***In* J-Y. Girard's Logic Dictionnary in "Locus Solum", MSCS, vol. 11/3, Cambridge UP, 2001.**

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LAPLACE (or, *Reflections on incompleteness*)
A major mathematician, known for his seminal work in Infinitesimal Analysis, Astronomy, Probability Theory. Laplace proposed a paradigm for the mathematical analysis of Physics, the so called « laplacian determinism ». In this perspective, the systems of (differential) equations could "completely" describe the physical world. More precisely, if one wanted to know the state of the physical world in a future moment, with a given approximation, than it could suffice to know the current state of affairs up to an approximation of a *comparable order of magnitude*. By formally computing a solution of the intended equations, or by suitable approximations by Fourier series (as it will be said later), one could *deduce* (or predict or decide) the future state, up to the expected level of approximation.

Poincaré, as a consequence of his famous theorem on the three bodies problem, proved that *minor variations* of the initial conditions could give *enormous changes* in the final result or, even, that the solutions could depend discontinuously on the initial conditions. Then, predictability, as "completeness w.r.t. the world" of a suitable set of differential equations, failed.

About one century later, D. Hilbert resumed Laplace's program in a different context. He first set the basis for the rigorous notion of « formal system », as well as for the distinction between « theory » and « metatheory ». He then conjectured that the key system for Number Theory, Peano's first order Arithmetic (where he had interpreted Geometry, 1899), was *complete* w.r. t. the intended structure of numbers (or that any first order assertion about the « world of numbers » could be *decided* by formal or « potentially mechanisable » tools).

A few soon reacted to Hilbert's program, such as the "lone wolf" among Hilbert's students, Hermann Weyl, who (though hesitantly) conjectured in Das Kontinuum, 1918 (!), the incompleteness of formal arithmetic (end of §.3). He also stressed in several places that the idea of mechanization of Mathematics trivializes it and misses the reference to meaning and structures. Besides Weyl (and Poincaré and a few others), Wittgenstein is another thinker who criticized Hilbert's program. For him «Hilbert's metamathematics will turn out to be a disguised Mathematics» [Waismann, 1931], since «[A metamathematical proof] should be based on entirely different principles w.r. t. those of the proof of a proposition ... in no essential way there may exist a meta-mathematics» (see Wittgenstein, Philo. Rem., § 153; quoted in [Shanker,1988]) and ... «I may play chess according to certain rules. But I may also invent a game where I play with the rules themselves. The pieces of the game are then the rules of chess and the rules of the game are, say, the rules of logic. In this case, I have *yet another game*, not a *metagame*» [Wittgenstein, 1968; p. 319].

As for formal Arithmetic, the key theory for finitistic foundationalism, these remarks may be now understood in the light of Gödel's Representation Lemma [Gödel, 1931]: by this very technical result, one may encode the metatheory of arithmetic into arithmetic itself, thus the "rules of the metagame" are viewed just as ... rules of the "arithmetical game". Moreover, many proofs, which entail the consistency of Arithmetic, such as (Tait-)Girard proof of "normalization" of Impredicative Type Theory ([Girard et al., 1989]), need a blend of metalanguage and language; or even purely combinatorial statements, such as Friedman's Finite Form of Kruskal's theorem, provably require the same entangled use of metatheory, theory and semantics, by the impredicative notions involved (see [Harrington et al., 1985]); an indirect confirmation of Wittgenstein's philosophical insight (and Weyl's, as for incompleteness).

Both Laplace and Hilbert program, which are strictly parallel and contributed to positivist philosophies in physics and in mathematics, opened the way to very relevant mathematical work: when precise and robust, even wrong programs may have extraordinary developments (such as XIX century Analysis, partly motivated by the laplacian « calculus of perturbations », or the rigorous notion of mechanisable computation of the '30's and its fall-out: actual computers, as purely formal symbol pushers). However, the corresponding incompleteness theorems, Poincaré's and Gödel's or more recent « concrete » ones, such as the two mentioned above, should finally take us away for the underlying philosophies, also to go further with mathematics. Poincaré's result, for example, is at the origin of beautiful and new mathematical theories (the geometry of dynamical systems), where qualitative predictions replace quantitative ones and the "mathematical understanding" does not need to coincide with completeness or predictability by formal tools. In mathematical logic we are not yet at a similar revolution, but the basis are being set towards breaking the metaphysics of the relevant, but artificial, organization of the discourse proposed by Hilbert, the theory/metatheory frame. Similarly, we have to overcome the believe that language ''predicates'' about the world: language and structures (of mathematics, of physics) are in permanent *resonance*. They construct themselves while singling out concepts and objects, in a permanent tension, which requires a parallel analysis of the foundation of these disciplines.

One further step is being now taken, well beyond the laplacian philosophy of formalism in Logic and Cognition. XX century physics departed from the Newtonian ''causal lawfulness'' of nature (and the mysterious instantaneous action at distance, gravitation) and stressed the geometric structuring of the world, by "geodetic principles". By this, it moves also towards unification with Quantum Mechanics (non-commutative geometry). In a sense, it is the structure of space and *location only* that matter, similarly as in the geometry of deduction in "*Locus Solum*".

For further remarks and for the references quoted above, see the downloadable papers:

G. Longo "The reasonable effectiveness of Mathematics and its cognitive roots" To appear in "**New Interactions of Mathematics with Natural Sciences**" (L. Boi ed.), Springer 2005.

G. Longo. "Laplace, Turing and the "imitation game" impossible geometry: randomness, determinism and programs in Turing's test". *Invited lecture,* Conference on **Cognition, Meaning and Complexity**, Roma, June 2002 (version française, **Intellectica**, n. 35, 2002/2, pp. 131-162.