Quelles garanties avec la cryptographie ?

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Outline

Security of Communications

One ever wanted to exchange information securely

With the all-digital world, security needs are even stronger. . .

In your pocket

But also at home

Cryptography

3 Historical Goals

- Confidentiality: The content of a message is concealed
- Authenticity: The author of a message is well identified
- Integrity: Messages have not been altered

between a sender and a recipient, against an adversary.

Also within groups, with insider adversaries

Cannot address availability, but should not affect it!

First Encryption Mechanisms

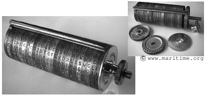
The goal of encryption is to hide a message

Scytale
Permutation

Substitutions and permutations
Security relies on
the secrecy of the mechanism

⇒ **How to widely use them?**

Alberti's disk
Mono-alphabetical Substitution

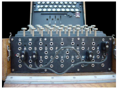


Wheel – M 94 (CSP 488)
Poly-alphabetical Substitution

Use of a (Secret) Key

A shared information (secret key) between the sender and the receiver parameterizes the public mechanism

Enigma:
choice of the connectors
and the rotors



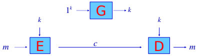
Security **looks** better: but broken (Alan Turing *et al.*)

⇒ **Security analysis is required**

Modern Cryptography

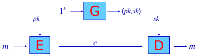
Secret Key Encryption

One secret key only shared by Alice and Bob:
this is a common parameter for both E and D



Public Key Cryptography [Diffie-Hellman – 1976]

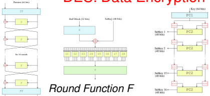
- Bob's public key is used by Alice as a parameter to E
- Bob's private key is used by Bob as a parameter to D



DES and AES

Still substitutions and permutations,
but considering various classes of attacks (statistic)

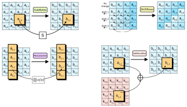
DES: Data Encryption Standard



"Broken" in 1998 by brute force:
too short keys (56 bits)!

⇒ **No better attack granted a safe design!**

New standard since 2001: Advanced Encryption Standard



Longer keys: from 128 to 256 bits

Criteria: Security arguments against many attacks

What does security mean?

Practical Secrecy

Perfect Secrecy vs. Practical Secrecy

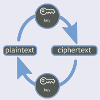
- No information about the plaintext *m* can be extracted from the ciphertext *c*, even for a powerful adversary (unlimited time and/or unlimited power): perfect secrecy
  $\Rightarrow$ information theory
- In practice: adversaries are limited in time/power
  $\Rightarrow$ complexity theory

We thus model all the players (the legitimate ones and the adversary) as Probabilistic Polynomial Time Turing Machines:

computers that run programs

Provable Security

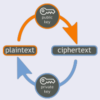
Symmetric Cryptography



The secrecy of the key guarantees the secrecy of communications

To be proven

Asymmetric Cryptography



The secrecy of the private key guarantees the secrecy of communications

To be proven

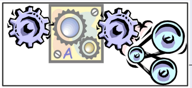
What is a Secure Cryptographic Scheme?

- What does security mean?
  $\rightarrow$ Security notions have to be formally defined
- How to guarantee above security claims for concrete schemes?
  $\rightarrow$ Provable security

Provable Security

- if an adversary is able to break the cryptographic scheme
- then one can break a well-known hard problem

hard instance $\rightarrow$



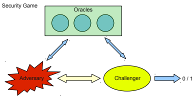
$\rightarrow$ solution

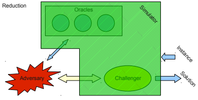
General Method

Computational Security Proofs

To prove the security of a cryptographic scheme, one needs

- a formal security model (security notions)
- a reduction: if one (Adversary) can break the security notions, then one (Simulator + Adversary) can break a hard problem
- acceptable computational assumptions (hard problems)





Proof by contradiction

Integer Factoring

Records

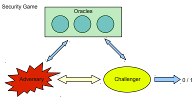
Given $n = pq$ $\longrightarrow$ Find p and q

Digits	Date	Bit-Length
130	April 1996	431 bits
140	February 1999	465 bits
155	August 1999	512 bits
160	April 2003	531 bits
200	May 2005	664 bits
232	December 2009	768 bits

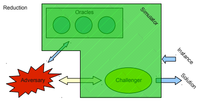
Complexity

768 bits $\to 2^{64}$ op.	3072 bits $\to 2^{128}$ op.
1024 bits $\to 2^{80}$ op.	7680 bits $\to 2^{192}$ op.
2048 bits $\to 2^{112}$ op.	15360 bits $\to 2^{256}$ op.

Reduction



Adversary running time t



Algorithm running time $T = f(t)$

- Lossy reduction: $T = k^3 \times t$

Modulus Bit-length	Adversary Complexity	Algorithm Complexity	Best Known Complexity	
$k = 1024$	$t < 2^{80}$	$T < 2^{110}$	2^{80}	✗
$k = 2048$	$t < 2^{80}$	$T < 2^{113}$	2^{112}	✗
$k = 3072$	$t < 2^{80}$	$T < 2^{115}$	2^{128}	✓

- Tight reduction: $T \approx t$
  With $k = 1024$ and $t < 2^{80}$, one gets $T < 2^{80}$ ✓

One-Way Functions

One-Way Functions

- $\mathcal{F}(1^k)$ generates a function $f : X \to Y$
- From $x \in X$, it is easy to compute $y = f(x)$
- Given $y \in Y$, it is hard to find $x \in X$ such that $y = f(x)$

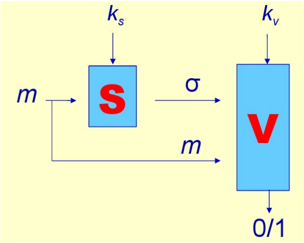
RSA Problem [Rivest-Shamir-Adleman 1978]

- Given $n = pq$, e and $y \in \mathbb{Z}_n^\star$
- Find x such that $y = x^e \bmod n$

This problem is hard without the prime factors p and q
It becomes easy with them: if $d = e^{-1} \bmod \varphi(n)$, then $x = y^d \bmod n$

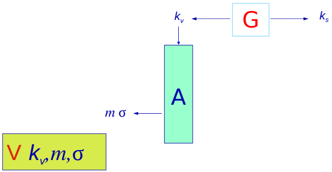
This problem is assumed as hard as integer factoring:
the prime factors are a trapdoor to find solutions
$\Rightarrow$ trapdoor one-way permutation

Signature



Goal: Authentication of the sender

EUF – NMA: Security Game



$$\mathbf{Succ}^{\mathrm{euf}}_{\mathcal{SG}}(\mathcal{A}) = \Pr[(k_s, k_v) \leftarrow \mathcal{G}(); (m, \sigma) \leftarrow \mathcal{A}(k_v) : \mathcal{V}(k_v, m, \sigma) = 1]$$

should be negligible.

$\mathcal{A}$ knows the public key only $\Rightarrow$ **No-Message Attack (NMA)**

EUF – NMA

One-Way Function

- $\mathcal{G}(1^k)$: $f \stackrel{R}{\leftarrow} \mathcal{F}(1^k)$ and $x \stackrel{R}{\leftarrow} X$, set $y = f(x)$, $k_s = x$ and $k_v = (f, y)$
- $\mathcal{S}(x, m) = k_s = x$
- $\mathcal{V}((f, y), m, x')$ checks whether $f(x') = y$

Under the one-wayness of $\mathcal{F}$, $\mathbf{Succ}^{\mathrm{euf-nma}}(\mathcal{A})$ is small.

But given one signature, one can "sign" any other message!

Signatures are public! $\Rightarrow$ **Known-Message Attacks (KMA)**

The adversary has access to a list of messages-signatures

EUF – KMA

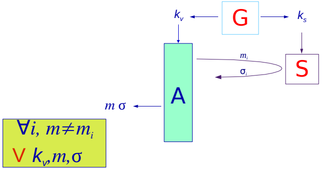
One-Way Functions

- $\mathcal{G}(1^k)$: $f \stackrel{R}{\leftarrow} \mathcal{F}(1^k)$, and $\vec{x} = (x_{1,0}, x_{1,1}, \ldots, x_{k,0}, x_{k,1}) \stackrel{R}{\leftarrow} X^{2k}$, $y_{i,j} = f(x_{i,j})$ for $i = 1, \ldots, k$ and $j = 0, 1$, $k_s = \vec{x}$ and $k_v = (f, \vec{y})$
- $\mathcal{S}(\vec{x}, m) = (x_{i,m_i})_{i=1,\ldots,k}$
- $\mathcal{V}((f, \vec{y}), m, (x'_i))$ checks whether $f(x'_i) = y_{i,m_i}$ for $i = 1, \ldots, k$

Under the one-wayness of $\mathcal{F}$, $\mathbf{Succ}^{\mathrm{euf-nma}}(\mathcal{A})$ is small.
With the signature of $m = 0^k$, I cannot forge any other signature.

With the signatures of $m = 0^k$ and $m' = 1^k$, I learn $\vec{x}$: the secret key
Messages can be under the control of the adversary!
$\Rightarrow$ **Chosen-Message Attacks (CMA)**

EUF – CMA



The adversary has access to any signature of its choice:
Chosen-Message Attacks (oracle access):

$$\mathbf{Succ}^{\mathrm{euf-cma}}_{\mathcal{SG}}(\mathcal{A}) = \Pr\left[\begin{array}{l}(k_s, k_v) \leftarrow \mathcal{G}(); (m, \sigma) \leftarrow \mathcal{A}^{\mathcal{S}(k_s,\cdot)}(k_v) : \\ \forall i, m \neq m_i \wedge \mathcal{V}(k_v, m, \sigma) = 1\end{array}\right]$$

The $\mathcal{RSA}$ Signature

[Rivest-Shamir-Adleman 1978]

The $\mathcal{RSA}$ Signature

The RSA signature scheme $\mathcal{RSA}$ is defined by

- $\mathcal{G}(1^k)$: p and q, two random primes, and an exponent v
  $n = pq$, $k_s \leftarrow s = v^{-1} \bmod \varphi(n)$ and $k_v \leftarrow (n, v)$
- $\mathcal{S}(k_s, m)$: the signature is $\sigma = m^s \bmod n$
- $\mathcal{V}(k_v, m, \sigma)$ checks whether $m = \sigma^v \bmod n$

Theorem (The Plain $\mathcal{RSA}$ is not $\mathbf{EUF-NMA}$)

The plain RSA signature is not secure at all!

Proof.

Choose a random $\sigma \in \mathbb{Z}_n^\star$, and set $m = \sigma^v \bmod n$.
By construction, σ is a valid signature of m □

Full-Domain Hash Signature

[Bellare-Rogaway – Eurocrypt '96]

Full-Domain Hash $\mathcal{RSA}$ Signature

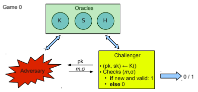
The FDH-RSA signature scheme is defined by

- $\mathcal{G}(1^k)$: p and q, two random primes, and an exponent v
  $n = pq$, $k_s \leftarrow s = v^{-1} \bmod \varphi(n)$ and $k_v \leftarrow (n, v)$
- $\mathcal{H}$ is a hash function onto $\mathbb{Z}_n^\star$
- $\mathcal{S}(k_s, m)$: the signature is $\sigma = \mathcal{H}(m)^s \bmod n$
- $\mathcal{V}(k_v, m, \sigma)$ checks whether $\mathcal{H}(m) = \sigma^v \bmod n$

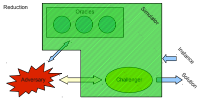
Theorem (Security of the FDH-RSA)

The FDH-RSA is $\mathbf{EUF-CMA}$ under appropriate assumptions on $\mathcal{H}$, and assuming the RSA problem is hard

FDH-RSA Security



Adversary running time t



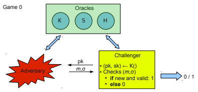
Algorithm running time $T = f(t)$

Initial reduction: $T \approx q_H \times t$ [Bellare-Rogaway – Eurocrypt '96]
(where q_H is number of Hashing queries $\approx 2^{60}$)

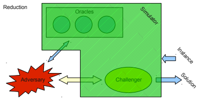
$k = 1024$	(2^{80})	$t < 2^{80}$	$T < 2^{140}$	✗
$k = 2048$	(2^{112})	$t < 2^{80}$	$T < 2^{140}$	✗
$k = 3072$	(2^{128})	$t < 2^{80}$	$T < 2^{140}$	✗

$\Longrightarrow$ large modulus required!

Improved Security



Adversary running time t



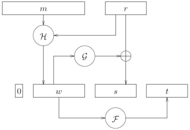
Algorithm running time $T = f(t)$

By exploiting the random self-reducibility of RSA: $(xr)^e = x^e r^e \bmod n$
$\Longrightarrow$ Improved reduction: $T \approx q_S \times t$ [Coron – Crypto '00]
(where q_S is the number is Signing queries $\leq 2^{30}$)

With $k = 2048$ and $t < 2^{80}$, one gets $T < 2^{110}$ ✓
(Best algorithm in 2^{112})

RSA-PSS (PKCS #1 v2.1)

[Bellare-Rogaway – Eurocrypt ’96]



- m is the message to encrypt
- r is the additional randomness to make encryption probabilistic

After the transformation, $w\|s\|t$ goes in the plain RSA

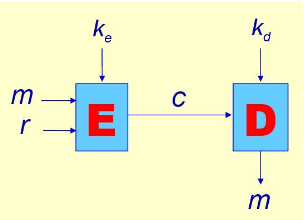
Theorem (EUF-CMA Security [Bellare-Rogaway – Eurocrypt ’96]**)**

RSA-PSS is **EUF-CMA** *secure under the RSA assumption*

Security reduction between $\mathbf{EUF-CMA}$ and the RSA assumption: $T \approx t$

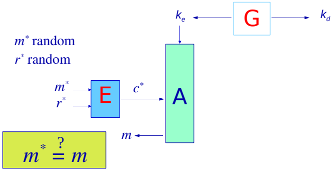
$\Longrightarrow$ 1024-bit RSA moduli provide 2^{80} security

Public-Key Encryption



Goal: Privacy/Secrecy of the plaintext

$\mathbf{OW-CPA}$: Security Game



$$\mathbf{Succ}_{\mathcal{S}}^{\mathrm{ow-cpa}}(\mathcal{A}) = \Pr\left[\begin{array}{c}(k_d, k_e) \leftarrow \mathcal{G}(); m^* \stackrel{R}{\leftarrow} \mathcal{M}; c = \mathcal{E}(k_e, m^*, r^*) : \\ \mathcal{A}(k_e, c^*) \to m^*\end{array}\right]$$

should be negligible.

$\mathbf{OW-CPA}$: Is it Enough?

The $\mathcal{RSA}$ Encryption [Rivest-Shamir-Adleman 1978]

- $\mathcal{G}(1^k)$: p and q, two random primes, and an exponent e: $n = pq$, $sk \leftarrow d = e^{-1} \bmod \varphi(n)$ and $pk \leftarrow (n, e)$
- $\mathcal{E}(pk, m) = c = m^e \bmod n$; $\mathcal{D}(sk, c) = m = c^d \bmod n$

$\mathcal{RSA}$ encryption is $\mathbf{OW-CPA}$, under the RSA assumption

$\mathbf{OW-CPA}$ Too Weak

- $\mathcal{G}' = \mathcal{G}$; $\mathcal{E}'(pk, m = m_1\|m_2) = \mathcal{E}(pk, m_1)\|m_2 = c_1\|c_2$
- $\mathcal{D}'(sk, c_1\|c_2)$: $m_1 = \mathcal{D}(sk, c_1)$, $m_2 = c_2$, output $m = m_1\|m_2$

If $(\mathcal{G}, \mathcal{E}, \mathcal{D})$ is $\mathbf{OW-CPA}$: then $(\mathcal{G}', \mathcal{E}', \mathcal{D}')$ is $\mathbf{OW-CPA}$ too

But this is clearly not enough: **half or more of the message leaks!**

OW − CPA: Is it Enough?

For a "yes/no" answer or "sell/buy" order,
one bit of information may be enough for the adversary!
How to model that no bit of information leaks?

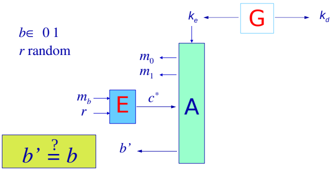
Perfect Secrecy vs. Computational Secrecy

- **Perfect secrecy**: the distribution of the ciphertext is perfectly independent of the plaintext
- **Computational secrecy**: the distribution of the ciphertext is computationally independent of the plaintext

Idea: No adversary can distinguish a ciphertext of m_0 from a ciphertext of m_1.

Probabilistic encryption is required!

IND − CPA: Security Game



$$(k_d, k_e) \leftarrow \mathcal{G}(); (m_0, m_1, \text{state}) \leftarrow \mathcal{A}(k_e);$$

$$b \overset{R}{\leftarrow} \{0,1\}; c^* = \mathcal{E}(k_e, m_b, r); b' \leftarrow \mathcal{A}(\text{state}, c^*)$$

$\mathbf{Adv}_{\mathcal{S}}^{\text{ind-cpa}}(\mathcal{A}) = 2 \times \Pr[b' = b] - 1$ should be negligible.

ElGamal Encryption

[ElGamal 1985]

The ElGamal Encryption ($\mathcal{EG}$)

- $\mathcal{G}(1^k)$: $\mathbb{G} = \langle g \rangle$ of order q, $sk = x \overset{R}{\leftarrow} \mathbb{Z}_q$ and $pk \leftarrow y = g^x$
- $\mathcal{E}(pk, m, r) = (c_1 = g^r, c_2 = y^r m)$
- $\mathcal{D}(sk, (c_1, c_2)) = c_2 / c_1^x$

The ElGamal encryption is **IND − CPA**, under the **DDH** assumption

Decisional Diffie-Hellman Problem

For $\mathbb{G} = \langle g \rangle$ of order q, and $x, y \overset{R}{\leftarrow} \mathbb{Z}_q$,

- Given $X = g^x$, $Y = g^y$ and $Z = g^z$, for either $z \overset{R}{\leftarrow} \mathbb{Z}_q$ or $z = xy$
- Decide whether $z = xy$

This problem is assumed hard to decide in appropriate groups $\mathbb{G}$!

ElGamal is IND − CPA: Proof

Let $\mathcal{A}$ be an adversary against $\mathcal{EG}$: $\mathcal{B}$ is an adversary against **DDH**:
let us be given a **DDH** instance $(X = g^x, Y = g^y, Z = g^z)$

- $\mathcal{A}$ gets $pk \leftarrow X$ from $\mathcal{B}$, and outputs (m_0, m_1)
- $\mathcal{B}$ sets $c_1 \leftarrow Y$
- $\mathcal{B}$ chooses $b \overset{R}{\leftarrow} \{0,1\}$, sets $c_2 \leftarrow Z \times m_b$, and sends $c = (c_1, c_2)$
- $\mathcal{B}$ receives b' from $\mathcal{A}$ and outputs $d = (b' = b)$
- $2 \times \Pr[b' = b] - 1$
  $= \mathbf{Adv}_{\mathcal{EG}}^{\text{ind-cpa}}(\mathcal{A})$, if $z = xy$
  $= 0$, if $z \overset{R}{\leftarrow} \mathbb{Z}_q$

ElGamal is IND − CPA: Proof

As a consequence,

- $2 \times \Pr[b' = b | z = xy] - 1 = \mathbf{Adv}^{\text{ind-cpa}}_{\mathcal{EG}}(\mathcal{A})$
- $2 \times \Pr[b' = b | z \xleftarrow{R} \mathbb{Z}_q] - 1 = 0$

If one subtracts the two lines:

$$\begin{aligned} \mathbf{Adv}^{\text{ind-cpa}}_{\mathcal{EG}}(\mathcal{A}) &= 2 \times \begin{pmatrix} \Pr[d = 1 | z = xy] \\ -\Pr[d = 1 | z \xleftarrow{R} \mathbb{Z}_q] \end{pmatrix} \\ &= 2 \times \mathbf{Adv}^{\mathbf{ddh}}_{\mathbb{G}}(\mathcal{B}) \leq 2 \times \mathbf{Adv}^{\mathbf{ddh}}_{\mathbb{G}}(t) \end{aligned}$$

IND − CPA: Is it Enough?

The ElGamal Encryption [ElGamal 1985]

- $\mathcal{G}(1^k)$: $G = \langle g \rangle$ of order q, $sk = x \xleftarrow{R} \mathbb{Z}_q$ and $pk \leftarrow y = g^x$
- $\mathcal{E}(pk, m, r) = (c_1 = g^r, c_2 = y^r m)$; $\mathcal{D}(sk, (c_1, c_2)) = c_2 / c_1^x$

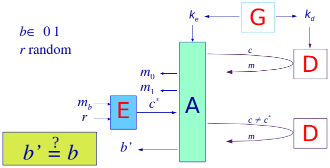
Private Auctions

All the players P_i encrypt their bids $c_i = \mathcal{E}(pk, b_i)$ for the authority; the authority opens all the c_i; the highest bid b_I wins

- **IND − CPA** guarantees privacy of the bids
- Malleability: from $c_i = \mathcal{E}(pk, b_i)$, without knowing b_i, one can generate $c' = \mathcal{E}(pk, 2b_i)$: **an unknown higher bid!**

IND − CPA does not imply Non-Malleability

IND − CCA: Security Game



The adversary can ask any decryption of its choice:

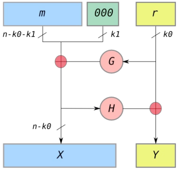
$\Rightarrow$ **Chosen-Ciphertext Attacks (CCA)**

Theorem (NM vs. CCA [Bellare-Desai-Pointcheval-Rogaway – Crypto ’98]**)**

The chosen-ciphertext security implies non-malleability
$\Longrightarrow$ *the highest security level*

RSA-OAEP (PKCS #1 v2.1) [Bellare-Rogaway – Eurocrypt ’94]

The $\mathcal{RSA}$ encryption is **OW − CPA**, under the RSA assumption, but even not **IND − CPA**: **need of randomness and redundancy**



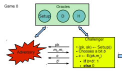
- m is the message to encrypt
- r is the additional randomness to make encryption probabilistic
- $00 \ldots 00$ is redundancy to be checked at decryption time

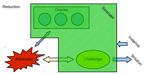
After the transformation, $X \| Y$ goes in the plain RSA

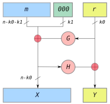
Theorem (IND-CCA Security [Fujisaki-Okamoto-Pointcheval-Stern – Crypto ’01]**)**

RSA-OAEP is IND-CCA secure under the RSA assumption

RSA-OAEP Security Proof [Fujisaki-Okamoto-Pointcheval-Stern – Crypto '01]





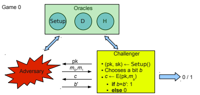


$c = f(X \| Y)$

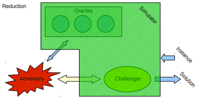
More precisely, to get information on m, encrypted in $c = f(X\|Y)$, one must have asked $\mathcal{H}(X) \Longrightarrow$ partial inversion of f

For RSA: partial inversion and full inversion are equivalent
(but at a computational loss)

RSA-OAEP Security [Fujisaki-Okamoto-Pointcheval-Stern – Crypto '01]



Adversary running time t



Algorithm running time $T = f(t)$

If there is an adversary that distinguishes, within time t, the two ciphertexts with overwhelming advantage (close to 1), one can break RSA within time $T \approx 2t + 3{q_H}^2 k^3$
(where q_H is number of Hashing queries $\approx 2^{60}$)

$k = 1024$	(2^{80})	$t < 2^{80}$	$T < 2^{152}$	✘
$k = 2048$	(2^{112})	$t < 2^{80}$	$T < 2^{155}$	✘
$k = 3072$	(2^{128})	$t < 2^{80}$	$T < 2^{158}$	✘

$\Longrightarrow$ large modulus: $>$ 4096 bits!

REACT-RSA Security [Okamoto-Pointcheval – CT-RSA '01]

REACT-RSA

- $\mathcal{G}(1^k)$: p and q, two random primes, and an exponent e:
  $n = pq$, $sk \leftarrow d = e^{-1} \bmod \varphi(n)$ and $pk \leftarrow (n, e)$
- $\mathcal{E}(pk, m, r) =$
  $$(c_1 = r^e \bmod n, c_2 = G(r) \oplus m, c_3 = H(r, m, c_1, c_2))$$
- $\mathcal{D}(sk, (c_1, c_2, c_3))$: $r = c_1^d \bmod n$, $m = c_2 \oplus G(r)$,
  if $c_3 = H(r, m, c_1, c_2)$ then output m, else output $\perp$

Security reduction between $\mathbf{IND} - \mathbf{CCA}$ and the RSA assumption:
$T \approx t$

$\Longrightarrow$ 1024-bit RSA moduli provide 2^{80} security

Conclusion

With provable security, one can precisely get:

- the security games one wants to resist against any adversary
- the security level, according to the resources of the adversary

But, it is under some assumptions:

- the best attacks against famous problems (integer factoring, etc)
- no leakage of information excepted from the given oracles

Cryptographers' goals are thus

- to analyze the intractability of the underlying problems
- to define realistic and strong security notions (games)
- to correctly model the leakage of information (oracle access)
- to design schemes with tight security reductions

Implementations and uses must satisfy the constraints!