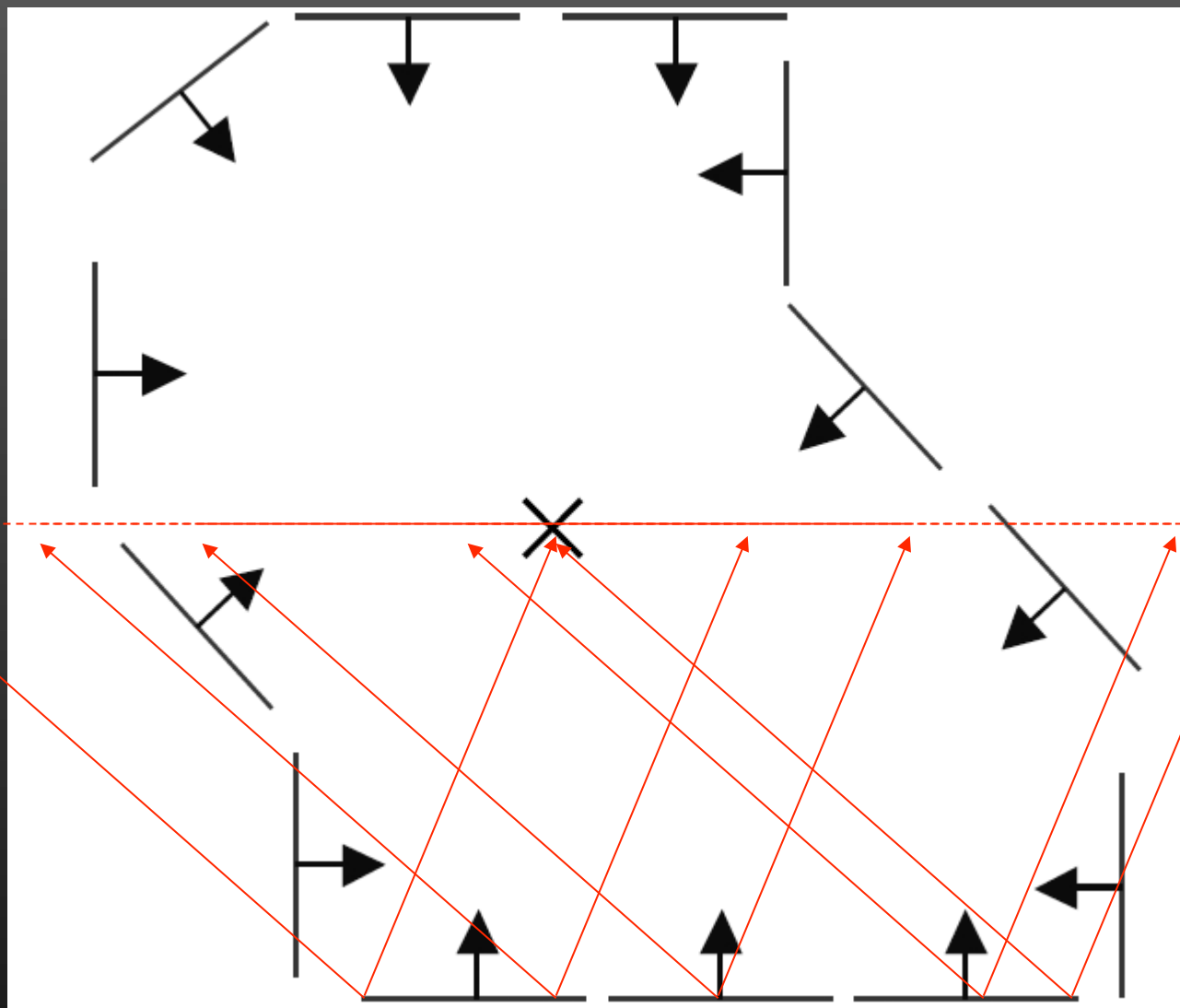
displacement vectors for model points

# Example



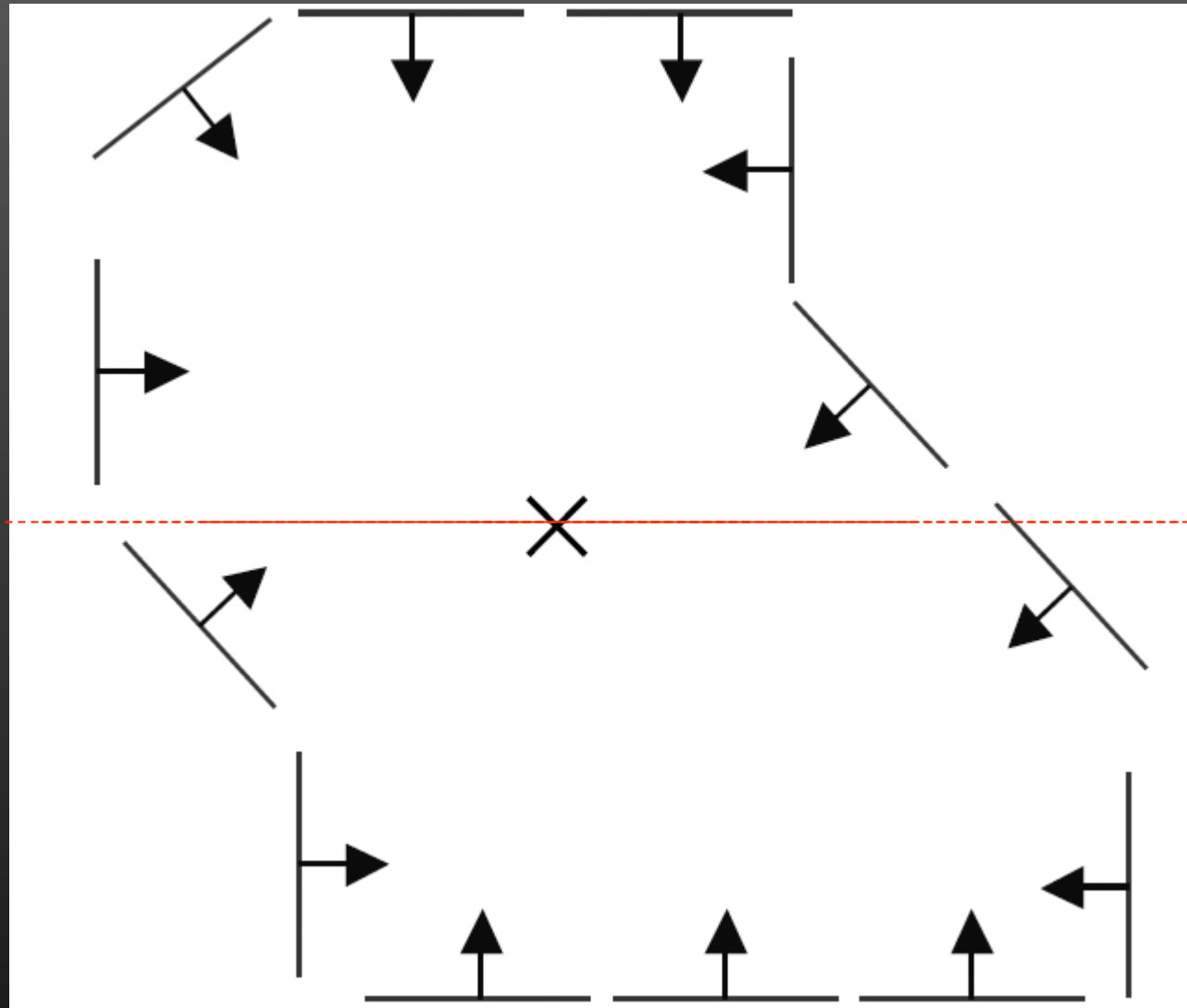
range of voting locations for test point

# Example



range of voting locations for test point

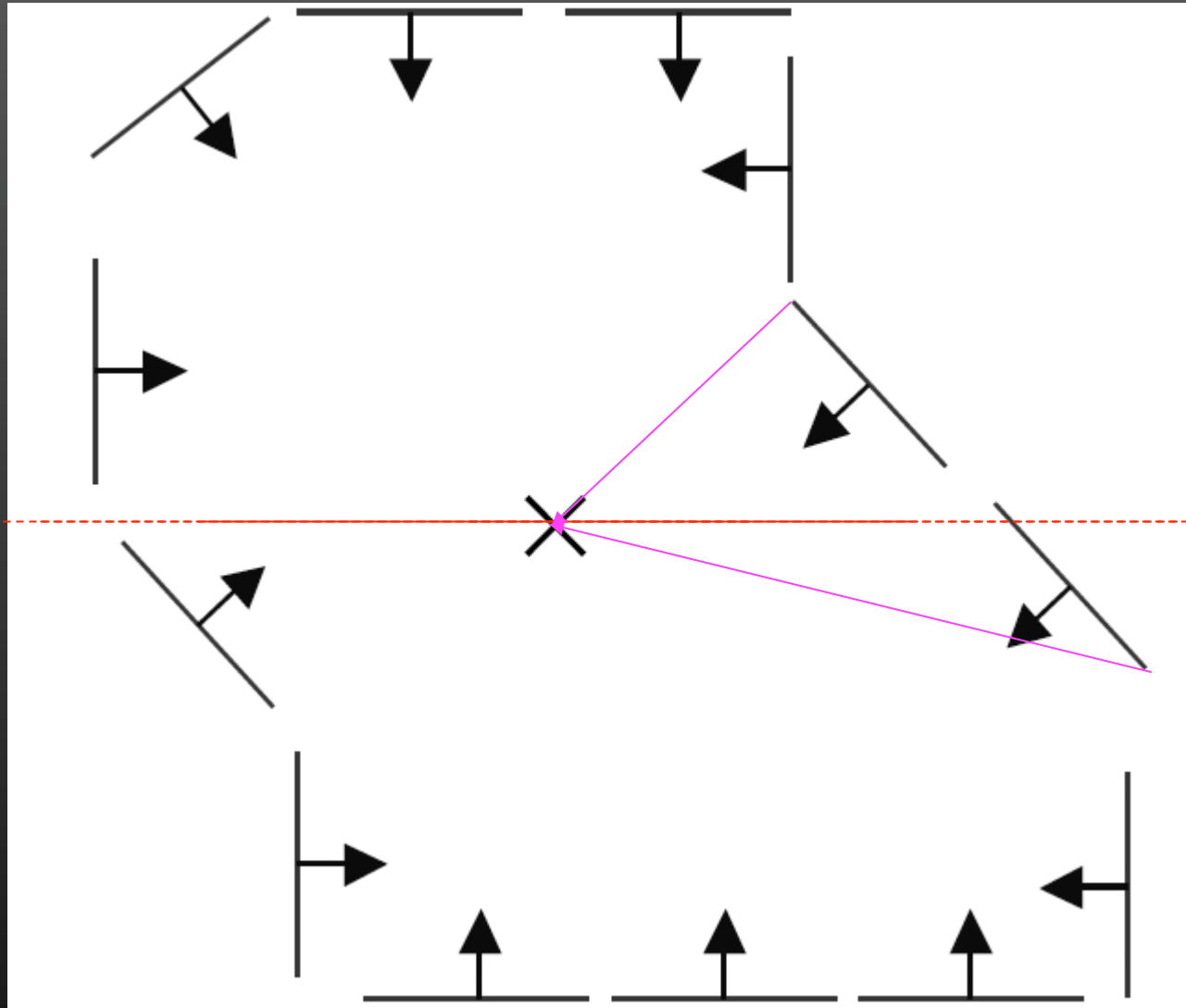
# Example



votes for points with  $\theta \uparrow$

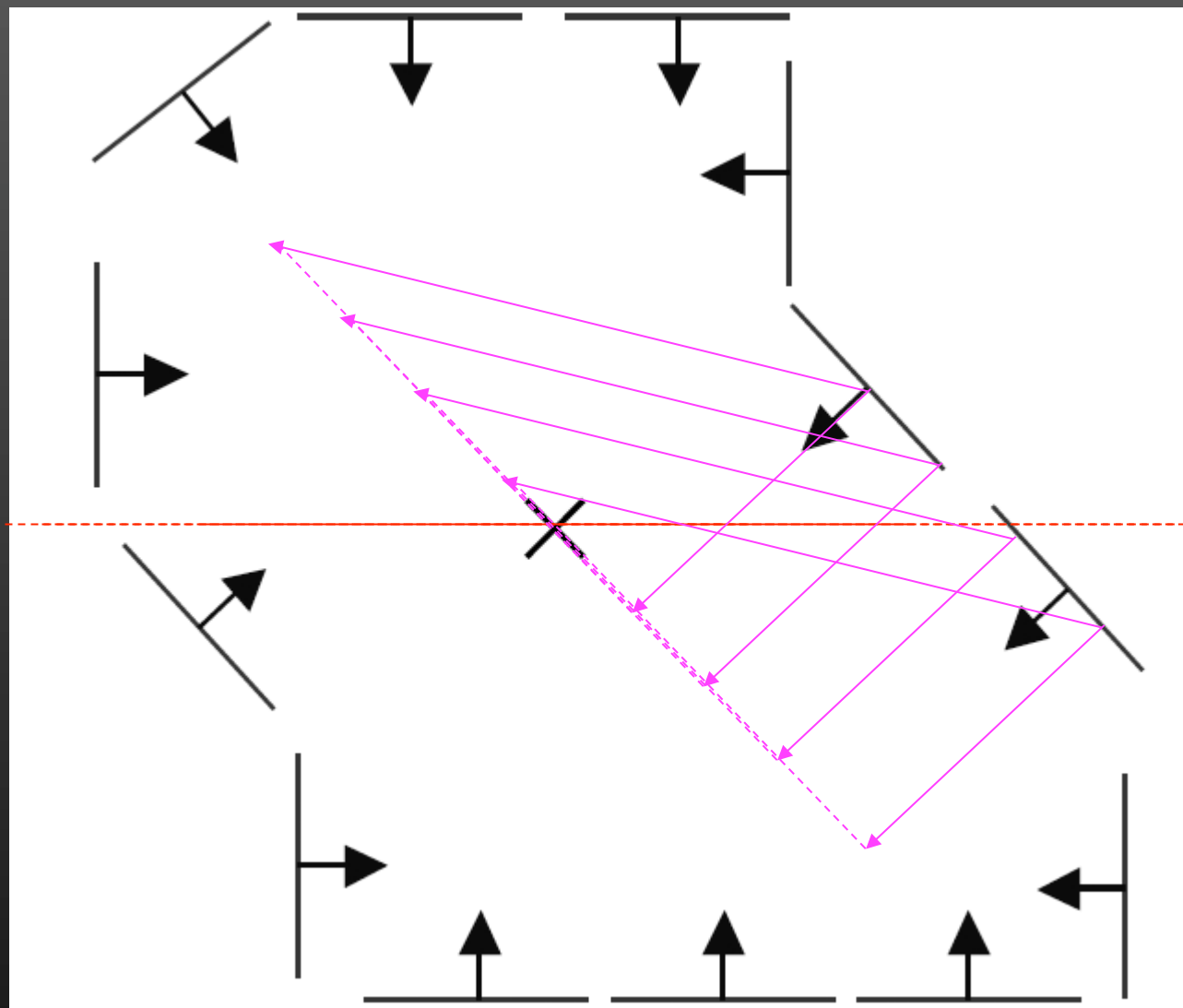


# Example



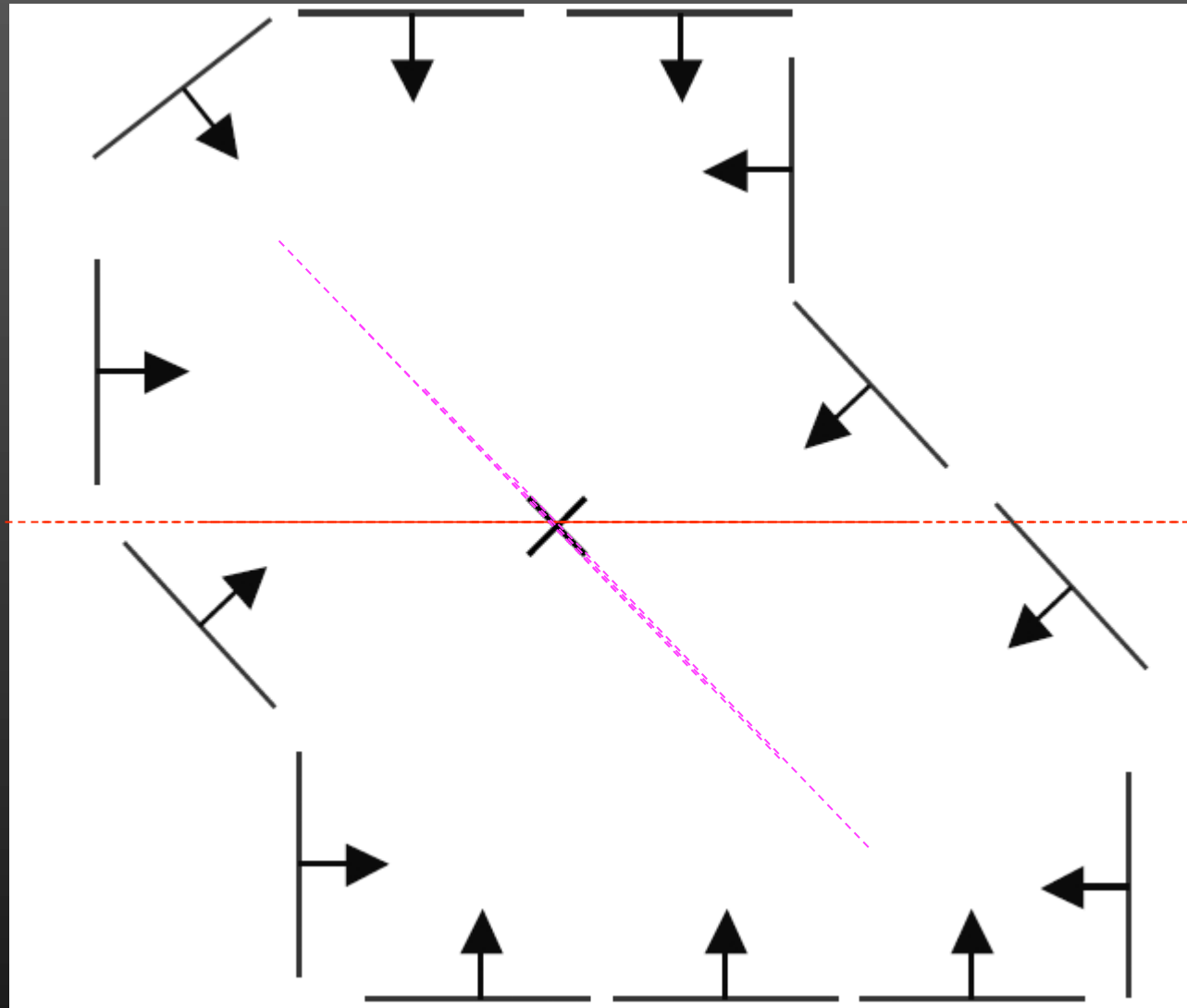
displacement vectors for model points

# Example



range of voting locations for test point

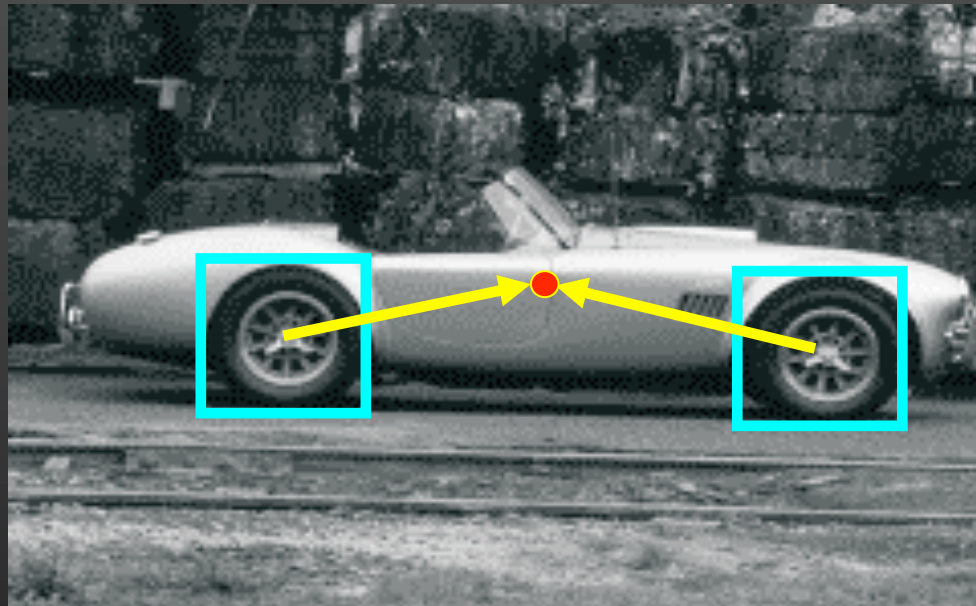
# Example



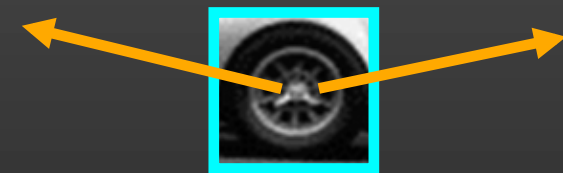
votes for points with  $\theta \neq \pi/4$

# Application in recognition

- Instead of indexing displacements by gradient orientation, index by “visual codeword”



training image



visual codeword with  
displacement vectors

B. Leibe, A. Leonardis, and B. Schiele,  
[Combined Object Categorization and Segmentation with an Implicit Shape Model](#), ECCV Workshop on Statistical Learning in Computer Vision 2004

# Application in recognition

- Instead of indexing displacements by gradient orientation, index by “visual codeword”

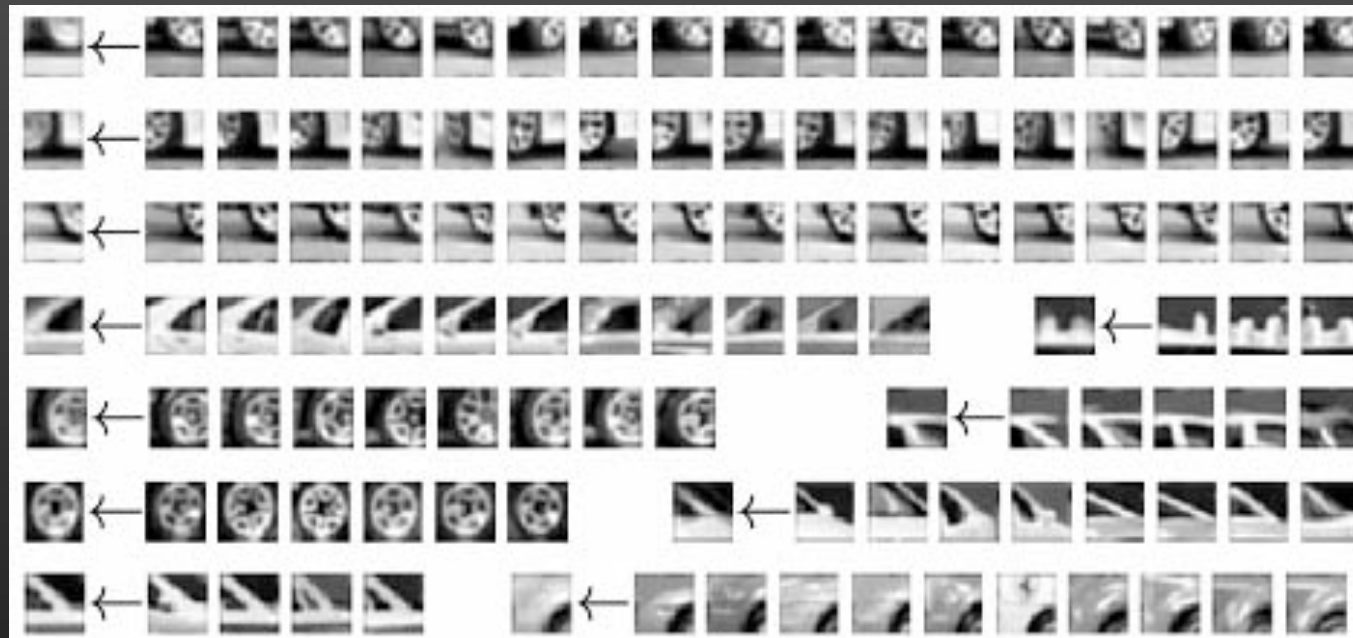


test image

B. Leibe, A. Leonardis, and B. Schiele,  
[Combined Object Categorization and Segmentation with an Implicit Shape Model](#), ECCV Workshop on Statistical Learning in Computer Vision 2004

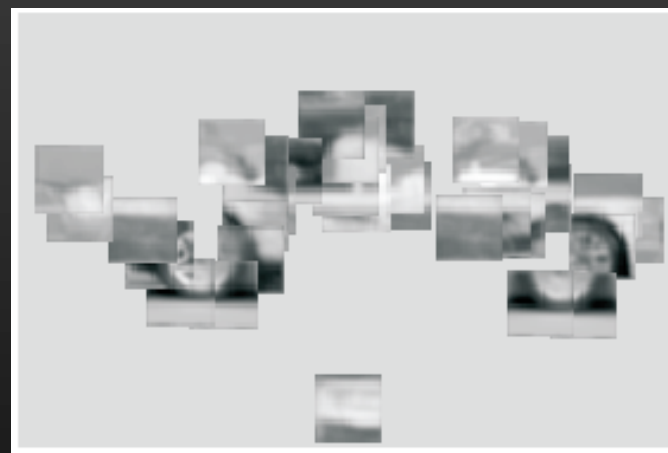
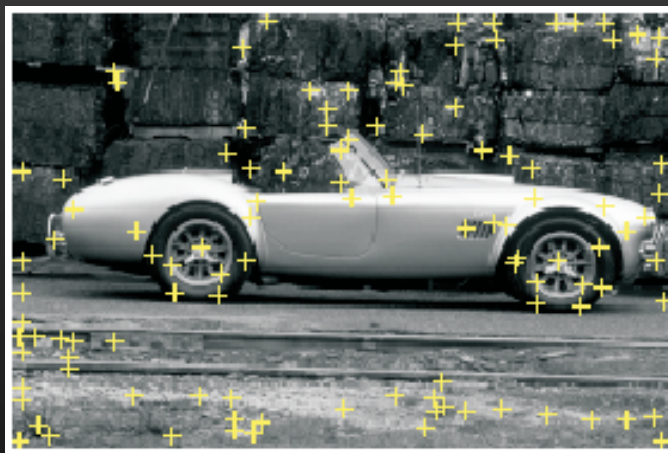
# Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering



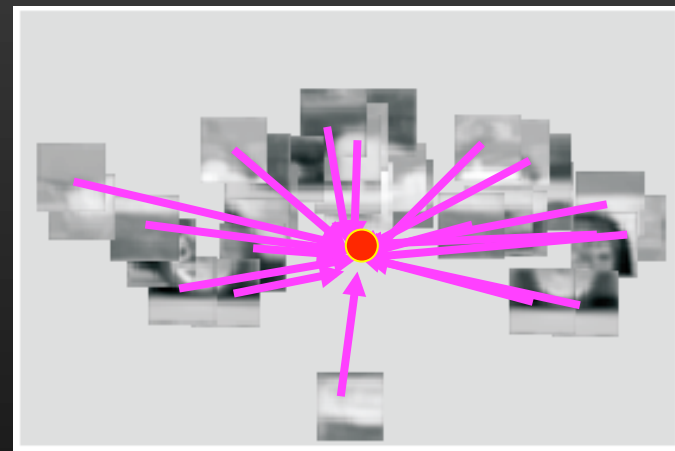
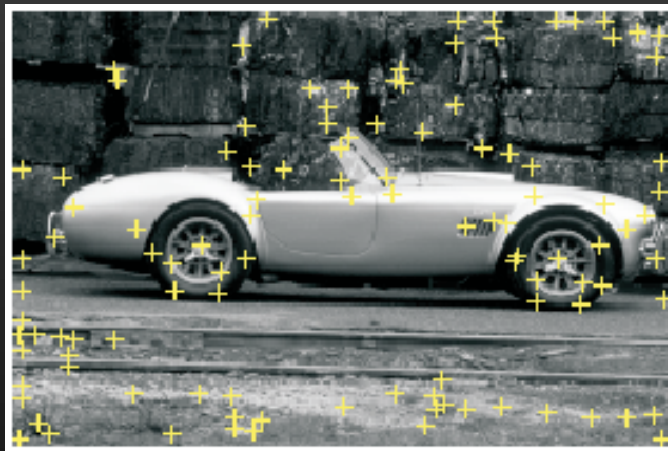
# Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry



# Implicit shape models: Training

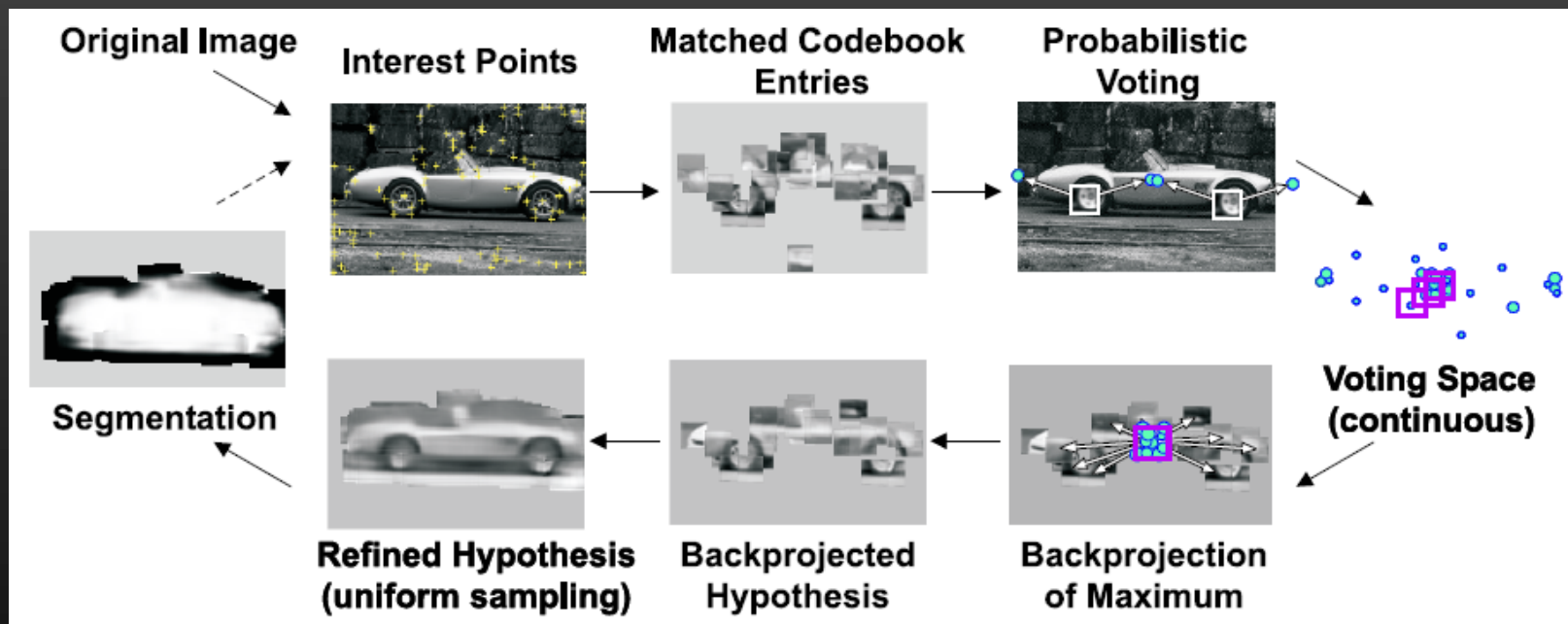
1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry
3. For each codebook entry, store all positions it was found, relative to object center





# Implicit shape models: Testing

1. Given test image, extract patches, match to codebook entry
2. Cast votes for possible positions of object center
3. Search for maxima in voting space
4. Extract weighted segmentation mask based on stored masks for the codebook occurrences



# Implicit shape models: Notes

- Supervised training
  - Need reference location and segmentation mask for each training car
- Voting space is continuous, not discrete
  - Clustering algorithm needed to find maxima
- How about dealing with scale changes?
  - Option 1: search a range of scales, as in Hough transform for circles
  - Option 2: use scale-invariant interest points
- Verification stage is very important
  - Once we have a location hypothesis, we can overlay a more detailed template over the image and compare pixel-by-pixel, transfer segmentation masks, etc.