Image Cosegmentation

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Image segmentation

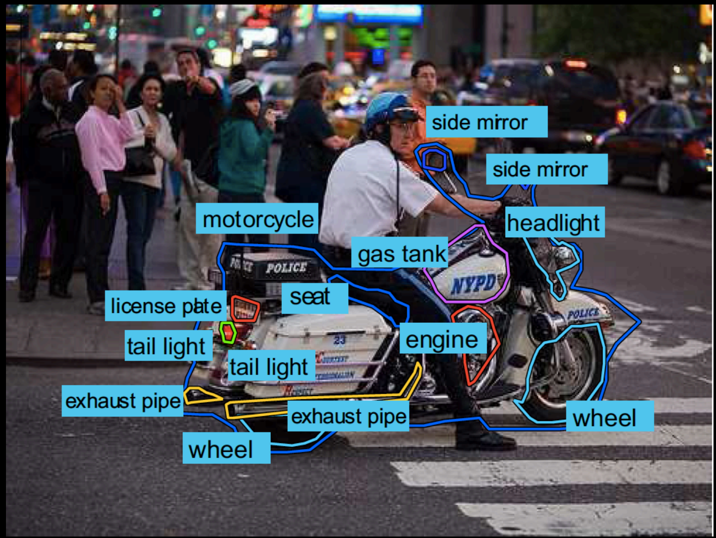
(Fowlkes & Malik, 2004)

Computer graphics applications

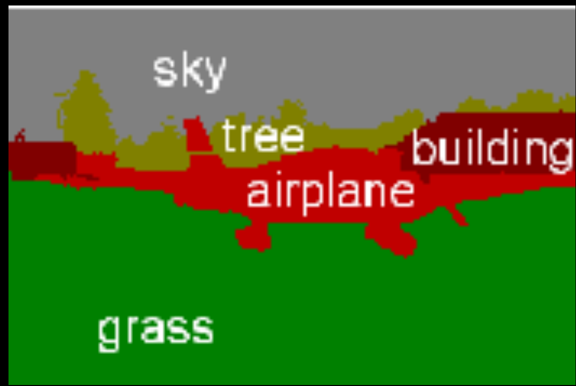
(Rhemann et al., CVPR'09)

(Rother et al., Siggraph'04)

Supervised segmentation (scene labelling)

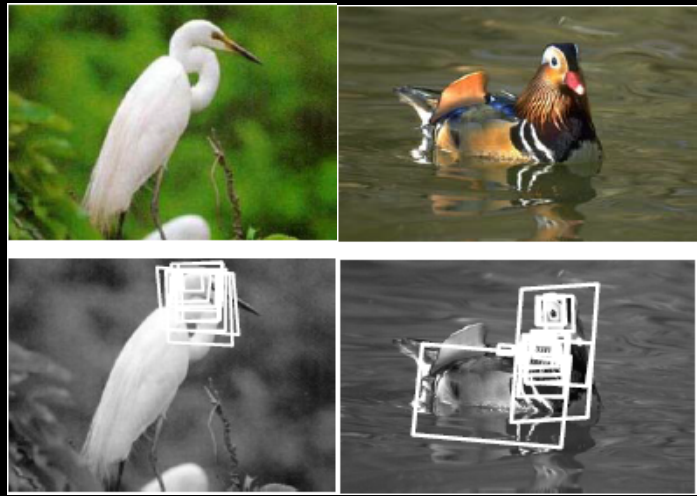


(Farhadi et al., CVPR'10)

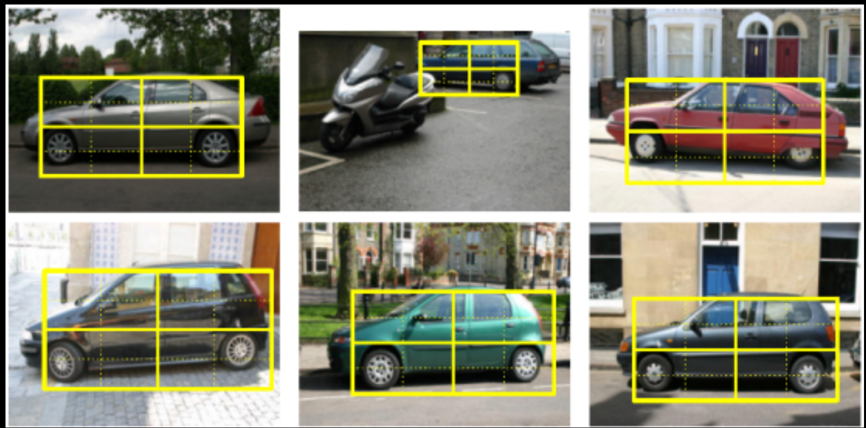


(Ladicki et al., ECCV'10)

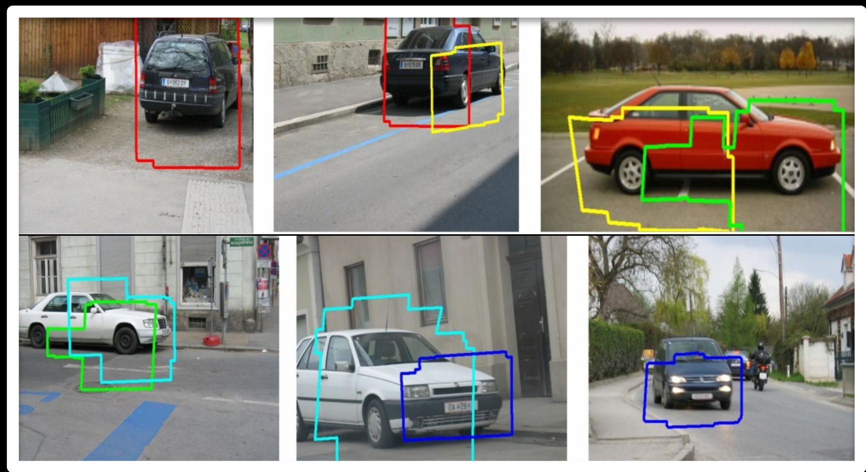
Weakly supervised learning for object recognition



(Lazebnik, Schmid, Ponce, ICCV'05)

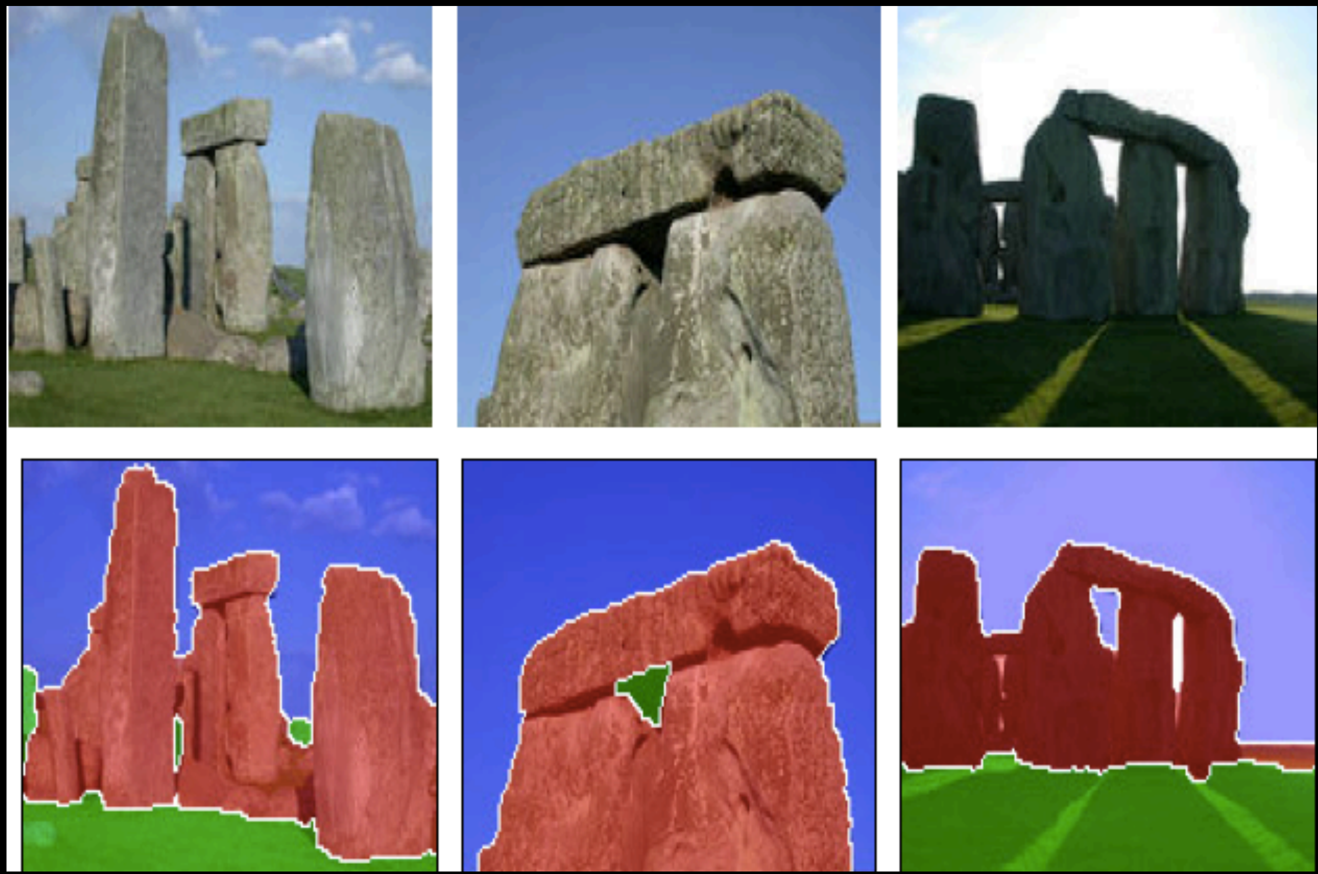


(Chum & Zisserman, CVPR'07)



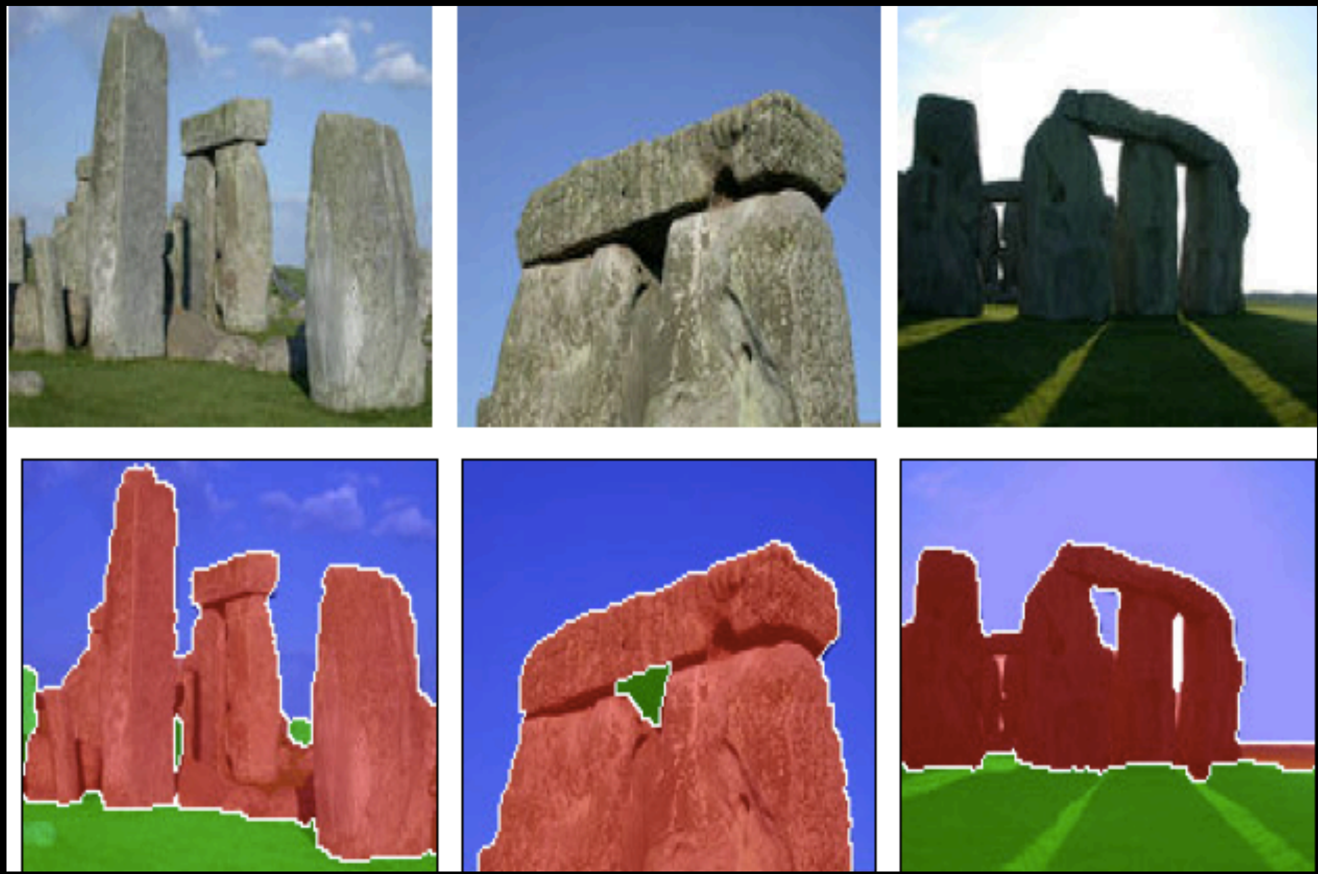
(Kushal, Schmid, Ponce, CVPR'07)

Cosegmentation



Definition: Divide a set of images assumed to contain K « object » classes into visually consistent regions while maximizing class separability across images.

Cosegmentation



Definition: Divide a set of images assumed to contain the same « foreground objects » into foreground and background regions.
(Rother, Kolgomorov, Minka, Blake, CVPR'06)

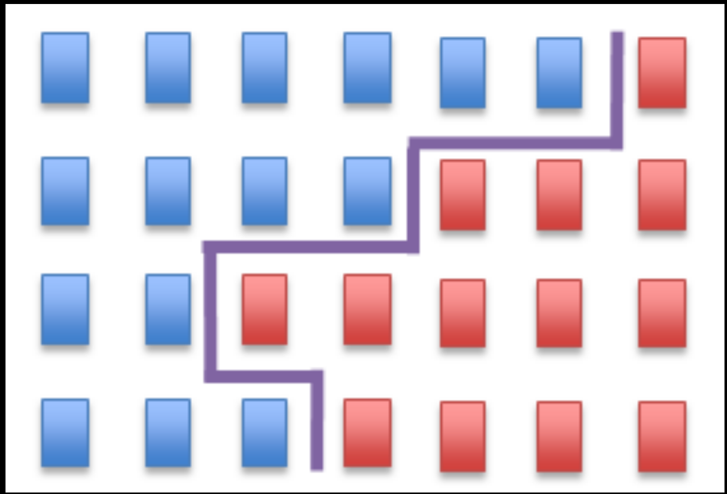
Related work

- Rother, Kolgomorov, Minka, Blake (CVPR'06)
- Hochbaum, Singh (ICCV'09)
- Vicente, Kolgomorov, Rother (ECCV'10)
- Vicente, Rother, Kolgomorov (CVPR'11)
- Kim, Xing, Fei-Fei, Kanade (ICCV'11)
- Mukherjee, Singh, Peng (CVPR'11)
- Chai, Rahtu, Lempisky, van Gool, Zisserman (ECCV'12)

- Duchenne, Laptev, Sivic, Bach, Ponce (ICCV'09)
- Joulin, Bach, Ponce (CVPR'10)
- Joulin, Bach, Ponce (CVPR'12)

- Xu, Neufeld, Larson, Schurrmans (NIPS'05)
- Bach & Harchaoui (NIPS'07)

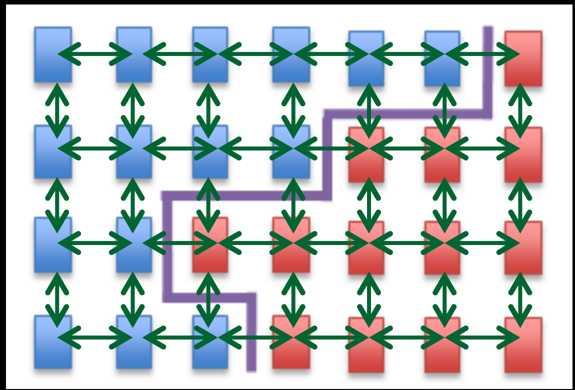
Notation



or superpixels

- Each image i is reduced to a subsampled grid of pixels.
- For the n-th pixel, we denote by:
  - x_n its d-dimensional feature vector.
  - y_n the K-vector such as $y_{nk} = 1$ if the n-th pixel is in the k-class and 0 otherwise.

Normalized cuts



$$\min \sum_{i=1}^{k} \frac{\mathrm{cut}(A_i, \overline{A}_i)}{\mathrm{vol}(A_i)}$$

Similarity matrix $$W_{nm} = \exp(-\lambda_p \|p_n - p_m\|_2^2 - \lambda_c \|c_n - c_m\|^2)$$

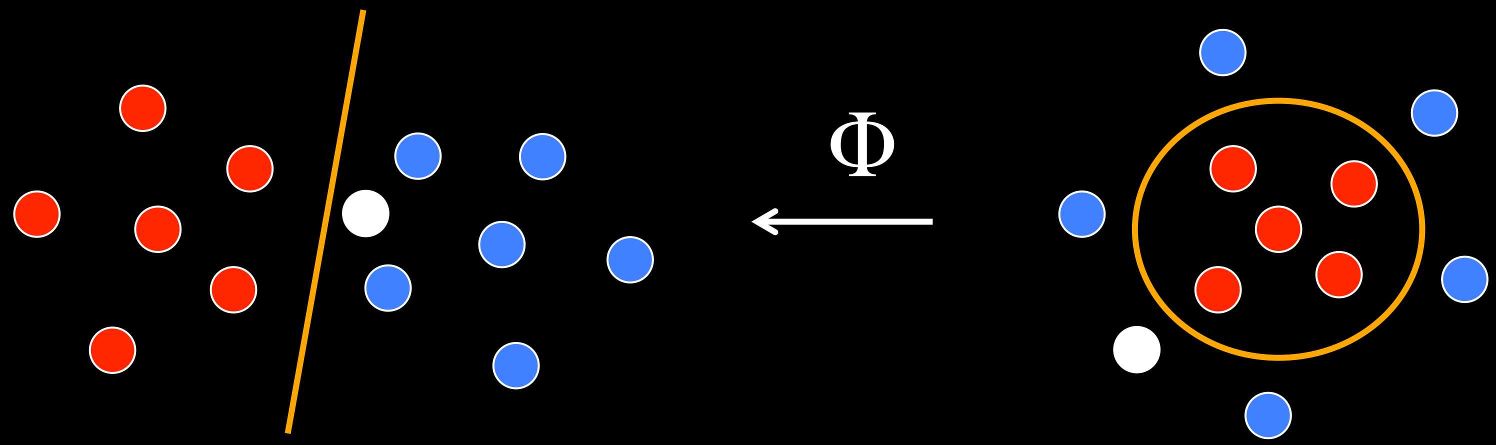
Laplacian matrix $$L = I - D^{-1/2} W D^{-1/2}$$

$$\min_{\substack{y \in \{0,1\}^{N \times K} \\ y 1_K = 1_N \\ y^\top D^{1/2} 1_n = 0}} \mathrm{tr}(y^T L y)$$

- Solve the relaxed version as an eigenvalue problem.
- Round up the solution using k-means

(Shi & Malik'97, Ng et al.'01, Arbelaez et al.'11, von Luxburg'07)

Supervised classification

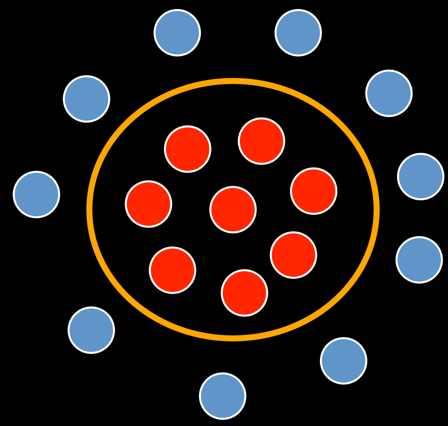


$$\min_{\substack{w \in \mathbb{R}^{K \times d},\\ b \in \mathbb{R}^{K}}} \frac{1}{N} \sum_{n=1}^{N} \ell(y_n, w\phi(x_n) + b) + \frac{\lambda}{2K} \|w\|_F^2$$

$$\Rightarrow \quad k(x, y) = \Phi(x) . \Phi(y)$$

(Schölkopf & Smola, 2001; Shawe-Taylor & Cristianini, 2004; Wahba, 1990)

Discriminative clustering



$$\min_{\substack{w \in \mathbb{R}^{K\times d},\\ b \in \mathbb{R}^K \\ y \in \{0,1\}^{N\times K} \\ y 1_K = 1_N}} \frac{1}{N}\sum_{n=1}^{N} \ell(y_n, w\phi(x_n) + b) + \frac{\lambda}{2K}\|w\|_F^2$$

(Xu et al., 2004; de Bie & Cristianini, 2006; Bach & Harchaoui, 2007)

Discriminative clustering: DIFFRAC

When using the square loss

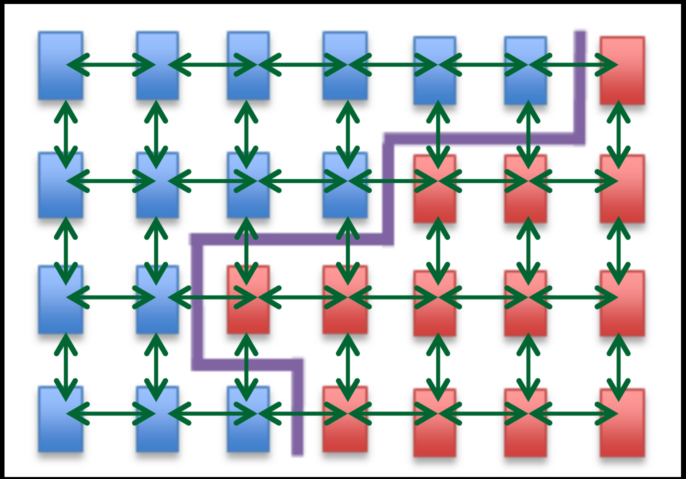
$$\min_{\substack{w \in \mathbb{R}^{K \times d},\\ b \in \mathbb{R}^K\\ y \in \{0,1\}^{N \times K}\\ y 1_K = 1_N}} \frac{1}{N} \sum_{n=1}^{N} \ell(y_n, w\phi(x_n) + b) + \frac{\lambda}{2K} \|w\|_F^2$$

$$\min_{\substack{y \in \{0,1\}^{N \times K}\\ y 1_K = 1_N}} \operatorname{tr}(y^T C y)$$

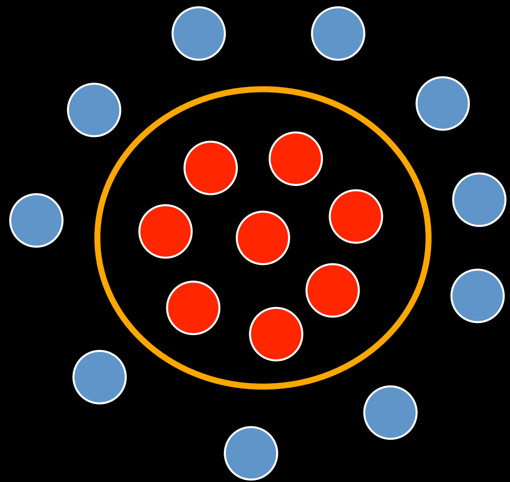
with $C = \lambda(I_N - \frac{1}{N} 1_N 1_N^T)(N\lambda I_N + \mathbf{K})^{-1}(I_N - \frac{1}{N} 1_N 1_N^T)$

(Bach & Harchaoui, NIPS'07)

Binary cosegmentation



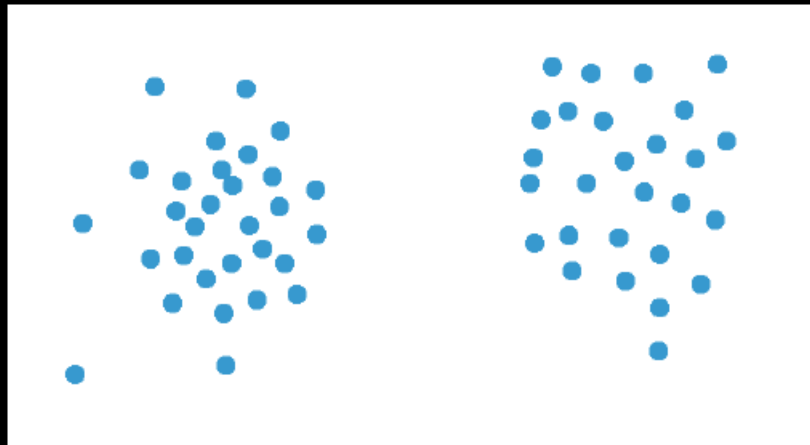
$$E_B(y) \quad = \quad \frac{\mu}{N}\mathrm{tr}(y^T L y)$$



$$E_U(y) = \mathrm{tr}(y^T C y)$$

(Joulin, Bach, Ponce, CVPR'10)

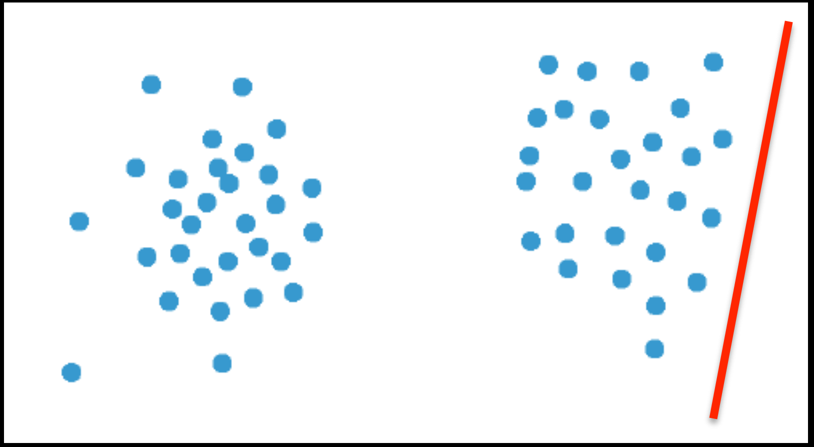
Cluster size constraints



$$\min_{\substack{y \in \{0,1\}^{N \times K}, \\ y 1_K = 1_N}} \mathrm{tr}(y^T C y)$$

(K=2 here)

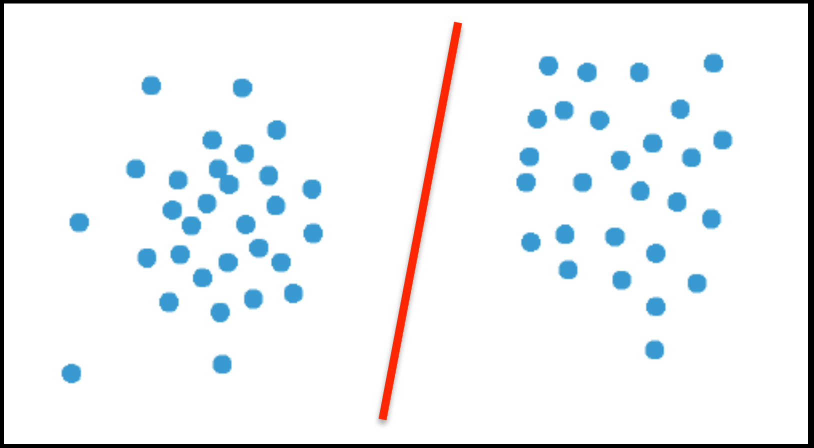
Cluster size constraints



$$\min_{\substack{y \in \{0,1\}^{N \times K}, \\ y 1_K = 1_N}} \operatorname{tr}(y^T C y)$$

(K=2 here)

Cluster size constraints



$$\min_{\substack{y \in \{0,1\}^{N \times K}, \\ y 1_K = 1_N}} \operatorname{tr}(y^T C y)$$

(K=2 here)

under the constraint:

$$\lambda_0 (\delta_i^\top \delta_i) 1_N \leqslant y y^\top \delta_i \leqslant \lambda_1 (\delta_i^\top \delta_i) 1_N$$

where:

- $\lambda_0 = 0.05$ and $\lambda_1 = 0.95$,
- $\delta_i \in \mathbb{R}^n$: indicator vector of the i-th image

$$
\begin{aligned}
\min_{y \in \{0,1\}^{N \times K}} \quad & \operatorname{tr}\left(yy^T\left(C + \frac{\mu}{N}L\right)\right), \\
\text{subject to} \quad & \lambda_0(\delta_i^\top \delta_i)1_N \leqslant yy^\top \delta_i \leqslant \lambda_1(\delta_i^\top \delta_i)1_N
\end{aligned}
$$

$$\begin{array}{ll} \min_{y\in\{0,1\}^{N\times K}} & \mathrm{tr}(yy^T(C+\frac{\mu}{N}L)), \\ \text{subject to} & \lambda_0(\delta_i^\top\delta_i)1_N \leqslant yy^\top\delta_i \leqslant \lambda_1(\delta_i^\top\delta_i)1_N \end{array}$$

Reparameterize by equivalence matrix $Y=yy^T$
to obtain an equivalent continuous problem:

$$\begin{array}{ll} \min_{Y\in\mathcal{E}} & \mathrm{tr}\Big(Y\big(C+\frac{\mu}{N}L\big)\Big), \\ \text{subject to} & \forall i,\ \lambda_0 1_N \leqslant Y\delta_i \leqslant \lambda_1 1_N \\ & \boxed{\mathrm{rank}(Y)=1}, \\ & Y\geq 0. \end{array}$$

makes Y binary

$$\mathcal{E}=\{Y\in\mathbb{R}^{N\times N}\ ,\ Y=Y^T\ ,\ \mathrm{diag}(Y)=1_N\ ,\ Y\succeq 0\}$$

$$\begin{aligned} \min_{y \in \{0,1\}^{N \times K}} \quad & \operatorname{tr}(yy^T(C + \frac{\mu}{N}L)), \\ \text{subject to} \quad & \lambda_0(\delta_i^\top \delta_i)1_N \leqslant yy^\top \delta_i \leqslant \lambda_1(\delta_i^\top \delta_i)1_N \end{aligned}$$

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$$\begin{aligned} \min_{Y \in \mathcal{E}} \quad & \operatorname{tr}\big(Y(C + \frac{\mu}{N}L)\big), \\ \text{subject to} \quad & \forall i,\ \lambda_0 1_N \leqslant Y\delta_i \leqslant \lambda_1 1_N \\ & \boxed{\operatorname{rank}(Y) = 1}, \\ & Y \geq 0. \end{aligned}$$

nonconvex!

$$\mathcal{E} = \{Y \in \mathbb{R}^{N \times N},\ Y = Y^T,\ \operatorname{diag}(Y) = 1_N,\ Y \succeq 0\}$$

$$\min_{y\in\{0,1\}^{N\times K}} \quad \mathrm{tr}(yy^T(C+\frac{\mu}{N}L)),$$
$$\text{subject to} \quad \lambda_0(\delta_i^\top\delta_i)1_N \leqslant yy^\top\delta_i \leqslant \lambda_1(\delta_i^\top\delta_i)1_N$$

Reparameterize by equivalence matrix $Y=yy^T$
to obtain an equivalent continuous problem:

$$\min_{Y\in\mathcal{E}} \quad \mathrm{tr}\Big(Y(C+\frac{\mu}{N}L)\Big),$$
$$\text{subject to} \quad \forall i,\ \lambda_0 1_N \leqslant Y\delta_i \leqslant \lambda_1 1_N$$

$$\mathcal{E}=\{Y\in\mathbb{R}^{N\times N}\ ,\ Y=Y^T\ ,\ \mathrm{diag}(Y)=1_N\ ,\ Y\succeq 0\}$$

Dropping the rank constraint yields a convex problem
over positive semidefinite matrices, or SDP

$$\min_{y\in\{0,1\}^{N\times K}} \quad \operatorname{tr}(yy^T(C+\frac{\mu}{N}L)),$$
$$\text{subject to} \quad \lambda_0(\delta_i^\top\delta_i)1_N \leqslant yy^\top\delta_i \leqslant \lambda_1(\delta_i^\top\delta_i)1_N$$

Reparameterize by equivalence matrix Y=yy^T
to obtain an equivalent continuous problem:

$$\min_{Y\in\mathcal{E}} \quad \operatorname{tr}\big(Y(C+\frac{\mu}{N}L)\big),$$
$$\text{subject to} \quad \forall i,\ \lambda_0 1_N \leqslant Y\delta_i \leqslant \lambda_1 1_N$$

$$\mathcal{E} = \{Y \in \mathbb{R}^{N\times N},\ Y = Y^T,\ \operatorname{diag}(Y) = 1_N,\ Y \succeq 0\}$$

- General purpose toolboxes would solve this problem in $O(N^7)$
- Bach and Harchaoui (NIPS 2007): $O(N^3)$.
- Modified version of Journée et al. (2008): $O(N^2)$.

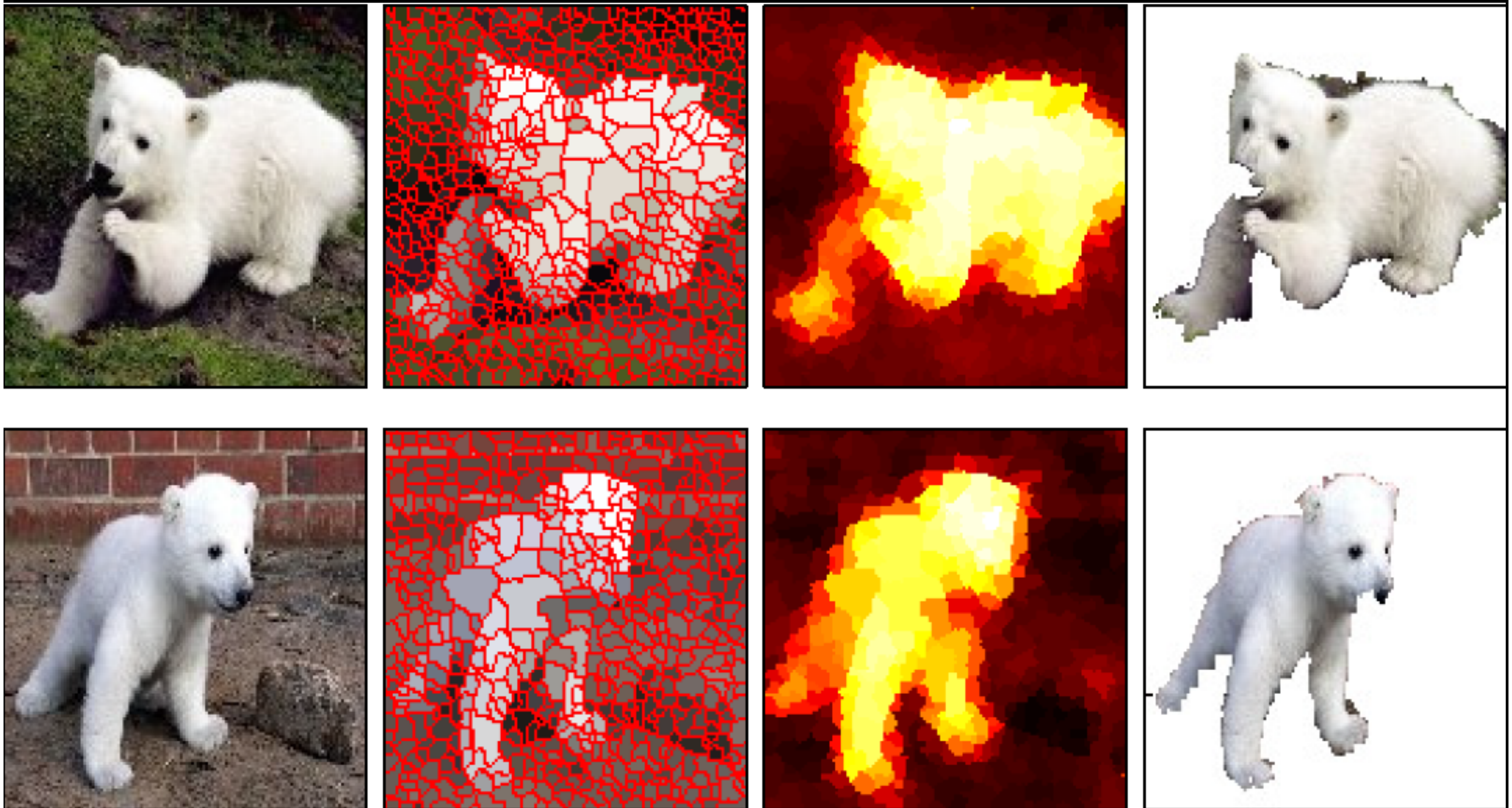
$$\min_{y \in \{0,1\}^{N\times K}} \quad \mathrm{tr}(yy^T(C + \frac{\mu}{N}L)),$$
$$\text{subject to} \quad \lambda_0(\delta_i^\top \delta_i)1_N \leqslant yy^\top \delta_i \leqslant \lambda_1(\delta_i^\top \delta_i)1_N$$

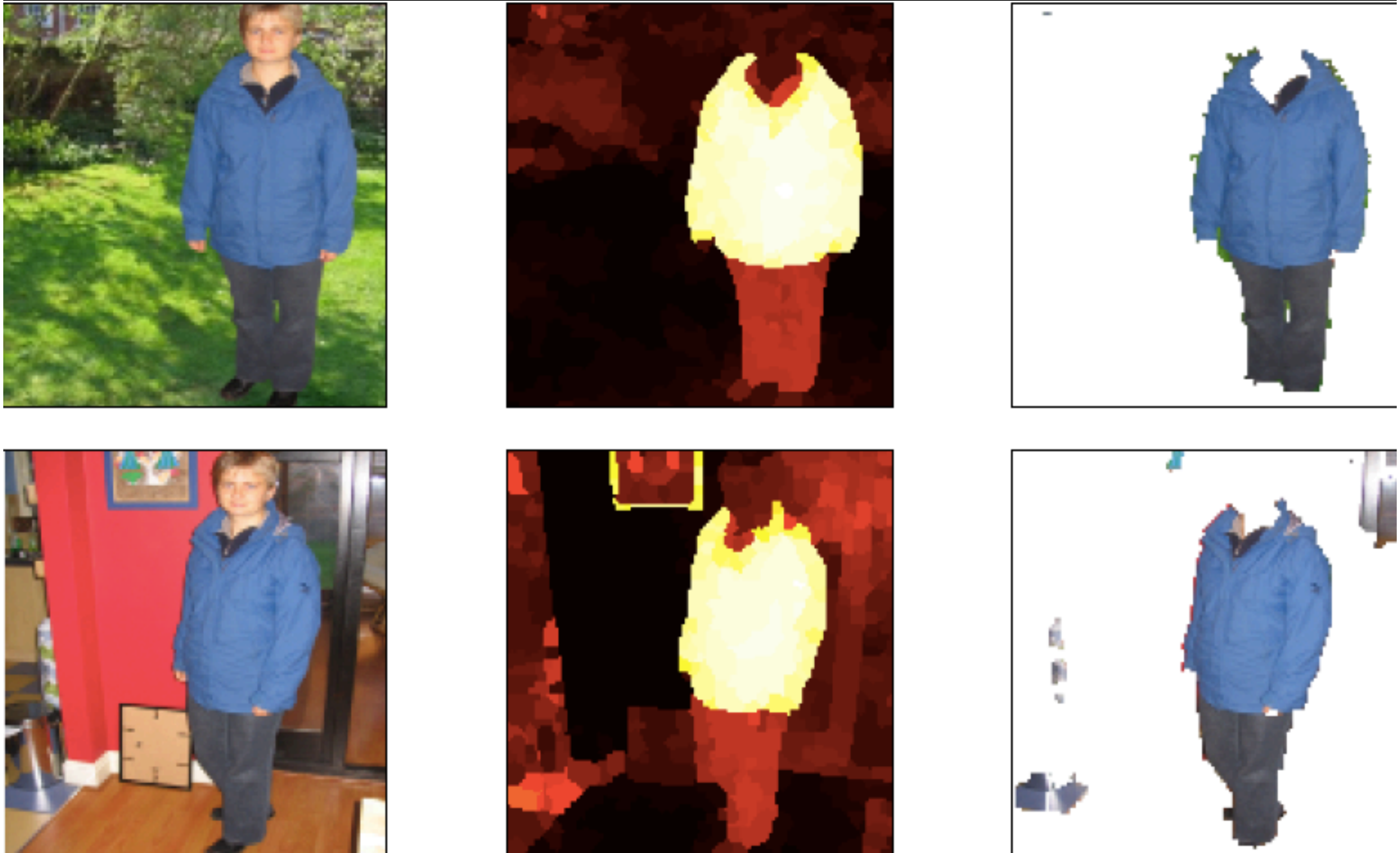
Reparameterize by equivalence matrix $Y=yy^T$ to obtain an equivalent continuous problem:

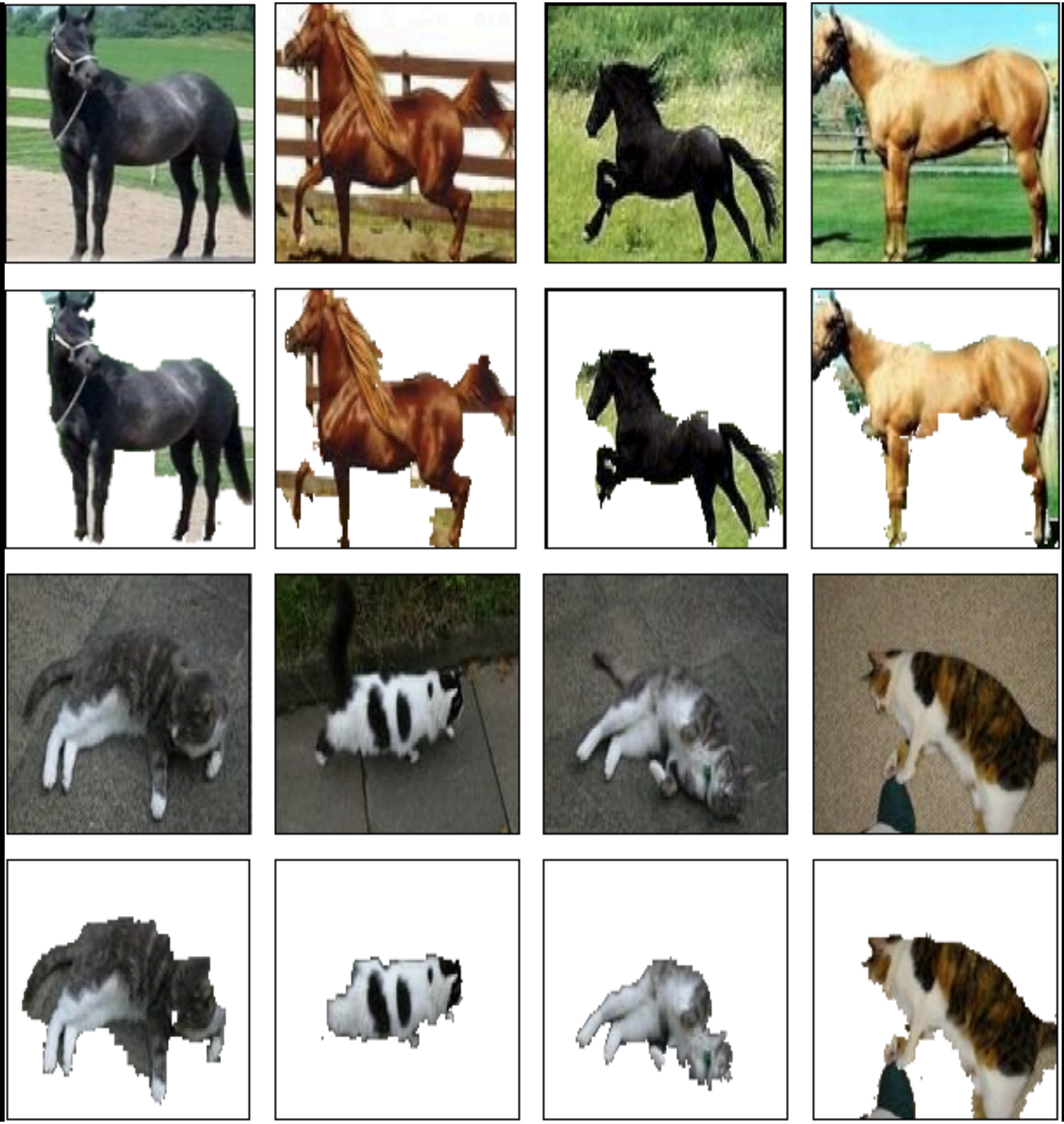
$$\min_{Y \in \mathcal{E}} \quad \mathrm{tr}\big(Y(C + \frac{\mu}{N}L)\big),$$
$$\text{subject to} \quad \forall i,\ \lambda_0 1_N \leqslant Y\delta_i \leqslant \lambda_1 1_N$$

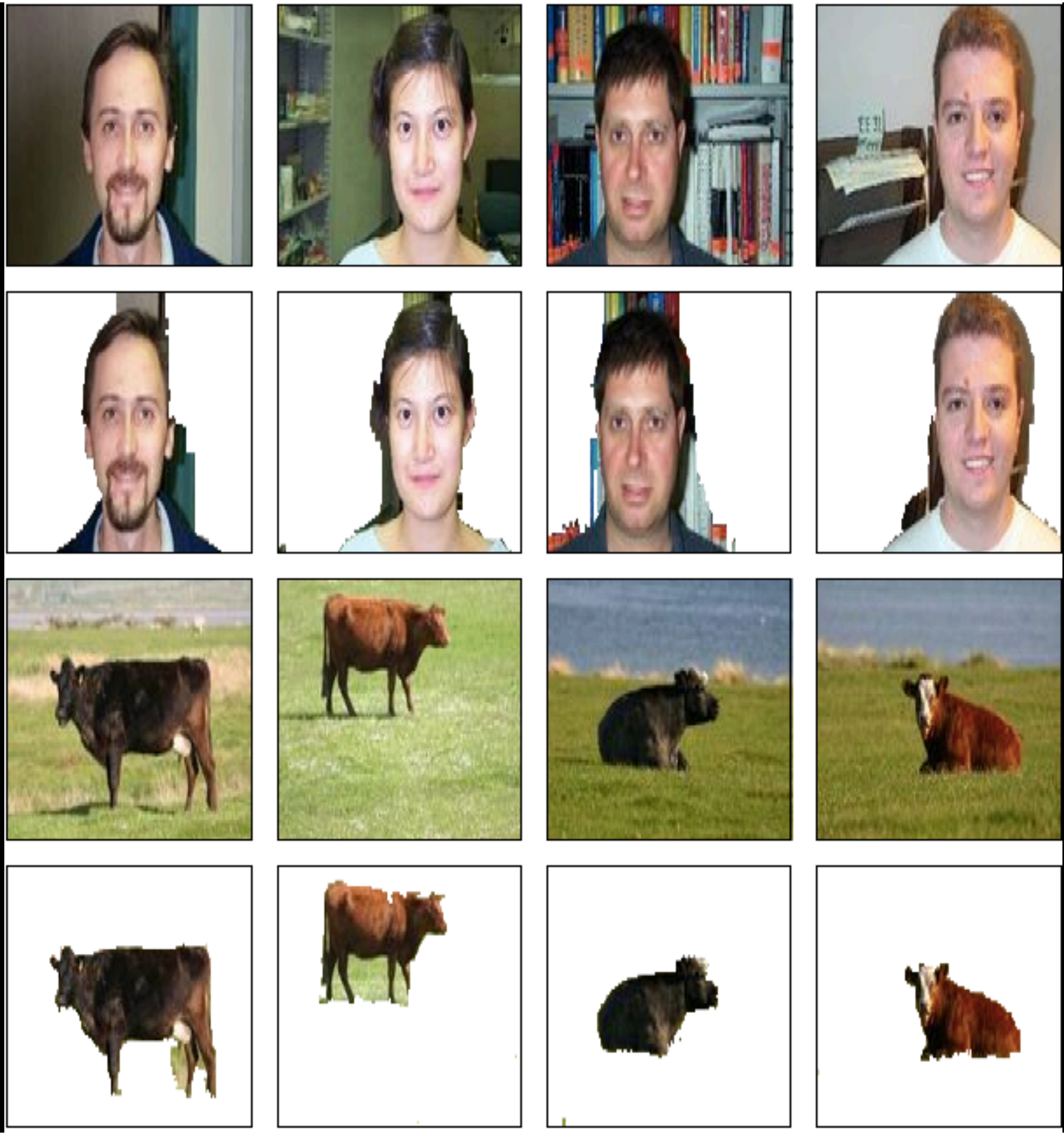
$$\mathcal{E} = \{Y \in \mathbb{R}^{N\times N}\ ,\ Y = Y^T\ ,\ \mathrm{diag}(Y) = 1_N\ ,\ Y \succeq 0\}$$

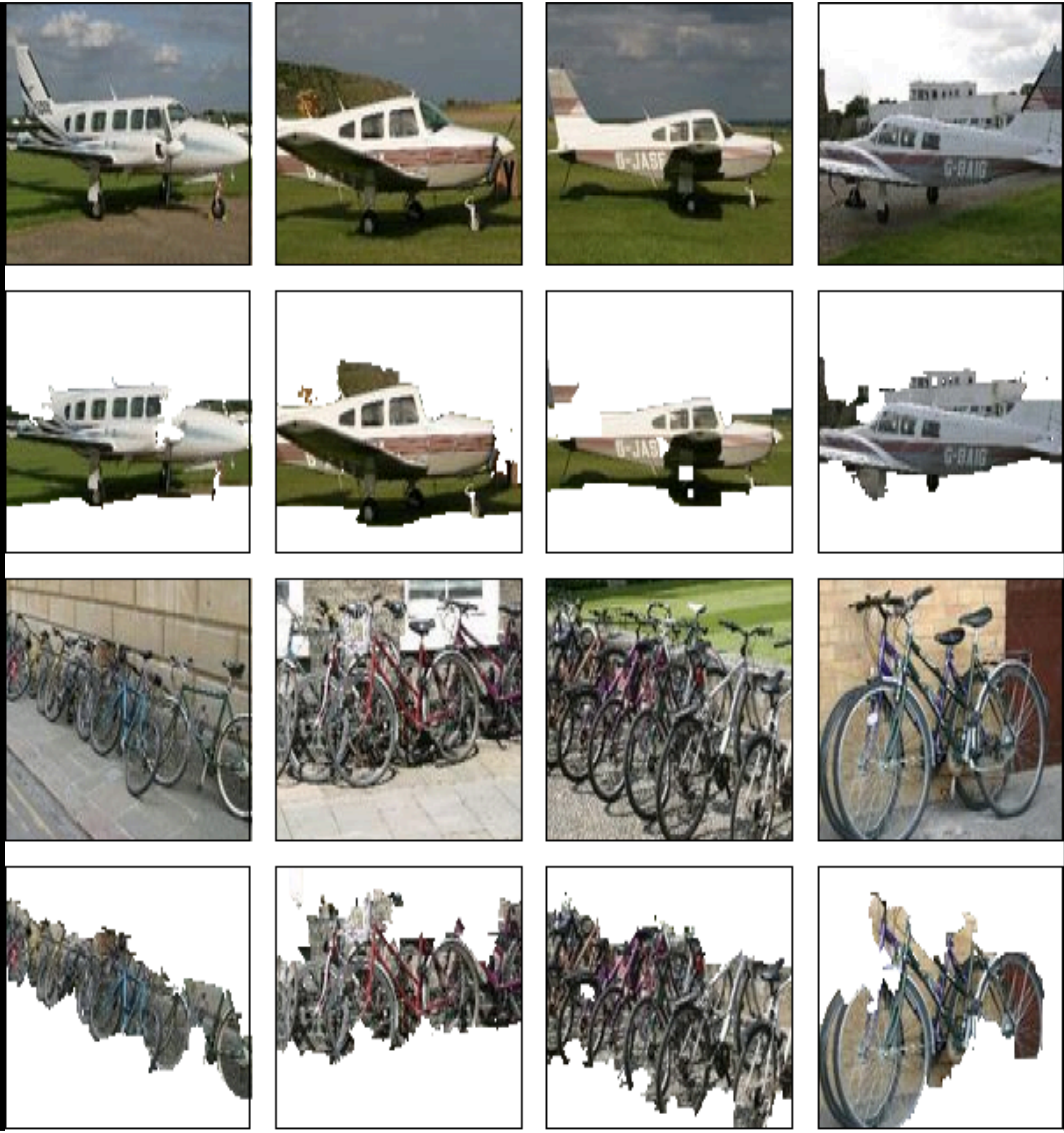
- Low-rank optimization on quotient manifold (Journée et al.'08)
- Eigendecomposition to project onto rank-1 solution
- Rounding by thresholding a 0
- Graph cuts to clean up the result










G-BAIG
G-BAIG

From two to multiple classes

Optimization problem

$$\min_{\substack{y\in\{0,1\}^{N\times K},\\ y1_K=1_N}} \left[\min_{\substack{w\in\mathbb{R}^{d\times K},\\ b\in\mathbb{R}^K}} E_U(y,w,b) \right] + E_B(y) - H(y)$$

- Discriminative term with softmax loss

$$E_u(y) = -\sum_{n=1}^{N}\sum_{p=1}^{K} y_{np} \log\left(\frac{\exp(w_p^T\phi(x_n)+b_p)}{\sum_{k=1}^{K}\exp(w_k^T\phi(x_n)+b_k)}\right) + \frac{\lambda}{2P}\|w\|_F^2$$

- Spectral clustering grouping term

$$E_B(y) = \mathrm{tr}(y^T L y)$$

- Class balancing entropy term

$$H(y) = -\sum_{i\in\mathcal{I}}\sum_{k=1}^{K}\left(\frac{1}{N}\sum_{n\in\mathcal{N}_i} y_{nk}\right)\log\left(\frac{1}{N}\sum_{n\in\mathcal{N}_i} y_{nk}\right)$$

Optimization:

- Relax to a nonconvex continuous problem
- Initialize with quadratic approximation
- EM/block-coordinate descent procedure with quasi-Newton and projected gradient descent for the two convex steps
- Round up the solution

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Initialization: Use a quadratic Taylor expansion in the neighborhood of uniform class distribution

$$\frac{K}{2}\left[\mathrm{tr}(yy^T C) + \frac{2\mu}{NK}\mathrm{tr}(yy^T L) - \frac{1}{N}\mathrm{tr}(yy^T \Pi_I)\right]$$

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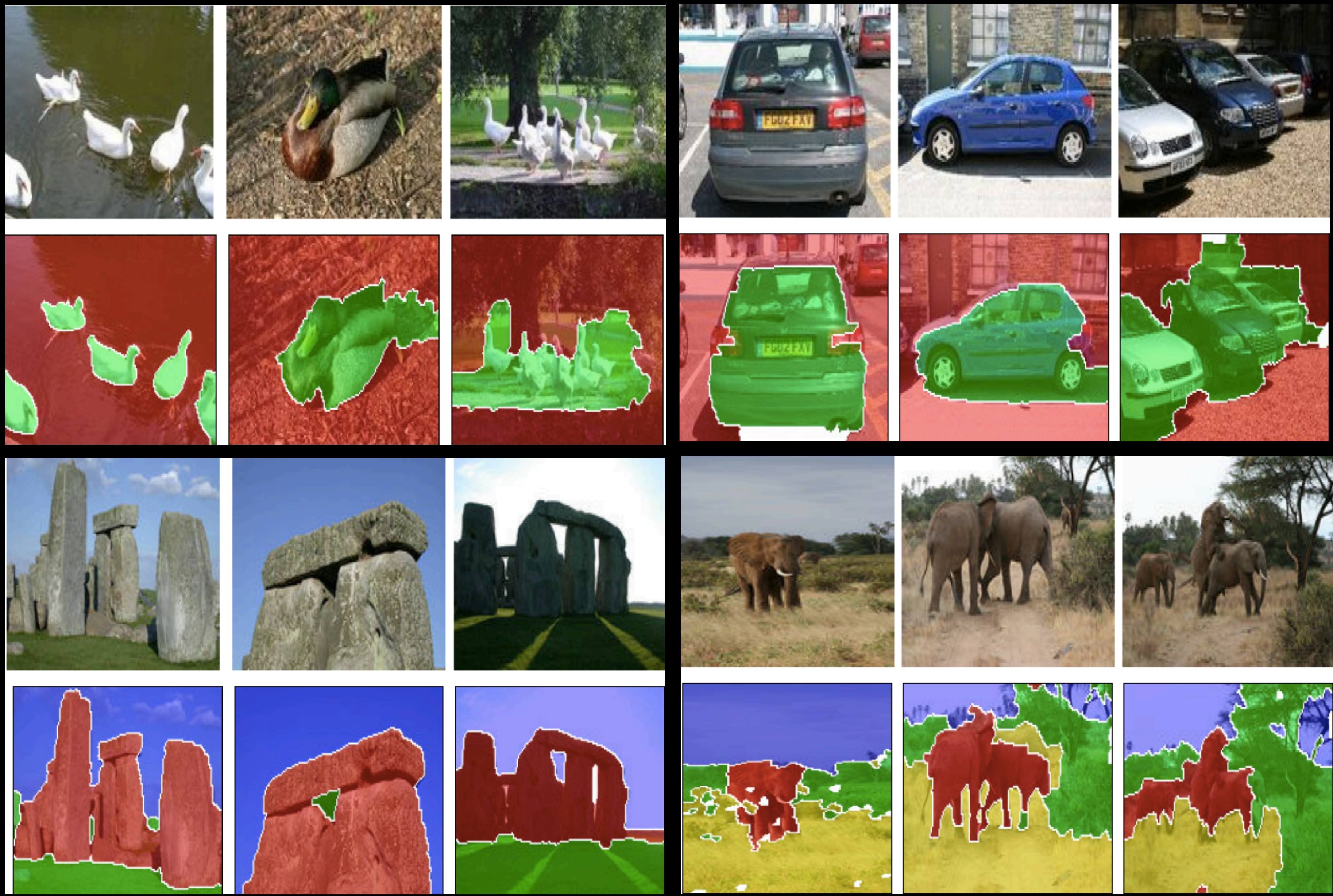
$$\frac{K}{2}\left[\mathrm{tr}(yy^T C) + \frac{2\mu}{NK}\mathrm{tr}(yy^T L) - \frac{1}{N}\cancel{\mathrm{tr}(yy^T \Pi_I)}\right]$$

$$\sum_{n \in \mathcal{N}_i} y_{nk} \leq 0.9 N_i \; ; \; \sum_{j \in \mathcal{I} \setminus i} \sum_{n \in \mathcal{N}_j} y_{nk} \geq 0.1(N - N_i)$$

Some examples



Failure cases

Evaluation

Binary evaluation: MSRC

images	class	Ours	[8]	[5]	[7]
30	Bike	**43.3**	29.9	42.3	42.8
30	Bird	**47.7**	29.9	33.2	-
30	Car	**59.7**	37.1	59.0	52.5
24	Cat	**31.9**	24.4	30.1	5.6
30	Chair	**39.6**	28.7	37.6	39.4
30	Cow	**52.7**	33.5	45.0	26.1
26	Dog	**41.8**	33.0	41.3	-
30	Face	**70.0**	33.2	66.2	40.8
30	Flower	**51.9**	40.2	50.9	-
30	House	51.0	32.2	50.5	**66.4**
30	Plane	21.6	25.1	21.7	**33.4**
30	Sheep	**66.3**	60.8	60.4	45.7
30	Sign	**58.9**	43.2	55.2	-
30	Tree	**67.0**	61.2	60.0	55.9
	Average	**50.2**	36.6	46.7	40.9

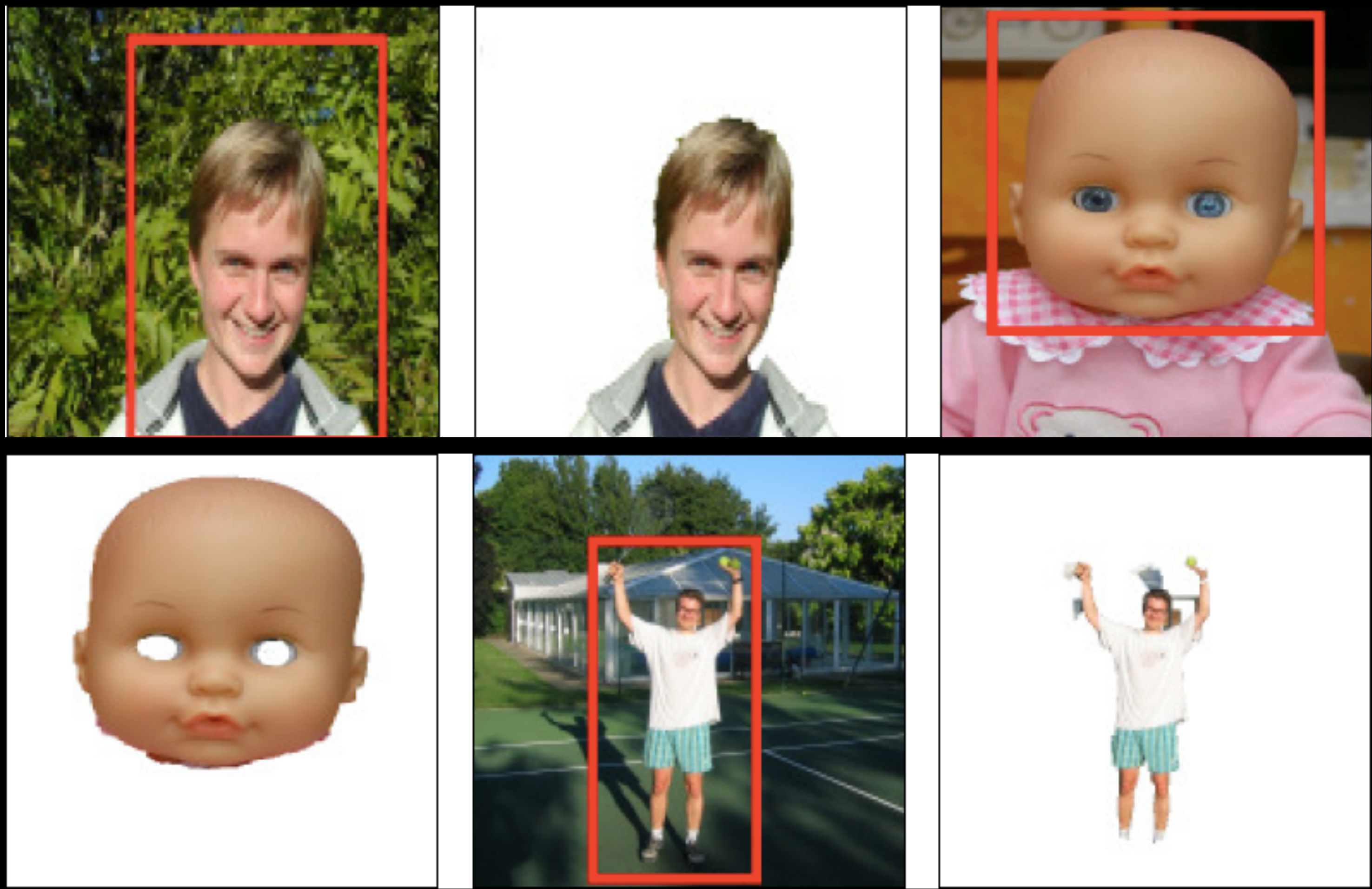
- Intersection over union score
- Evaluated on the main object class
- Matlab, 30mn-1hr for 30 images

[5] Joulin et al. (CVPR'10)
[7] Mukherjee et al. (CVPR'11)
[8] Kim et al. (ICCV'11)

Multi-class evaluation

dataset	images	class	K	Ours	multiclass Joulin et al. [5]	Kim et al. [8]	Joulin et al. [5]
iCoseg	25	Baseball player	5	**62.2**	53.5	51.1	24.9
	5	Brown bear	3	75.6	**78.5**	40.4	28.8
	15	Elephant	4	**65.5**	51.2	43.5	23.8
	11	Ferrari	4	**65.2**	63.2	60.5	48.8
	33	Football player	5	**51.1**	38.8	38.3	20.8
	7	Kite Panda	2	57.8	58.0	**66.2**	58.0
	17	Monk	2	**77.6**	76.9	71.3	76.9
	11	Panda	3	**55.9**	49.1	39.4	43.5
	11	Skating	2	**64.0**	47.2	51.1	47.2
	18	Stonehedge	3	**86.3**	85.4	64.6	62.3
MSRC	30	Plane	3	**45.8**	39.2	25.2	25.1
	30	Face	3	**70.5**	56.4	33.2	66.2
		Average		**64.8**	58.1	48.7	43.9

Extension: Interactive cosegmentation



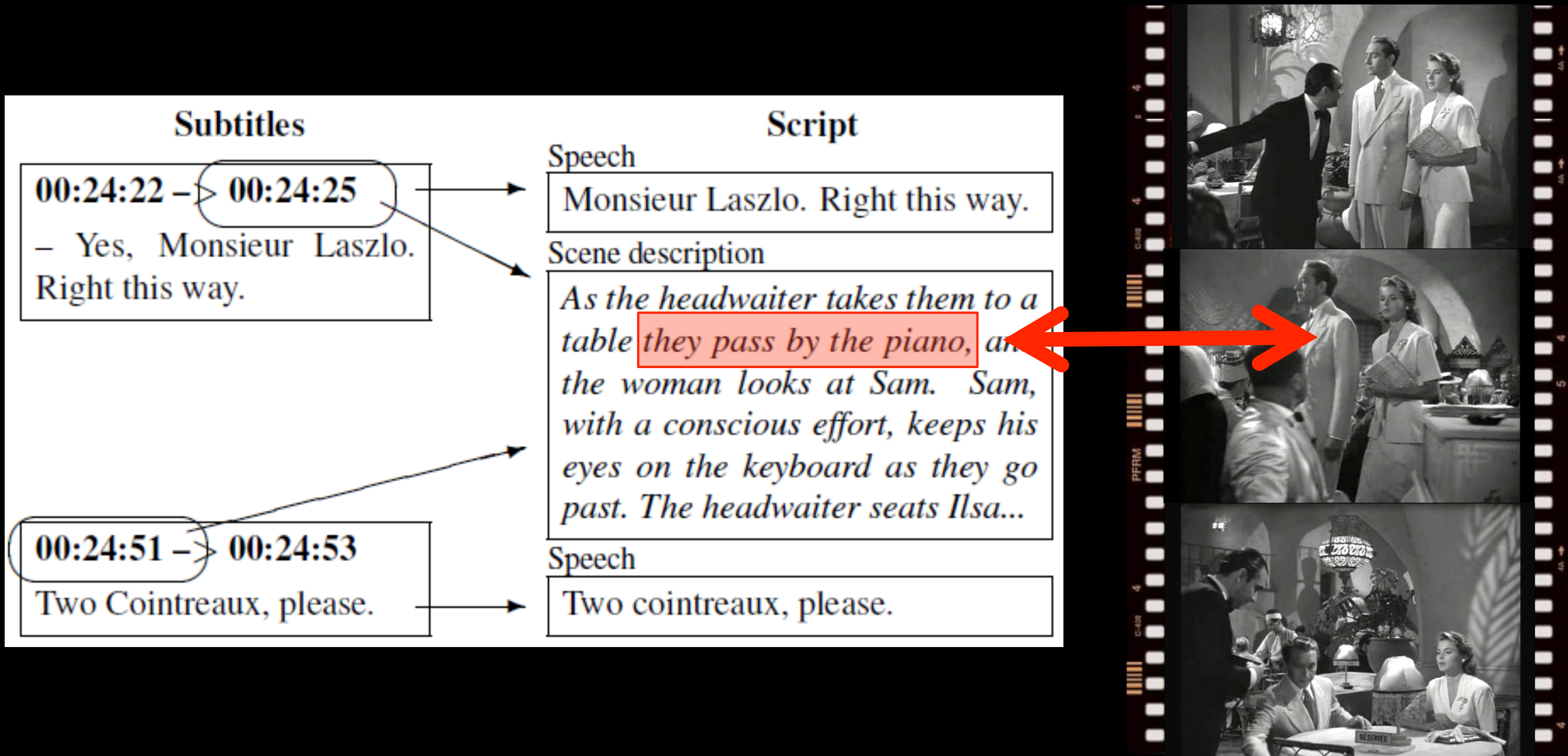
Use entropy term to distribute pixels to FG, BG in the box, and BG outside

Cosegmentation of a video shot

Weak supervision is the rule for video

(Sivic, Everingham. Zisserman, CVPR'09)

Video and text



(Duchenne, Bach, Laptev, Sivic, Ponce, ICCV 2009)

Discriminative clustering for temporal action localization

(Duchenne, Laptev, Sivic, Bach, Ponce, ICCV'09)

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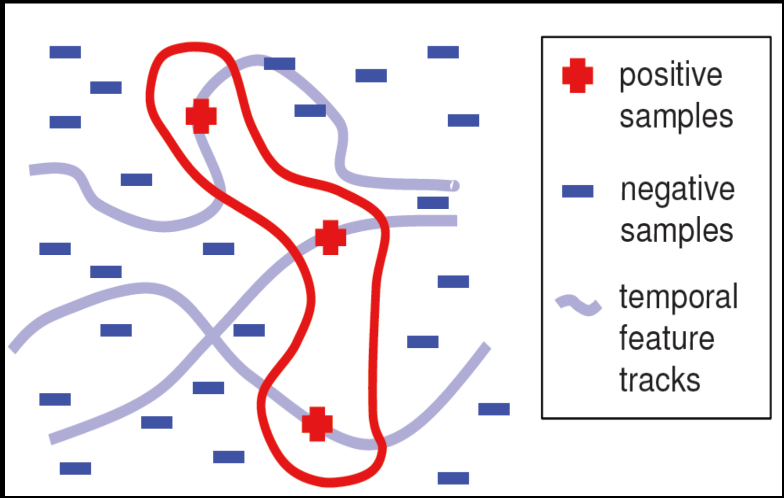


$$\min_{\substack{w \in \mathbb{R}^{K\times d},\\ b\in\mathbb{R}^K\\ y\in\{0,1\}^{N\times K}\\ y1_K=1_N}} \frac{1}{N}\sum_{n=1}^{N} \ell(y_n, w\phi(x_n)+b) + \frac{\lambda}{2K}\|w\|_F^2$$

Optimization:

- Negatives are fixed, random video intervals.
- Block-coordinate descent, alternating between training an SVM with positive intervals fixed, and computing the optimal positive intervals given the SVM parameters.

(Duchenne, Laptev, Sivic, Bach, Ponce, ICCV'09)

Automatic Annotation of Human Actions in Video

ICCV 2009 DEMO

O.Duchenne, I.Laptev, J.Sivic, F.Bach and J.Ponce

Temporal detection of actions OpenDoor and SitDown in episodes of
The Graduate, The Crying Game, Living in Oblivion

CAUSE MOTION

He throws his napkin on the table , gets up and leaves the room .

Bobby hurls the man onto the ground and turns around to go after the second man , struggling with Elton .

CHANGE POSTURE

Paul sits down besides them .

MANIPULATION

He and Rick grab the journals at the same time .

JOHN pulls a Polaroid from his pocket .

FRAMENET frames found by SEMAFOR

PERCEPTION EXPERIENCE

He 's seen someone he recognises .

He can hear music inside .

https://framenet.icsi.berkeley.edu/
http://code.google.com/p/semafor-semantic-parser/

SELF MOTION

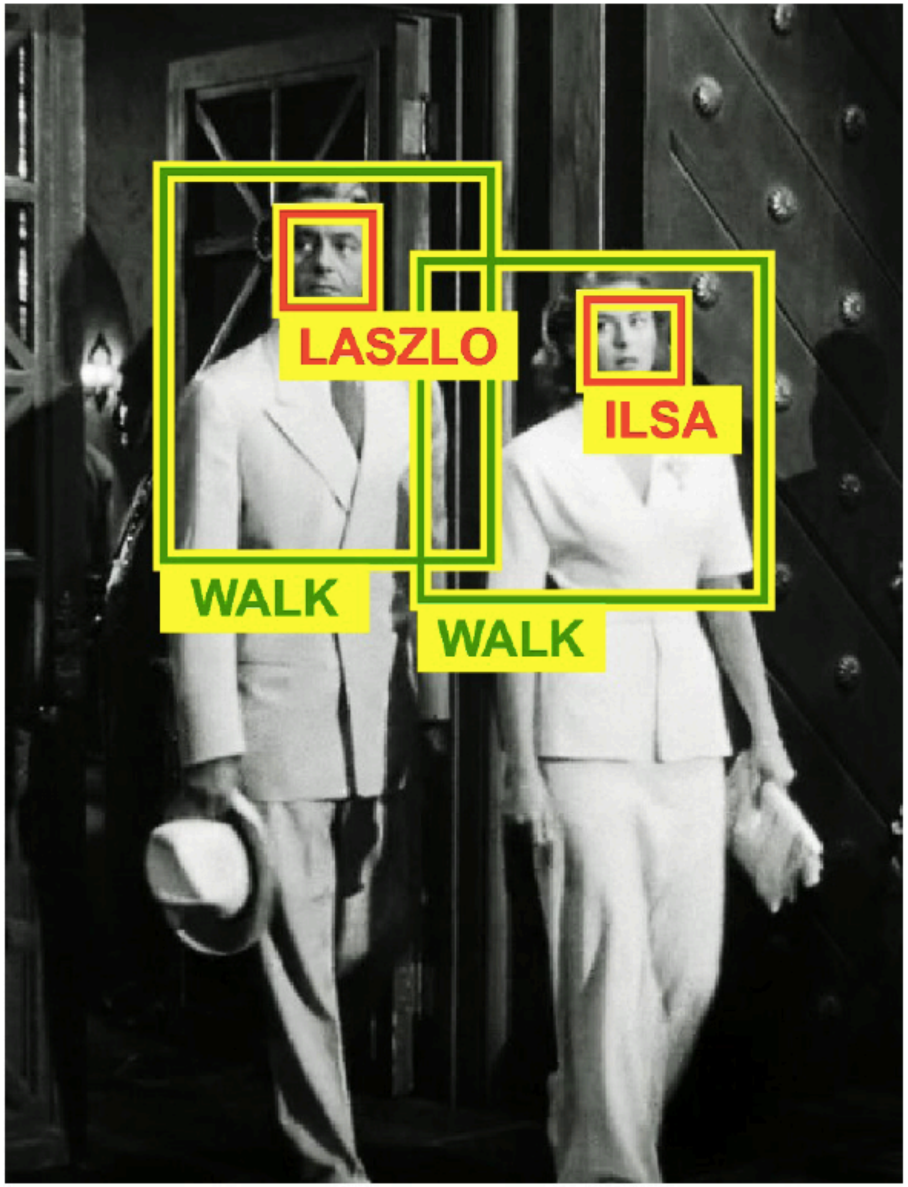
Dan walks along the walkway and Bubba and Forrest follow .

They walk up the block with the dog .

HAGEN has hurried into the Den to get the phone ; the OTHERS move in .

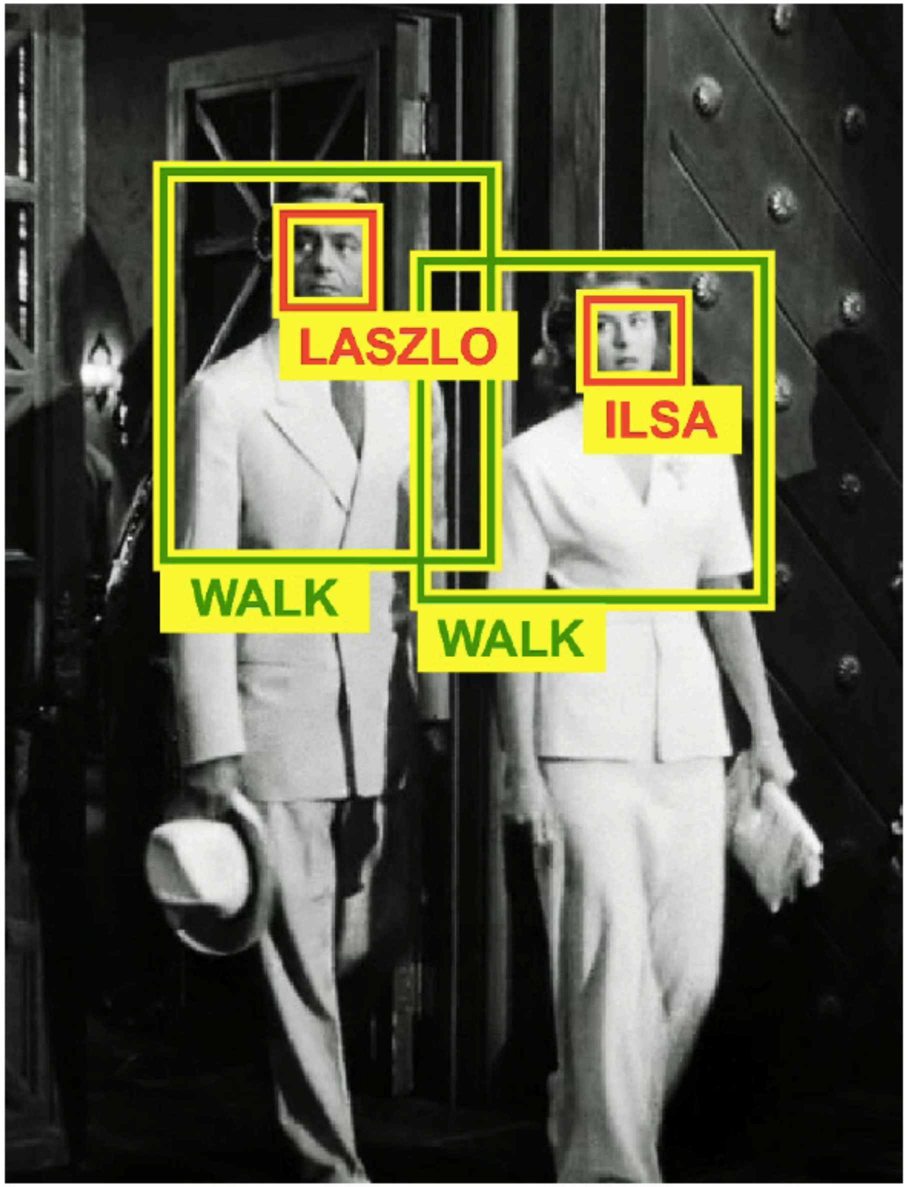
Then , as viewed from his position at the cabin door , Ellie appears standing on the rail ; and with a professional dive , she leaps into the water .

Can we identify characters and what they do?



(Bojanowski, Bach, Laptev, Ponce, Schmid, Sivic, 2013)

This is a structured cosegmentation problem



(Bojanowski, Bach, Laptev, Ponce, Schmid, Sivic, 2013)

Conventional discriminative clustering (Bach & Harchaoui, 2007)

$$\min_{Z,f} \frac{1}{N} \sum_{i \in I} \sum_{n \in \mathcal{N}_i} \ell(z_n, f(\phi(x_n))) + \Omega(f)$$

$$\min_{Z,w,b} \frac{1}{N} \|Z - \phi(X)w - b\|_F^2 + \lambda \;\; Tr(w^T \; w)$$

$$\min_{Z} \; Tr(ZZ^T A(X, \lambda))$$

Two-class discriminative clustering

$$\min_{Z,T} \; Tr(ZZ^T A(X, \lambda_1)) + Tr(TT^T B(X, \lambda_2))$$

under the constraints

$$\forall i \in I \;, \; \forall (p,a) \in \Lambda_i \;, \; \sum_{n \in \mathcal{N}_i} z_{np} \, t_{na} \geq 1$$

$$\forall (p, \varnothing) \in \Lambda_i \;, \; \sum_{n \in \mathcal{N}_i} z_{np} \frac{1}{A} \geq 1$$

$$\forall (\varnothing, a) \in \Lambda_i \;, \; \sum_{n \in \mathcal{N}_i} \frac{1}{P} t_{na} \geq 1$$

Conventional discriminative clustering (Bach & Harchaoui, 2007)

$$\min_{Z,f} \frac{1}{N} \sum_{i \in I} \sum_{n \in \mathcal{N}_i} \ell(z_n, f(\phi(x_n))) + \Omega(f)$$

$$\min_{Z,w,b} \frac{1}{N} \|Z - \phi(X)w - b\|_F^2 + \lambda \ \ Tr(w^T \ w)$$

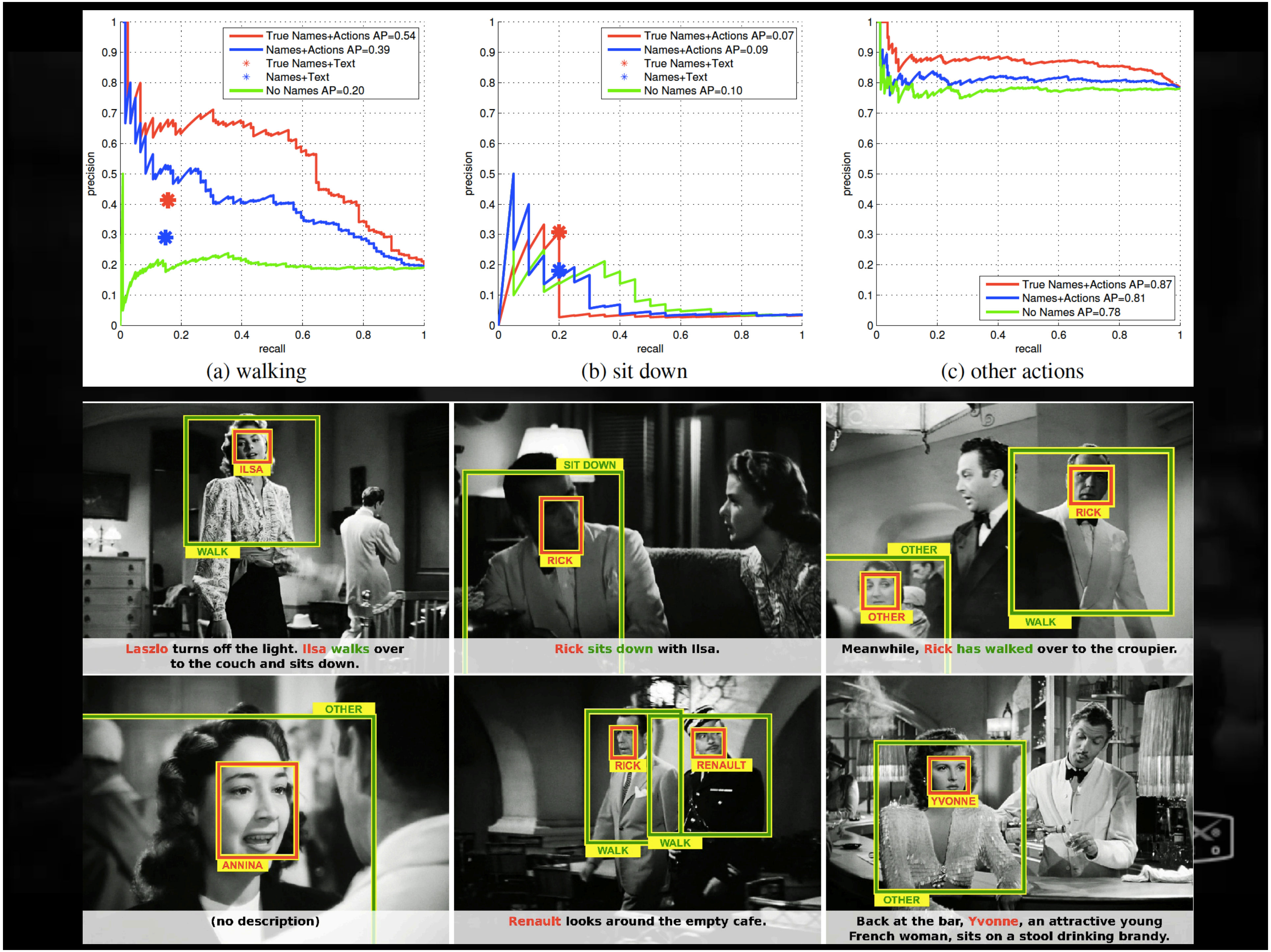
$$\min_{Z} \ Tr(ZZ^T A(X, \lambda \ ))$$

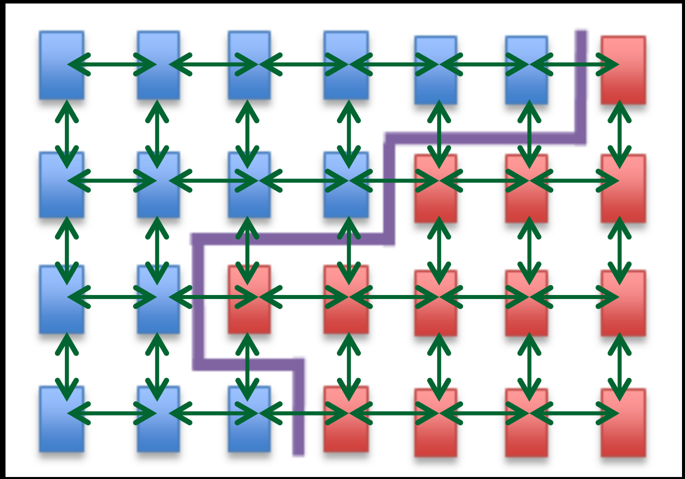
Two-class discriminative clustering

Optimization:

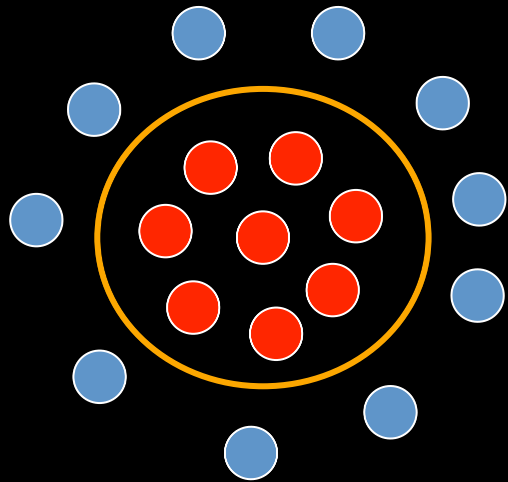
- Relax to continuous problem
- Block-coordinate descent, solving a convex QP program under linear constraints at each step, initialized with uniform T
- Round up the solution

Related to MIL (Vijayanarasimhan and Grauman'08) and ambiguous labelling (Cour et al.'09)

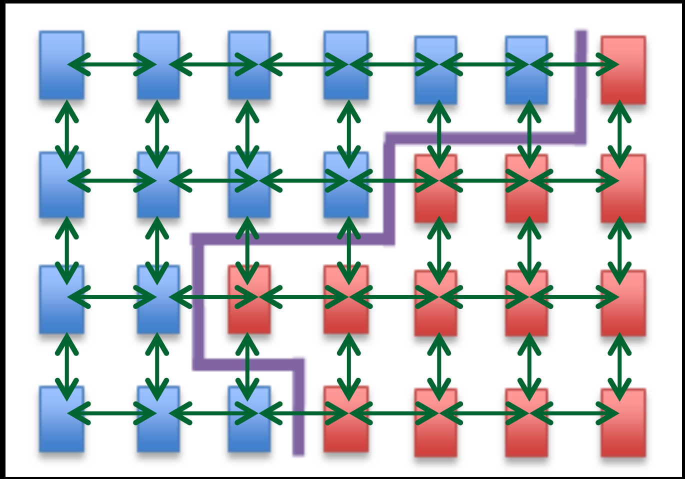

True Names+Actions AP=0.54
Names+Actions AP=0.39
True Names+Text
Names+Text
No Names AP=0.20
precision
recall
(a) walking
True Names+Actions AP=0.07
Names+Actions AP=0.09
True Names+Text
Names+Text
No Names AP=0.10
(b) sit down
True Names+Actions AP=0.87
Names+Actions AP=0.81
No Names AP=0.78
(c) other actions
ILSA
WALK
Laszlo turns off the light. Ilsa walks over
to the couch and sits down.
SIT DOWN
RICK
Rick sits down with Ilsa.
OTHER
OTHER
RICK
WALK
Meanwhile, Rick has walked over to the croupier.
OTHER
ANNINA
(no description)
RICK
RENAULT
WALK
WALK
Renault looks around the empty cafe.
YVONNE
OTHER
Back at the bar, Yvonne, an attractive young
French woman, sits on a stool drinking brandy.



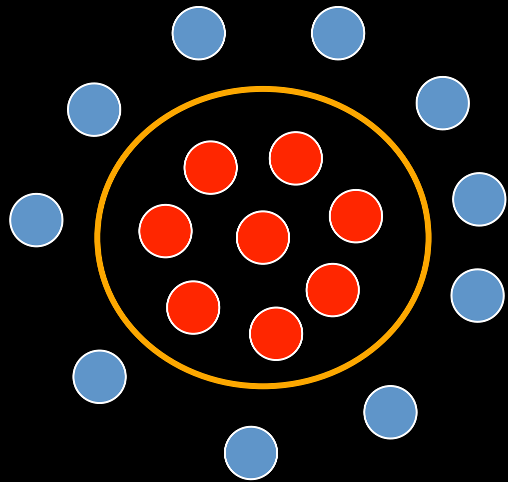
Within each image, we enforce grouping constraints



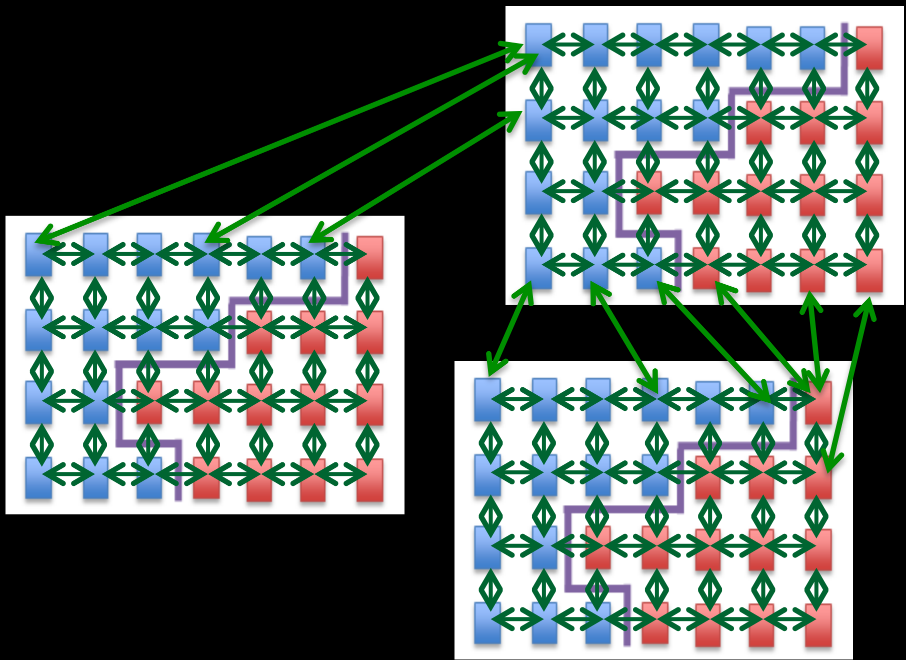
Across images, we discriminate among classes



Within each image, we enforce grouping constraints



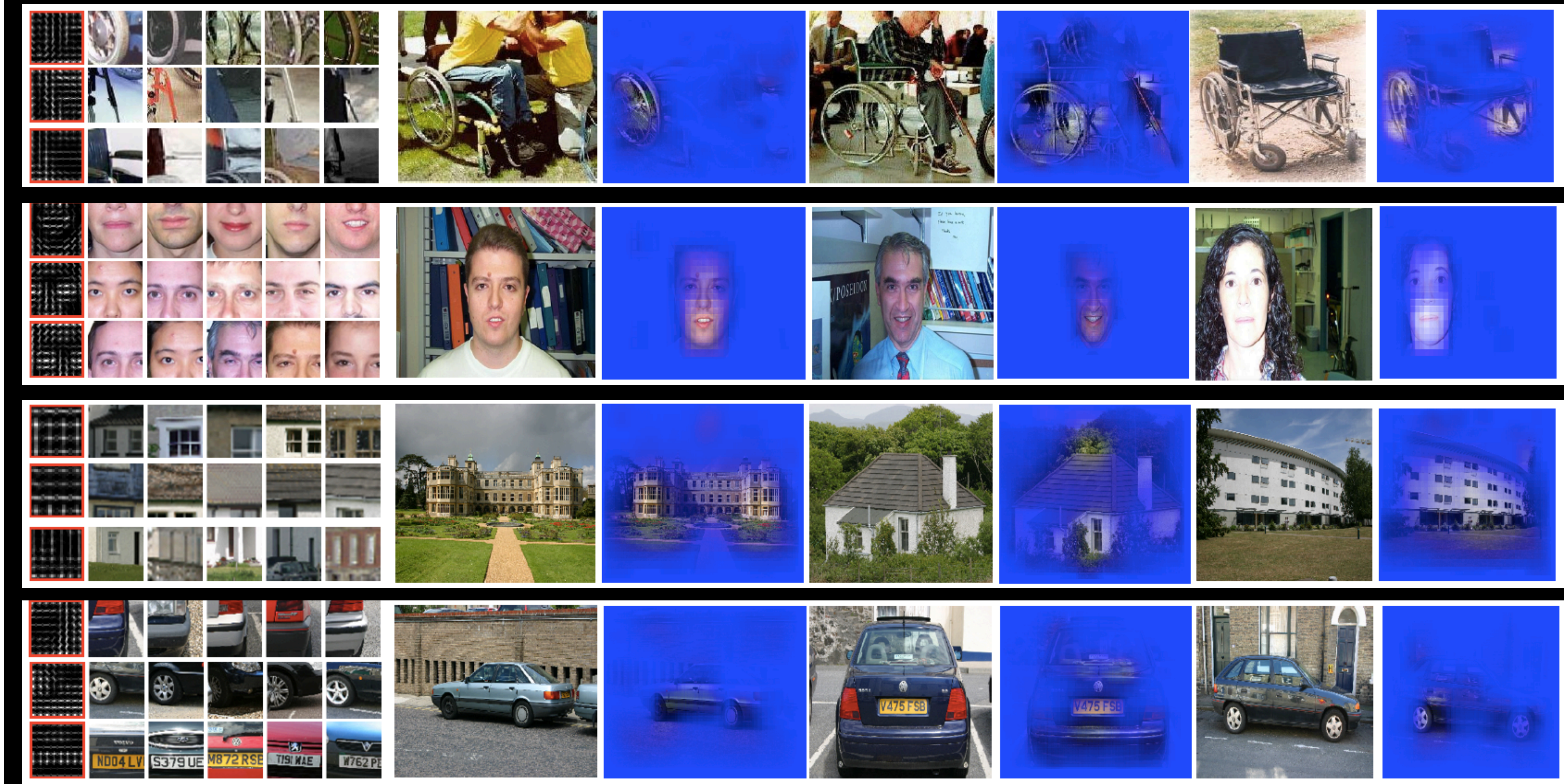
Across images, we discriminate among classes



But we don't model the fact that common classes occur over different images

(Rother et al., CVPR'06)
(Vicente, et al., CVPR'11)

Discriminative part models
(Sun and Ponce, 2013)



(Nevatia & Binford'72; Brooks'81; Ioffe & Forsyth'00; Fergus et al.'03; Felzenszwalb & Huttenlocher'03 Lazebnik et al.'04; Kushal et al.'07; Felzenszwalb et al.'08)

Datasets	Images	[16]	[17]	[19]	[25]	Ours_init	Ours
Bike	30	42.3	43.3	29.9	42.8	46.5	**50.7**
Bird	30	33.2	**47.7**	29.9	–	22.8	31.0
Car	30	59.0	59.7	37.1	52.5	55.0	**61.5**
Cat	24	30.1	31.9	24.4	5.6	36.5	**48.0**
Chair	30	37.6	39.6	28.7	39.4	39.4	**48.9**
Cow	30	45.0	**52.7**	33.5	26.1	38.2	45.6
Dog	26	41.3	41.8	33.0	–	32.4	**46.6**
Face	30	**66.2**	70.0	33.2	40.8	48.4	50.3
Flower	30	50.9	51.9	40.2	–	50.2	**75.7**
House	30	50.5	51.0	32.2	**66.4**	51.1	61.5
Plane	30	21.7	21.6	25.1	**33.4**	28.2	28.1
Sheep	30	60.4	66.3	60.8	45.7	47.8	**65.2**
Sign	30	55.2	58.9	43.2	–	50.9	**69.9**
Tree	30	60.0	67.0	61.2	55.9	55.8	**70.1**
Average		46.7	50.2	36.6	–	43.1	**53.8**

[16]: [Joulin et al.'10]
[17]: [Joulin et al.'12]
[19]: [Kim et al.'11]
[25]: [Mukherjee et al.'11]

Using discriminative parts for cosegmentation (Sun and Ponce, 2013)

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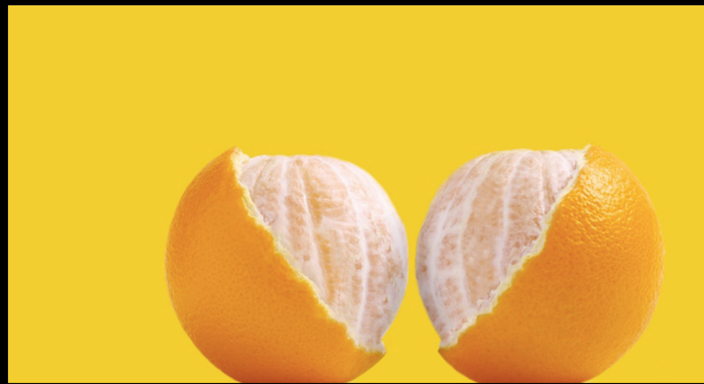
And just because it will be good for you: Look up Jan Koenderink's latest book



http://www.gestaltrevision.be/en/resources/clootcrans-press