Large Language Models and Network Emergence[1]

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1 - Large Language Model or how to cheat at random

A Large Language Model (LLM) is a probability distribution over the strings in a language, that is the probability that a given string is a string that is part of a language. Thus, a LLM can compute the probability P for a sequence of words, P(W1, ..., Wn), the conditional probability of a word Wn following W1, ..., Wn-1, P(Wn / W1, ..., Wn-1). Then, it can easily deduce from the net the probabilities of word ordering:

P(Yes I love my cat) > P(Cat love yes my I)

as well as that

P(cheese / Mary favorite food is) > P(wood / Mary favorite food is)

The basis of the analysis boils down to constructing lists of tokens, both words or sentences, on the grounds of probabilities – a very "intelligent" (?) decision making. The most trivial way is to take the most probable token at each step, i.e. highest P(Wn / W1, ..., Wn-1). This is *beam search*, that is breadth-first search to build a search tree, i.e. searching according to the probabilities of a sequence of tokens.

However, there is a rub. As long as the Data Base is stable, and even the web may be relatively stable for most searches, if one asks the same question to the machine after some time, one gets *exactly* the same answer, in the sense of *exactly the same sequence of letters*, whatever complicated the question is. As the promotors of ChatGPT wanted to show its "creativeness", they improved the search by

Top-K sampling: at each step randomly select a token from the top k most likely tokens,
Top-P sampling: at each step randomly select a token from the set of most probable tokens whose combined probability is above P - by the increasing size of the Data Bases (from the 780 Giga of PaLM in 2022, to 13T of GPT-4 in 2023), this provides lots of possible choices;

that is, they *explicitly* introduced some randomness to *pretend* that the machine is "creative". Technically, many models today use non trivial results from Graph Theory, in particular Random Graphs (Janson, Rucinski, 2000). This theory has been extensively developed with relevant advances in recent years – below we discuss further rich mathematical frames. The algorithms are largely based on optimality techniques (optimized trade-off, maximal coupling...) and allow to describe critical transitions and other "emerging" structures in terms of scaling laws (van der Hofstad, 2016). The applications range from the analysis and administration of internet networks to

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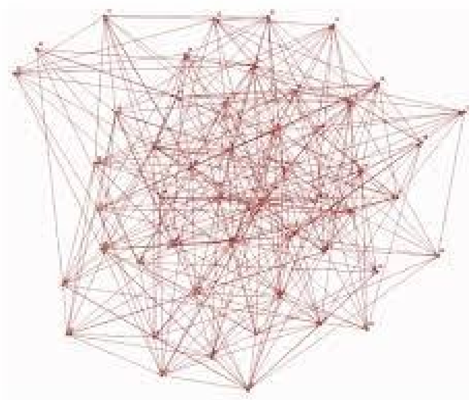
the recent AI applications we are discussing here, also based on statistical analyses and optimality methods on very large finite graphs, e.g. in LLM. On the immense data bases we have today, these techniques may be very useful: they may provide a greatly improved Search Engine on the web, immensely more powerful than Google, say, as dealing also with complex and long queries and producing long synthetic texts and images.

Yet, as Cristian Calude writes: “The key word is randomness: if we use the exact random prompt multiple times, we will get different essays each time!”. Randomness may be borrowed from the thermal fluctuation even in an isolated computer, or more easily from the net: how many requests (even or odd) are *now* at that node of the net? This would not be a problem, if these identically iterating machines, with some randomness artificially added on top of their identically iterating processes on Discrete Data Types, were not “sold” as “intelligent”, “creative”, soon expressing human consciousness… A better understanding and use of LLM may be promoted by looking more closely to some of its mathematics, beyond myth.

2 - Emergence in networks

Emergence has been thus understood in physics in different and very rich theoretical frames, broadly described as “theories of complex systems”. Unification of different scales by new theories has been another methodological tool.

A recent approach refers to the peculiar complex nets of interactions known as “spin glasses” theories (Parisi et al., 1987; here is an image of a typical network “a la Parisi”). This approach analyzes the coupling between different spins that can be more or less intense - attractive or repulsive - depending on the material and the distance that separates them. They are modeled statistically as Ising spins (plus or minus one) coupled by random constants representing disorder. These constants evolve slowly as impurities diffuse and the spin glass changes in time - their couplings are then called frozen, or time-independent (quenched). The couplings force the behavior of each element or node in the network according to the state of the neighboring elements.



Parisi showed how these interactions may lead to self-organizing forms, in physics and, most surprisingly, that they may model some animals’ collective behavior. So, the peculiar dynamics of flocks of birds and school of fishes may be described in the mathematical terms of networks of spin glasses. These networks constitute then emergent collective behaviors and yield many possible dynamic equilibria. In short, the mathematical analysis is based on “rugged landscapes”, so that minor fluctuations of one or a few elementary components of the network may lead to very different global trajectories. Landscapes or the space of all possible landscapes are mathematically pre-given or pre-conceived.

A more classical approach to similar structures is *Graph Theory*, in particular the theory of random graphs mentioned above (Janson, Rucinski, 2000), and its advances in recent years. The approach is largely based on optimality techniques (optimized trade-off, maximal coupling…) and on analyses of critical transitions (van der Hofstad, 2016). The emergent phenomena have the usual physico-mathematical nature and follow “optimal” paths, enriched by some statistics (e.g. use the connection given by the highest probabilities), in huge and pre-given phase spaces. The paths and the results may be highly unpredictable, as most emerging phenomena in physics, from the forms of clouds or hurricanes and flames (self-organizing far from equilibrium phenomena) to the collective behaviors in rugged landscapes mentioned above. We stress in this text, and more extensively here[2], the difference between the unpredictability in the many emergent phenomena described in the inert, including AI systems, and the production of novelty in the living state of matter (and, indirectly, provide hints in cognition and in historical sciences).

3 - Pre-given phase or state spaces

In all the previous examples, once the phenomenal level of emergent phenomena is identified, both the phase or state spaces of the elementary dynamics and the space of the global level of the new forms or structures (the spaces of pertinent observables and parameters) are mathematically pre-supposed or pre-given. From the possible forms in Turing and Thom, to Parisi's rugged landscapes, to the correlation graphs of LLMs, the global dynamics take place in a pre-described space of all possible trajectories – that is, the set of all possible emerging observables and the intended parameters are mathematically pre-given, in the most general sense. Thus, the unpredictability of the trajectory is given within a (possibly huge) space of possibilities. The infinity of all possible trajectories and of their spaces in not a problem, in mathematics - the point is to be able to describe it, a priori. The infinite phase space may even have an infinite number of dimensions, like some Hilbert spaces used for Schrödinger equation - it suffices that the infinite and/or infinite dimensional space has enough symmetries, like the Hilbert spaces, to be definable, a priori, by a finite number of formal properties or axioms and definitions (a finite writing).

In historical sciences, like biology or in the analysis of cognition, there is no way to describe a priori a non-existing phenotype or cognitive construct. Before the evolutionary production of the ears' structure, its form and *function* could not be pre-described, like the function as a radio box for the chassis of a chair in the XVIII century. As these structures are ex-apted from previous forms and functions, they are far from optimal: the historical constraints canalize and delimit the new forms and their functionality – like the box of a radio ex-apted from a chair, in Jacob's example, and the "physically absurd" structure of the internal ear in large vertebrates[3]. A component of the changing space of possibilities of human concepts may refer, by analogy, to ex-aptation, overloading, recombination… of previous concepts and linguistic constructions. In short, a cell is from a cell, yet an eukaryote derived from the symbiosis of an archea and a bacterium is a totally new organism, it is not the emergence of order from molecular disorder, as in various forms of self-organizations observed in physics; similarly, a conceptual construction is grounded on previous linguistic and life experiences, but it may yield a radically new observable. In this sense, in biology and cognition, "emergence", as production of novelty, is historical. None is optimal, as this notion is mathematically well-defined only in a pre-given space of possibilities, a non-sense in historical sciences.

In short, the result of coin tossing is unpredictable, but it yields no emergence; the forms of hurricanes and clouds are unpredictable, emergent and optimal (they are geodetic surfaces in pre-given mathematical spaces), with no meaning, like LLM sequences of signs – yet we can interpret them: that cloud has the shape of an elephant, this sequence of signs statistically produced by ChatGPT is a nice poem… The forms of the rocks and the clouds in a Deposition from the Cross or in other paintings by Andrea Mantegna (XV century, Italy), instead, are drawn in order to express the despair of the Universe for the death of Christ, they are creative and meaningful. They are the

2 https://www.di.ens.fr/users/longo/files/EmergeCompareBioNovelty.pdf

3 An intelligent designer and programmer could do much better than the existing internal ears in large vertebrates, with an out-placed vestibular systems – in some invertebrates it is (more soundly?) related to the visual system (Bender, Frey, 2008) – but of course, this is just the contingency of a history and its constraints. As observed by Helmholtz, we should fire the designer of the vertebrates' eyes: the nerve fibers route *before* the retina, blocking some light and creating a blind spot where the fibers pass through the retina. In cephalopod eyes, the nerve fibers route *behind* the retina, and do not block light nor disrupt the retina (https://en.wikipedia.org/wiki/Cephalopod_eye). The different phylogenetic origins and embryogenesis of eyes in relation to the brain make intelligible these different structures, far from "optimality".

result of a historical formation of an artistic sense, that of new observable forms of nature in paintings, an invention of Italian renaissance.

4 – More on emergence in LLM *vs* production of novelty

The extensive use of immense data bases in recent applications of AI, such as LLM, in particular by techniques based on graphs or networks hinted above, leads to highly unpredictable emergent structures. The nodes in these networks may be the result of human activities, e.g. meaningful sentences written by humans, in different contexts. In a way, then, these nodes are "specific" and "historical", yet the mathematics of the network dynamics treats them as *generic* nodes, just labelled by different probabilities, thus the unpredictable "novelty" is actually an unpredictable emergent form in a graph, as in physical or mathematical dynamics. Thus, "emergence" in LLM is of the same mathematical type as emergence in physics: it uses variations of the same mathematical methods in immense networks of digital computers and data bases, it is analyzed in terms of geodetics (optimal paths) or statistical maximality – with some random variation.

The confusion of optimization methods and probabilities in a connectivity structure (typically, based on the number of direct connections in a random graph) with "meaning", in its human, historical, bodily… sense (Longo 2019), is a major conceptual mistake and may prevent the actual production of novelty in the potentially fantastic interaction humans/machines we may have today. The human "ex-aptation" or "oveloading" of concepts or linguistic practices, their "metaphorical transfer" into new ones, or their radical invention, are theoretically remote from the new sentence added on the grounds of maximality or probabilistic criteria (the largest numbers of connections, the shortest paths to reach it in a graph… with some stochasticity added on top). These criteria force averages or emergence in conformity to mean fields (and "common sense"), the opposite of the human invention of "new points of view or new forms of sense", of our "taking a risk" by inventing new meanings. In a sense, we do this every day or whenever facing a new situation in changing historical contexts and ecosystems. Let me just recall some major "productions of scientific novelty", such as assuming the perspective of the Sun to analyze the Solar system (in no way this may be deduced from the "data", see (Longo 2023)), writing radically new equations and their geometric (Newton) or algebraic (Leibniz) calculus that unify falling bodies and planets' movement, assuming curving spaces to understand gravitation and inertia at once... just to mention a few theoretical inventions rich of history and sense and proposed against average thinking and common sense. These risky inventions were later followed by major changes also in the meaning of their linguistic expressions, by the invention, say, of Differential Geometry. The production of conceptual novelty, including new perspectives in organizing reality around us (such as space by new geometries, non-euclidean ones, say) is at the core of scientific inventions, often grounded on metaphysical or even religious commitments. For example, the invention of the "perspective" in early Renaissance painting, is a theologico-pictorial decision where the presence of the infinity of God in Annunciations is "metaphorically" suggested by the projective point (Arasse 1999); this invention is at the origin of the mathematical re-organization of space at the core of the scientific revolution and its mathematics (Longo, Longo 2020). In general, any new and relevant proof in Mathematics requires the invention of new concepts and structures, of new "perspectives", strongly embedded in a historical context of meaning. As for more examples, infinitary or geometric judgements are at the core of the proofs of interesting statements of formal Number Theory that are formally unprovable – some difficult, recent "concrete incompleteness" results (Longo 2011). But even when considering formal proofs in the abstract frame of Type Theory, formally implementable in a computer, very basic Classical Logic yields practically non-mechanisable proofs for LLM: proofs "per absurdum" require connections in graphs by paths of a length and structure that goes beyond the limits of current (and possible?) programming methods in LLM (Oldenburg 2023). We, human mathematicians, bypass the problem by "establishing connections by new meanings", not already present as paths in pre-given data bases, or by inventing new, non-existing symmetries, like young Gauss or proof-theorists in (Longo 2011), or new forms of the infinite in Mathematics, a concept to which theologians and painters gave a robust meaning by "showing" it to us in perspectival paintings, as a visible convergence or perspectival *point* (Longo, Longo 2020).

Formal, statistical and optimality methods are all provably incomplete; their abuse may also prevent human production of novelty, by imposing mean field criteria (follow the average, use the most probable) and even slow down more effective progresses in the applications of these fantastic discrete state machines produced by the inventive mathematics of the 1930s by Gödel, Church and Turing. Awareness of the limitations, beyond the usual technological arrogance, may help to improve also these technologies. In particular, by missing the differences between emergent phenomena in optimal or statistical dynamics in pre-given phase spaces, on the one side, and the historical formation of novelty and sense in changing spaces of possibilities, in biology and cognition, on the other, researchers in AI discourage the invention of new uses of digital networks to be constructed in collaborative, non-competitive interfaces between humans and machines (Lassègue, Longo 2024). Our aim is to go beyond the political ideology of *control* and *replacement* of biological and cognitive processes by statistically more probable or by "optimal" paths (geodetics with "no alternatives") and by re-programming DNA (in spite of fifty years of failures), an ideology that still prevails. By developing an analysis that differentiates physical vs biological (and cognitive) theorizing, we aim instead to focus on the role of the scientific construction of knowledge and technologies by collaborating in the meaningful spaces of our communicating humanity, today greatly empowered by these digital devices and their networks, whose construction and use may be further improved by the scientific awareness of their limits (Longo 2023).

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https://www.di.ens.fr/users/longo/files/EmergeCompareBioNovelty.pdf)

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