1. Proof

Proposition 1. $\Psi(x)$ is a surrogate function of $\psi(x)$.

Proof. A function $g(x)$ is said to be a surrogate function of $f(x)$ provided

$$
\begin{align*}
  f(x) &\leq g(x), \forall x, y \in [0, \infty) \\
  f(y) &\leq g(y), \forall y \in [0, \infty)
\end{align*}
$$

Let us consider the following function:

$$
  f(x) = \frac{1 - \exp(-\nu x)}{\nu}.
$$

Since $f''(x) < 0$, this function is strictly concave, $\forall x, y$,

$$
  f(x) \leq f(y) + (x - y)f'(y),
$$

with equality holding at $x = y$. Thus, the surrogate function $g(x)$ is

$$
  g(x) = f(y) + (x - y)f'(y),
$$

Let us substitute $x$ and $y$ with $x^2$ and $y^2$, respectively, then

$$
  \psi(x) = f(x^2) \\
  \leq g(x^2),
$$

and

$$
  
$$

\[\Box\]

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2. More Results

Table 1. Quantitative Evaluation of Upsampled Depth Maps on the Middlebury Test Bed [8]

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Omega_{\text{alt}}$</th>
<th>$\Omega_{\text{disc}}$</th>
<th>$\Omega_{\text{all}}$</th>
<th>$\Omega_{\text{alt}}$</th>
<th>$\Omega_{\text{disc}}$</th>
<th>$\Omega_{\text{all}}$</th>
<th>$\Omega_{\text{alt}}$</th>
<th>$\Omega_{\text{disc}}$</th>
<th>$\Omega_{\text{all}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear Int.</td>
<td>10.40</td>
<td>46.30</td>
<td>3.29</td>
<td>37.10</td>
<td>11.80</td>
<td>35.30</td>
<td>14.60</td>
<td>35.80</td>
<td>13.07</td>
</tr>
<tr>
<td>GF [3]</td>
<td>9.87</td>
<td>43.20</td>
<td>2.74</td>
<td>26.50</td>
<td>15.50</td>
<td>37.50</td>
<td>15.50</td>
<td>34.40</td>
<td>19.14</td>
</tr>
<tr>
<td>Park et al.</td>
<td>6.14</td>
<td>28.00</td>
<td>1.03</td>
<td>10.10</td>
<td>7.88</td>
<td>22.20</td>
<td>8.10</td>
<td>19.20</td>
<td>14.23</td>
</tr>
<tr>
<td>Ours</td>
<td>2.39</td>
<td>10.40</td>
<td>0.55</td>
<td>5.50</td>
<td>7.39</td>
<td>20.30</td>
<td>5.24</td>
<td>12.40</td>
<td>10.04</td>
</tr>
</tbody>
</table>

References

Figure 1. Visual comparison of (×8) upsampled depth maps and point cloud reconstructions on books sequence in the Middlebury test bed [8].
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Figure 4. Visual comparison of (×8) upsampled depth maps and point cloud reconstructions on laundry sequence in the Middlebury test bed [8].
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Figure 9. Visual comparison of upsampled depth maps on books sequence in the Graz data sets [2].
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- **(a) Input image**
- **(b) WLS \[1\] (µ = 40)**
- **(c) WLS \[1\] (µ = 5)**
- **(d) RGF \[11\]**
- **(e) Ours**
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Figure 22. Examples of the texture removal for regular textures. (a) Input image, (b) Cov. M1 \[ \sigma = 0.3, r = 10 \], (c) RTV [9] \[ \lambda = 0.01, \sigma = 3 \], (d) RGF [11] \[ \sigma_s = 3, \sigma_r = 0.05, k = 5 \], (e) ours \[ \mathbf{u}^0 = \mathbf{u}_{i+}, \lambda = 2000, \sigma = 1, \mu = 5, \nu = 40, k = 10 \].
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Figure 27. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] $[\sigma = 0.3, r = 10]$), (c) RTV [9] $[\lambda = 0.01, \sigma = 9]$, (d) RGF [11] $[\sigma_s = 5, \sigma_r = 0.05, k = 5]$, (e) ours $[\mu^0 = 1, \lambda = 100, \sigma = 2, \mu = 5, \nu = 40, k = 5]$. 
Figure 28. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 \([\sigma = 0.3, r = 10]\), (c) RTV \([\lambda = 0.01, \sigma = 3]\), (d) RGF \([\sigma_x = 3, \sigma_r = 0.05, k = 5]\), (e) ours \([u^1 = 1, \lambda = 50, \sigma = 1, \mu = 5, \nu = 40, k = 5]\).
Figure 29. Examples of the texture removal for irregular textures. (a) Input image, (b) Cov. M1 [4] \(\sigma = 0.3, r = 10\), (c) RTV [9] \(\lambda = 0.01, \sigma = 6\), (d) RGF [11] \(\sigma_s = 3, \sigma_r = 0.1, k = 5\), (e) ours \(u^0 = 1, \lambda = 50, \sigma = 2, \mu = 5, \nu = 40, k = 5\).
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(a) RGB image
(b) Flash NIR image
(c) GF [3]
(d) Ours
Figure 33. RGB and flash NIR image restoration on window sequence [10]: (a) RGB image, (b) NIR image, (c) GF [3] [$r = 3$, $\varepsilon = 0.0004$], (d) ours [$u^0 = 1$, $\lambda = 15$, $\mu = 60$, $\nu = 30$, $k = 5$].
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