Objective and Epistemic Complexity in Biology

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Objective and Epistemic Complexity in Biology[1]

(Towards a "conceptual organization")

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1. Introduction

The current analysis of information and complexity mostly concern "artificial" systems of various natures. Particularly deep mathematical work has been developed during the XXth century on complexity and elaboration of information in and by artifacts, in particular in digital computers and their programs. The transfer of these analyses to natural phenomena encounters some crucial difficulties, which partly amount to the "objective" complexity of reality w.r. to artificial systems. Our (complex) artifacts, in general, from buildings and clocks to modern computers, are built by starting with elementary and *simple* bricks: with simple components we constructed St. Peter's Cathedral, the wonderful clocks and mechanical devices of the XVIIth and XVIIIth century and our modern computers (they are composed by simple logic ports and their formal/programming languages use very simple primitives). It is not so in "natural structures": strings, say, are elementary components in physics, but they are not simple; cells can't be split further if one wants to preserve the phenomenal level of life (they are elementary), but they are not "simple" either (even the most ancient prokaryotes are far form simple: organelles, symbiotic phenomena, complex metabolic activities ... are already present, see [Bailly, Longo, 2003a]). Even the elementary particles of natural languages are grounded on polysemy and heavily depend on contexts.

In our view, this is one of the facts that contribute to the enormous difficulties we encounter when trying to simulate a natural structure by an artifact, in particular for the purposes of a complexity measure: the elementary components are the first, yet very different, building blocks. But, of course, this is also a challenge for other scientific descriptions, in particular for mathematical ones. We are all aware of the difficulties, in microphysics, with entanglement and non-locality of elementary particles. Further on, it is as if the "emerging" new phenomenal levels (life over physics, language over life) were grounded on the complex synthesis of the underlying level, which seems to be partly "summarized" in the elementary components of the new natural structures, at the "higher" level (biological structures over physical ones, cognitive phenomena over life). Simulating (understanding the origin) of cells, as elementary, yet complex components, is the hardest issue in simulating (studying the origin of) life; translating the English "on" or "towards", in different contexts (space, time, metaphors...), is one of the hardest tasks in automatic translation and synthesis of natural languages.

Complexity, in particular, has been formally analised in depth by the mathematics of computational complexity, as properties of formal languages and their computations by machines, whose elementary components are very simple. This relevant part of mathematical computing has been focusing on (sequential) time and space and attained a major technical depth even if restricted to the flat conceptual (and physical) dimension of strings of 0 and 1's and their length. Natural complexity requires a different insight into its the elementary yet very complex grounds and even more when dealing with compound structures. G. Edelman

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and G. Tononi, in reference to the brain, call "complex" a structure which is, at once: « orderered *and* desordered, regular *and* irregular, variant *and* invariant, stable *and* instable », see [Edelman, Tononi, 2000]. We will add "differenciated *and* integrated" to this list by the analysis below of the levels of organisation and hint to the issues of "stability vs. instability, variation vs. invariance".

It is clear that our classification of "Nature" in three major phenomenal levels (physical, biological, cognitive) is rather arbitrary. Yet, it is made not just for convenience, but also on the grounds of history: mankind has been giving itself different methodological tools in the analysis of physical phenomena, biological ones and cognitive/relational experiences. Mathematics has been structuring Physics and largely grew out of it. Modern Biology was born when it could unify networks of samples in little or no mathematized theories (since Darwin and Claude Bernard): as we will try to hint in several places, conceptual invariants were proposed in order to grasp variation, yet the observation of individuality and variability remained at the core of biological theoretizing. Discursive presentations largely prevailed and prevail in Biology, Linguistic, Cognition and, more generally, in humanities. In spite of being a physicist and a mathematician, we do not see this as a "deficiency", as a lack of rigor in disciplines different from ours, but as an inherent difficulty, a challenge to be faced by attempting unified approaches that compare and synthesize, if possible, the objective and epistemic differences. Objective (some would say "ontological"), in the sense that the constituting of a new phenomenal level, along evolution, actually gives new "entities", whatever meaning one wants to attach to this word; epistemic, as our ways of knowing differ and not because biologists or humanists are incompetent, but because the historical knowing subject endeavors different "frictions" on reality, in making the manifold object matter intelligible. Or, in short, reality presents itself to us at different phenomenal levels and this as a result of our active construction of knowledge. Mathematics has had a normative role in physics (it *organized* reality, in particular since infinitesimal calculus and, later on, by the geometrization of Physics); it still must deserve a similar role in other disciplines, if possible. The common idea that the same mathematical tools that were so successful in Physics could play a similar role in Biology is mostly based on confusion between the *dependence* of biological phenomena on physical ones and their *reducibility*. The fact that life phenomena depend on physical matter is a basic *assumption* for any monist, often to be made explicit; the reduction to Physics is *theoretical operation*. That is, the first is the very assumption of modern science, the second must be actually implemented, and it is usually done w. r. to *existing physical theories*.

In this paper we will hint to a conceptual organization of some biological phenomena, which tries to unify different facts. This is a very modest attempt to "unify by concepts"; an attempt, which should always precede mathematization. Unfortunately, the pressure of times often compels to compute before understanding and conceptualizing. Yet, Mathematics itself is the result of a progressive conceptualization, where crucial notions and structures are the result of a difficult conceptual stabilizing of informal practices. That is, the perfectly stable conceptual invariants of Mathematics are the result of a "praxis", as active attempts to understand and organize the world. Greek Geometry, say, did not pop-out from an axiomatic approach, but was the result of a broad practice of abstraction over active measure and computations. The use of actual infinity by Newton and Leibniz, in the calculus of finite movement, speed and acceleration, followed centuries of debate on the difference of potential vs. actual infinity, often of a religious nature. Then the unification of sub-lunar and supra-lunar physics, Newton's major contribution as they were considered well separated ontologies, was obtained by radically new Mathematics, modern infinitesimal calculus. In general, unification of (apparently) different objective (or epistemic) levels is most likely to require new technical tools or even new conceptual organizations of the object matter; it is the result of a novel synthesis, not just of a transfer of (mathematical) techniques (even though a good practice of this transfer may eventually help in finding new tools). For this purpose, we try here to unify by concepts some biological structures and phenomena, and just hint to a possible mathematization. In this trans-disciplinary attempt, we plan to bring into Biology our experience of "theoretizing", which surely differs from the practice in Biology, yet it is applied on the grounds of a stringent dialogue with practitioners of this discipline.

Once more, a monist view, such as ours, should not make us blind: no doubt that "physical matter" is unique and that there is nothing else besides it in the world. Yet the methods we gave us to understand natural phenomena, along history, are not unified and we cannot pretend to unify them by a forced methodological monism. Even Physics, as a theoretical construction, is far from being unified (e.g. Quantum Mechanics and General Relativity Theory). In short, we are monists of the matter, not of the method, as the unity is in the matter, not necessarily in existing *theories*, where unity must be constructed. But the dialogue of disciplines may lead us to new ideas and methodological synthesis. We start here by focusing on specific issues, such as morphology and the related concept of complexity.

On coding and forms

The birth of the fantastic digital computers that are changing our life is grounded on some key methodological assumptions that are not always made explicit by researchers in the field. The "Laws of Thought" by Boole (1854), are the beginning of a trend that marked the perspective: encode thinking (and the world) into sequences of 0's and 1's and compute on them. But the "world" was actually begun to be encoded, in modern terms, by Hilbert, if we agree to give to the geometrization of space a crucial role in understanding reality. In “The foundations of Geometry”, by Hilbert, in 1899, all possible axiomatic approaches to Euclidean and non-Euclidean geometry were encoded into Arithmetic (Formal Number Theory) and their foundation was reduced to the problem of consistency of "potentially mechanizable" or formal deductions, at the time still an informal concept. When machines were first described, mathematically, in the '30s, many could finally dream of a complete encoding of reality (or its knowledge) into digital computers, these 0-1's arithmetic devices that finally made precise the notion of certainty as mechanical deduction from evident axioms. Elementary *and* simple axioms, of course.

This bold enterprise had a major fall-out, namely the concrete electronic devices that we all know, and originated two remarkable mathematical theories: Computability and Information Theory, both referring to the elaboration and transmission of strings of 0's and 1's. Since thought, as well as all "finite" structures in space and time, can be encoded by them, they seemed to provide a complete account of all possible processes, including biological and cognitive ones, in space and time.

Yet, something is missing. When encoding a geometric form or structure, a relevant part of the information may be lost as well as any intrinsic measure of complexity, in the sense above. That is, the relevant properties of a geometric structure may "depend on codings" or, more precisely, they may be "sensitive to coding", in the same sense as a dynamical system is "sensitive to limit conditions". As proposed in [Longo, 2003], following an idea of J.-Y. Girard, we may even take this as a (very) broad definition of geometric: sensitivity to coding. When encoding, say, the real plane, as a geometric (topological, metric) structure, into the real line by Cantor's technique, everything is lost, as neither topology nor metric is preserved (the bijection is everywhere discontinuous). That is, in the continuous case, the information of "what is close to what" disappears; in discrete or finitary cases, it requires a very heavy coding that turns immediate (very simple) information into a very complex one. Lots of coding are necessary to encode geometric evidence. This may be fatal for an analysis of complexity and may modify the understanding of the informational content. Clearly, the mathematical impossibility to preserve topological information by Cantor's encoding of an infinitary (continuous) structure becomes "just" an issue of complexity and amount of information, over discrete or finite structures. But we are discussing precisely of information and complexity, here. Yet, some existing attempts to measure morphological complexity are based on program complexity: the measure is given by the minimum length of the (or a) program that generates the intended form. This seems largely unsatisfactory, by the coding issue we just mentioned (compiled programs work on strings of digits) and by the strong assumption that any structure of reality is engendered (or may be simulated) by a computer program or alike. An assumption yet to be justified, in particular when referring to biological phenomena (see [Longo, 2003d]). In order to avoid this unjustified detour, we prefer to look at complexity "intrinsically" and, more precisely, as for life, we will focus on complexity as *nesting and interaction of organization levels* as "loci" for integration and differenciation (our main perspective, see below). In short, thus, our long term aim is to propose an "intrinsic theory" of morphological complexity. This paper is a first step in this direction.

2. Objective and Epistemic Complexity

In order to propose a sound notion of structural complexity, we emphasize two aspects of complexity, that we call **objective** and **epistemic**, respectively, and that will accompany our entire work. A reference to physical examples, which are easier to give than the biological ones, will, hopefully, help their understanding.

Objective Complexity. By this notion we mean the internal relations and the (possibly) mathematical description of the intended system. This includes:

- the topological structures, including different "stratifications" (superposing and nesting of topologies); in biology: the number of and the relations between levels of organization, see picture 1;
- singularities (number and equational descriptions);
- the (mathematical) nature of dynamical processes (equations, structural stability, that is attractors' basins, homeostatic areas)
- the (mathematical) structures of relations to the environment, including its internal representation (in biology: the relation to the ecosystem, including, say, chemiotactism, phototropism, cerebral maps)

Physical examples. A fully mathematized objective complexity of a physical system is given by the degree and the number of equations that represent it. Six or nine non-linear equations fully represent the mathematical (objective) complexity of two or three bodies, respectively, in a gravitational field: they entirely give the internal relations of the systems, namely Newton's gravitational law of (reciprocal) attraction. This complexity is not particularly high, but the example will yield a much higher amount the other sort of complexity, the epistemic one. Another example which further justifies our distinction, may be the one point system governed by the equation $x_{n+1} = k\ x_n(1 - x_n)$. Objectively and mathematically simple, this system may turn out very difficult to describe geometrically (epistemically hard), as a shape (form) generated over the interval [0,1] (see below).

Epistemic Complexity. By this notion we mean the global functions of a system, the external description of it as given by the knowing subject (thus "epistemic"). This includes:

- the representational graph of a process;
- the classification within a context;
- the anatomic description;
- physiological and relational functions;
- the mathematical modeling of the major (biological) invariants (metabolism, reproduction, numerical invariants, e.g. metabolic and functional clocks).

Physical examples. The epistemic complexities of the two or three bodies systems above radically differ. Two bodies will move along keplerian orbits, perfect ellipses, very easy to describe as solutions of the six equations' system. The three bodies' system instead, as known since Poincaré, presents some major difficulties in finding the solution and the orbits follow very complicated patterns: they provided the paradigm of a chaotic system. In short, any description of the evolving system is highly complicated, and its dynamic engenders a very complex pattern (possibly stabilizing into an attractor, whose structure highly depends on – is sensitive to - the initial conditions). But also the sequence $x_{n+1} = k\ x_n(1 - x_n)$ above, for some k, is sensitive to initial conditions and yields a chaotic behavior over [0,1]: for some values of k, minor changes of x_0 induce major variation on the sequence $\{x_n\}_n$, which also happens to be a dense subset of the space of values, [0,1]. Once more the resulting pattern and its description are epistemically very complex (in short, they are "hard to know and to give explicitly").

3. Complexity and an alternating invariant: life units and their organs

As a starting point towards Biology, we will now elaborate on and apply the "alternating organization" of living entities, first suggested in [Bailly *et al.*, 1993] and analyze its complexities. The approach in this section provides us with the most basic (but not simple!) example of complexity as *nesting* and *interaction* of levels of organization, that we consider a core aspect of the complexity of biological phenomena. The example should also help to make more precise the notion of "form" we refer to: the way matter is organized into a living unity is considered a form by us, or the morphology of a (many level and) living structure is part of that very broad concept of form we adopt here. In a sense, even the graph below is a form, as it spells out, by its structure, the epistemic complexity of the intended object of study. The resulting invariance may be seen in the iteration of the same pattern at different levels.

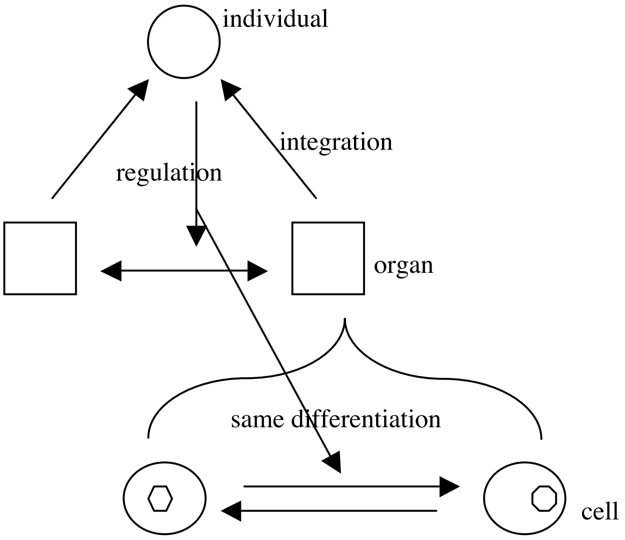
We first start by a diagrammatic description of an individual metazoan, to be extensively discussed below. In picture 1:

- The circles ○ represent a **living unit** (a metazoan or a cell);
- the squares □ represent **organs**.
- **Integration** (of organs into the individual) is described by upwards arrows ↗
- **Regulation** is represented by downwards arrows ↓ .
- Horizontal arrows → ↔ represent **interactions** (mainly energetic exchanges between organs; information – and energetic - exchanges between cells).

- The horizontal parenthesis ⏞ stays for **integration** of cells **into an organ**.

Note that the integration of organs into an individual (the upwards arrows) is a consequence of the embryogenetic unity which yields a functional integration. The integration of cells into organs (the horizontal brackets) is given by the common genetic expression. Regulation mechanisms are “downwards” effects and are crucial to the living unity, both as organic regulatory effects and as cellular regulations. In particular, the major functions of organs (respiratory, immune system ... but movement as well) are subject to neural and hormonal regulations. Similarly, cells and intra-cellular regulations include genetic, enzymatic ... regulations. The mechanisms for this may be physical or biochemical, dynamically governed by the values of parameters, by re-entry mechanisms which lead to a dynamic homeostasis, an “homeorhesis” as "regulation while changing", eventually deregulated by pathologies or death.

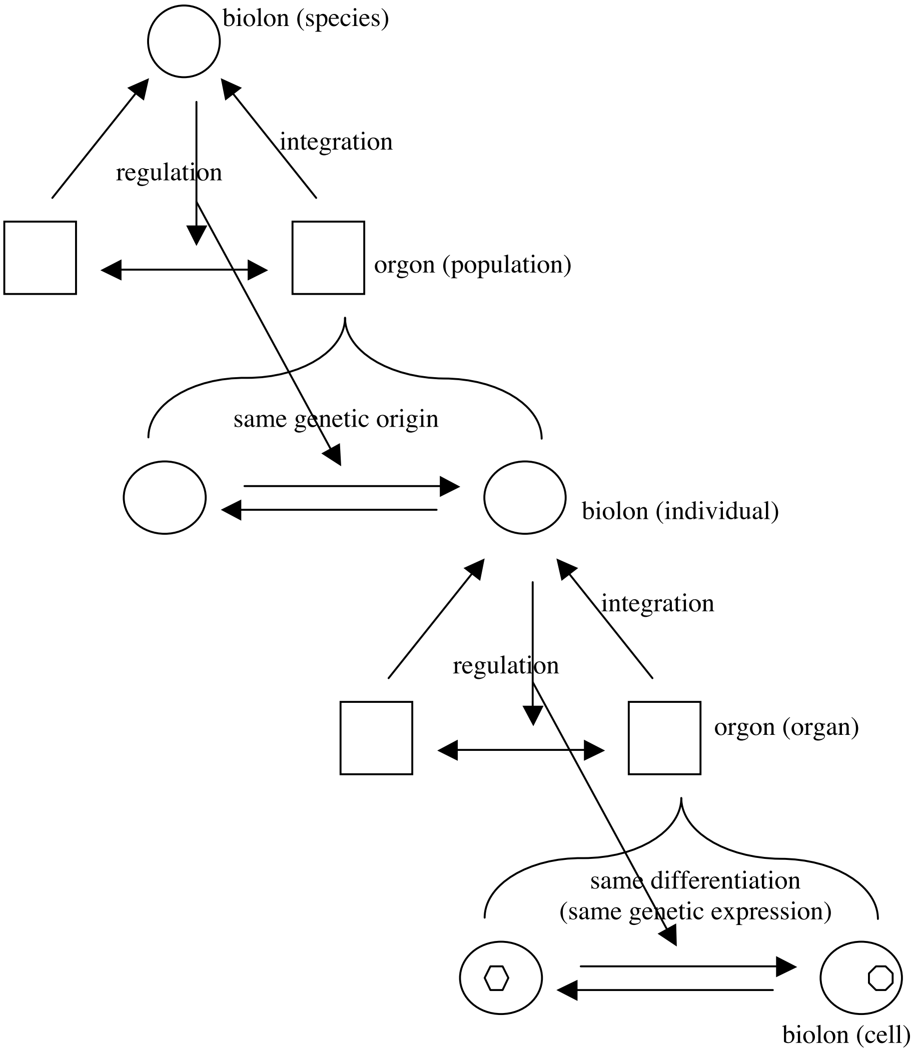
Finally, inside cells the squared-circles are *organelles*, some of which are former living units, the mitochondria. Their are the energetic exchangers, apparently originated by ancient parasites or early protozoan.



Picture 1

Our analysis of complexity aims here at a unified conceptual understanding of the nesting and interaction of various levels of organization in Biology. The interesting fact that the basic structure hinted by Picture 1 may be iterated at three levels of organization: living units apparently regulate and integrate each other in a conceptually similar way, as Picture 2 tries to depict.

In order to introduce a unified approach, by stressing invariance, we call **biolon** a living unit (a cell, an individual ... we will also extend this term to an entire species). Biolons are composed by **orgons** (the organelles of a cell, the organs of an individual plant or animal ... the organized populations of a species). We plan to justify this apparently arbitrarily unified terminology, in reference to common genetic origin and similarities in functionalities and behavior (including interaction, in an environment). But the first glance to Picture 2 should immediately evidentiate what we mean by the presence of invariants in the nested structure of interactions, proper to life. As we will point out, several levels within this nesting are deeply analyzed by (different) mathematical theories, each isolating the intended level from the interactions with the lower and higher levels. Yet, the mathematics of the *system* is still missing.



Picture 2

Before discussing in some more details this nested/iterated representation, observe the peculiar way this graph "collapses" at the bottom: a sort of "fixed point" is given by the identification of a primitive orgon (an organelle) with a formerly "autonomous" biolon (mitochondria have an independent genetic patrimony) and this within a biolon (a cell). Note also the enormous complexity of the elementary components of this hierarchical structure, the cell: it almost contains a "compressed image" of the entire graph, by its orgons, "internal-biolons" and their integratory and regulatory phenomena. That is, one may find in a cell complete genetic information, the main protein pathways, the basic paradigms for energetic exchanges ... in short, a cell almost has the objective complexity of the overall graph. Moreover, these complex elementary components establish very complex interactions, including early phenomena of symbiosis, which further develop into the epistemic complexity hinted by the structure in Picture 2.

Both the horizontal and the vertical structure of the graphs above can be better specified by analyzing more closely the different role in it of our notions of complexity, that is the interactions and the nesting of activities and forms that characterize the graph.

Horizontal structure:

Objective Complexity: *Genotype*; exchanges and reactions (energy and information; physiological reactions as for orgons, physical and bio-chemical reactions as for biolons - the horizontal arrows).

Epistemic Complexity: *Phenotype*; the (dynamic) structuring of orgons (e. g. as fractals, see below) and of biolons and their association (as networks of interactions or dynamical systems).

Vertical structure:

Objective Complexity: Relations between levels: *Integration* and *regulation* activities of biolons and orgons (by bio-chemical cascades, hormones, the neural systems - oblique and vertical arrows).

Epistemic Complexity: graph-structure (arrows and links, that is *the phenomenal appearance of the interactive nesting of objective complexities*) .

On the other hand, the **global structure** of the graph, up to the intended level, displays the epistemic complexity by the corresponding anatomy and physiology (of a cell, level I; a individual metazoan, level I and II; or the organization of a species, level I, II and III). The objective complexity of the whole is given by the "sum" of the interactions displayed, vertically and horizontally.

The horizontal *objective complexity* (and the intended exchanges and reactions), are directly influenced (regulated) by the vertical exchanges. Yet, vertical and horizontal exchanges belong to different interactive frames: the horizontal exchanges do not dramatically change during phylogenesis or ontogenesis, in reference to the environment (smooth limit conditions of the processes). The vertical ones, instead, depend on the formation of new levels, along the vertical organization of the graph. This yields a (possible) discontinuity along phylogenesis and ontogenesis (it corresponds to the formation of new levels of organization: new individuals and new organs).

From the graph, we may extract this further information:

1. The *objective complexity* is a matter of both:
   1.1 organization (and this is given by the topological structure of each level – what is close to what - and its nesting w. r. to the next ones) and
   1.2 functions (interactions within a level and between levels).

Similarly,

2. The *epistemic complexity* is a matter of both:
   2.1 organization (but this is anatomy, now) and
   2.2 functions (physiology).

The difference between these points should be clear. The "structure" in 1.1 refers to the local complexity, including the topological proximity and relative organization in relation to other levels. In 2.1, it is a matter of anatomic complexity (of an organ, of an individual entity – a biolon), as *described*, this is the sense of the "anatomic description". Similarly for functions: in 1.2 this refers to functional/internal exchanges (biochemical etc.), while 2.2 corresponds to the physiology of these exchanges, including in its relation to the environment, and their *description.*

Our tentative organization of the nested complexity of life, by pointing out some iterated invariants (the vertical and horizontal structure of the graph), may contribute to appreciate the relevance and the limits of some long lasting mathematical work in Biology. The Mathematics of fractals has been giving major contributions to the analysis of orgons and their shaping in the environment: from phyllotaxis, [Jean, 1994], to more recent investigations of the fractal structures of lungs and the vascular systems, [Mandelbrot 1982; Horsfield 1977; Lefèvre, 1983; Bailly *et al.* 1989, West 1997]. Energetic exchanges are at the core of these insightful physical analyses.

Biolons and their interactions, instead, have been mostly studied by the Mathematics of Dynamical Systems, from neural systems to phylogenesis. Co-evolutive dynamical systems are a most advanced frontier of this analysis: inside an orgon (the brain, the population of a species), biolons interact among each other and with the environment dynamically and co-constitute the phase space. Yet, this is exactly the challenge: phase transitions are very well handled by this approach, along the lines of Mathematical Physics, but vertical interactions, along the graph above, may cause a change of the *phase space*. That is, the interference, along the vertical arrows, between an upper and a lower level, each treated by different mathematical tools (fractals or some dynamics of interaction, typically), may yield a change in the pertinent variables or in the entire space of phases. For example, a latent potential, expressed as a novel orgon in some biolons of a population (the Panda's thumb, the hears of some reptiles, which developed from the double jaws' bone of their ancestors, about 200 millions years ago ...) may radically modify the behavior, thus the forms of interactions In our graph. This means that fruitful analyses of one horizontal level (orgons or biolons) may find their limits and face major mathematical difficulties when extending the interaction vertically, in particular since, at each level, researchers have been developing, and not by chance, their own specific insight and tools. The complexity of life is not only in the internal dynamicity of each level, but also in the dynamic unit of the nested structures, whose phenomenality is also due to the integrative and regulatory, vertical, activities we tried to evidentiate.

3.1. Complexity's variations along evolution.

One more issue should be mentioned, which further evidentiates the invariance in the graph of Picture 2 as well as its explicative role. Living beings basically use the same molecular structures and biochemical pathways to sustain and reproduce. Thus, we could represent, in our graph, similar (objective) interactions among biolons at the various levels. Let's now understand by this the apparently paradoxical claim, often accepted in biology, that the "complexity" of life did not change much along evolution: "everything" was already there, with bacteria (see [Gould 1997]). It is actually so, provided that one refers to *objective* complexity of *functions* and to the basic stability of the genes, throughout species.

Epistemic complexity instead dramatically changes along evolution and ontogenesis, both as for structures and functions (phenotype, anatomy, physiology, environmental interactions...). Biologists, who mostly care of these aspects, soundly disagree on the stability of life, as for complexity, claimed by those who focus on its objective side. Thus, in our approach, both views seem justified, provided that a clear distinction is made on what is observed; in short, objective complexity is essentially stable, while epistemic complexity grows.

One may further refine this viewpoint by introducing a degree n of infinitesimal variation of objective complexity, according to the intended biological level (lowest at the cellular level, highest at the level of species). If $\square^n$ is infinitely small, of degree n (i.e. relatively to the intended biological level and its observable quantities), then write:

$$\text{(Eqs 1)}\quad\begin{cases} \square K^{(n)}_{obj} \approx \square^{n} \text{ (which expresses the approximate stability of the objective complexity)} \\ \\ \square K^{(n)}_{epi} > 0 \text{ (which expresses an increasing epistemic complexity).} \end{cases}$$

Note that the first approximate equation may be compared to the physical situation, where energy is preserved while entropy is produced. The second inequality may be understood as a decreasing component of entropy, due to the growth of organization.

In synthesis, in this approach, the objective complexity in a paramecium, a mouse, and an elephant does not change much: same internal bio-chemical cascades and functions, similar relation (integrative, regulative) between the individual and its organs. Of course the epistemic complexity changes and even between the two mammals one may find important anatomic and physiological differences, up to very different behaviors in relation to the environment (the two mammals will share other "absolutes", the respiratory and circulatory clocks, but this will be discussed later).

4. Complexity and states.

In a previous section, we started by giving motivations (and examples) from physics to our conceptual distinction between objective and epistemic complexity. Once more now, Physics may help us to introduce concepts, even though the difficulty largely lies into their transfer to biological phenomena. Yet, clarifying some aspects of the issue in Physics may guide the distinction w.r. to the peculiarities of phenomenal life.

4.1 From gas to crystal, from local to global.

A crystal and a gas are at the opposite of the scale, as for order. Yet, the complexity of both states is relatively low: their objective complexity requires a simple mathematical description. Epistemically, either physical state yields simple structures: a perfect, iterated order, or a complete disorder.

The actual complexity shows up (grows) towards intermediate states (quasi-crystal, glass, viscous, liquid,...). These states blend order and disorder, by an average increase in complexity towards the middle state. The classical notion of entropy, as an increasing quantity with disorder (from crystal to gas) may provide a qualitative expression of this: let S_m and S_M the least and highest entropy, respectively (crystal vs. gas), then epistemic complexity may be *qualitatively* expressed as a function K of the entropy S of the system, namely

$$K = a(S/S_m - 1)(1 - S/S_M) .$$

But Physics suggests two further analytic elements concerning these changes of states: the notions of *phase transition* and of *correlation length.*

The average complexity variation qualitatively expressed above does not consider the changes from one state to the other that is the phase transitions as changes from gas, to liquid, to crystal. These go trough a *critical* state, which radically changes properties and parameters (a current mathematical image of a criticality is given by a maximal point in a curve: this is a (local) maximum for a potential – a ball on the mathematical top of a hill, say). In particular, phase transitions change the correlation length, that is the distance of (possible) causal relations between elements of the physical structure. In a crystal, this length is of the order of magnitude of the crystalline structure; in a perfect gas, it is very small (particles must almost touch each other to interact). When changing state, the global structure is entirely involved in the activities of the single particle: the local situation depends on (correlates to) the global. Mathematically, this may be expressed by the fact that the correlation length formally tends to infinity; physically, this means that the determination is global and no more local. Yet, the transition happens at a finite length, which is usually treated by renormalizing measures and parameters, at state or phase transition[2]. Of course, in Physics, a critical state cannot last for long.

In this simple example we have seen the notions of critical state, phase transition, correlation length at work. The scale of observation matters crucially: a critical point as maximum for the average complexity may be seen as a divergence point as for correlation length, at a smaller scale (the largest distance of possible causal interactions between elements of the system).

In conclusion, moving from local to global, in Physics, requires the passage through a critical state; this is understood as a mathematical divergence of a correlation length, thus as a critical point where infinitesimal variations yield finite changes (or finite variations lead to infinite changes, mathematically[3]). Our claim below is this may help to understand a biological state as a "long lasting" physical singularity, an "extended critical state", where, in particular, homeorhetic processes sustain a permanent tension between local and global. A physical situation that, mathematically, is understood as the locus of a divergence, a passage from finite to infinite, and as the singular point where the "pertinent object" changes (renormalization, see note 1): the local is integrated into an novel object, the global one (the living unit or biolon). Thus, *in physical terms*, the objective complexity measure of the least living unit, a cell, a global structure w.r. to its components, has an infinite value. That is, if one considers physical parameters as for measures (correlation length and related effects), living matter is in an ongoing critical state, as a permanent passage/tension local vs. global. Thus, a singular dynamic unity, it is infinitely more complex (in a mathematically precise sense) than any physical process, which may behave critically only in exceptional, short lasting, singularities.

4. 2 Life as an "extended critical state".

We are now ready to elaborate on an important theme introduced in [Bailly, 1991]. A critical state, in Physics, is a singularity in a process: that is, along a process, a position, a configuration, a "state" may be assumed *briefly* before the global state of the observed matter (radically) changes. A critical state may also be seen as a bifurcation or, in some cases, a catastrophe, in the sense of Thom, in particular if the intended change turns out to be irreversible. In a sense it is the opposite of a situation of equilibrium (the opposite sign in the

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[2] Renormalization is a fundamental mathematical technique that originated in Quantum Electrodynamics. From a conceptual view point, it is a "redifinition" of the object considered, by adding to its initial characteristics (its mass, typically) some classes of interactions that modify them. More broadly, the "renormalisation group" provided the mathematical tool used to represent the passage from local to global along critical transitions, also in other areas of Physics: it describes a change of measure and object, obtained by integrating the new interaction classes due to the transition.

[3] Another classical example in Physics is given by the critical transition from paramagnetic to ferromagnetic states in a spin system. When approaching the critical temperature Tc, an order appears (the global magnetic momentum), the thermic desorder becomes less relevant than the novel energetic component due to the interactions between spins – whose alignment prevails over the thermic energy. At the same time, the correlation length L increases up to the volume of the total system. Note though, that L is given by an increasing function of $(T/Tc - 1)^{-v}$, where v is a positive (critical) exponent (often close to 1/2); thus, L goes to infinity. Magnetic susceptibility, as well, goes to infinity, according to its own critical exponent. In this sense, at the critical point, finite changes induce infinite consequences (or infinitesimal changes, finite ones).

mathematical description); and it differs as well from "being far from equilibrium" as this situation, in general, does not imply *possibly different evolutions* of the system (bifurcations).

By definition, a system cannot stay "for long" in a critical state: the very development along time forces it beyond that state. It is a typical aspect of transition, where minor fluctuations, possibly below the level of observability, may lead towards radically different evolutions. The instantaneous nature of criticality is beautifully expressed in Mathematics by the divergence (towards infinity, see above) of some functional descriptions, according to the intended parameters. Or, also, as discussed above, by the maximality of complexity, which also yields instability.

The claim here is that life phenomena belong to a permanent, more precisely "extended", critical state. That is, a life unit, a biolon in our terminology, is critically unstable and it is kept in this situation, far from static equilibrium, by homeostasis, or, better, by homeorhesis. Or, the dynamic integration and regulation of its components (orgons, with their components, biolons), its "ago-antagonistic" relations [Bernard-Weil, 2002] within itself and the environment, sustain it in a physically implausible situation. Autopoiesis is another way to express this auto-constitutive dynamics, [Varela, 1989]. A conceptual organization which may be related to autopoiesis is that of several coupled endomorphisms, in mathematical terms; their "organizational closure" may then correspond to the limits of structurally stable attractors; the interior and the membranes may be understood as attractors' basins and borders, respectively.

As soon as integration or regulation fail or overcome the limits of the non-pathological state, everything collapses: entropy suddenly grows, disorder is death. Running on a tense rope is a good image of the ongoing being of a biolon: when control, as regulation and integration, decreases beyond a certain level (the critical borders of the extended critical state, as the acceptable limits of pathology) death ends this contingency, life, by a final, irreversible state transition. Of course, within the borders of the extended critical state, phase transitions, changes in correlation length, passages through singularities ... occur continually, but, within the intended limits, they are dealt with by the regulatory activities. Actually, they are an essential part of it: all the bio-chemical thresholds that contribute to exchanges within a biolon can be seen as elementary components of the global homeorhesis. More globally, life itself may be seen as a "extended physical singularity".

The physical paradigms helped us to formulate this notion, which is not of a physical nature. Again, monism should not make us blind: there is nothing else but (physical) matter in the world. Yet, we are facing different phenomenalities and we make them intelligible by conceptually (and mathematically) organizing them in different manners. A synthesis is far from obvious; it must be constructed by a novel conceptual (possibly mathematical) unity, a long-term goal.

5. Integration, regulation and causal regimes

A few more facts may stress the difference between physical and biological phenomena (but the list is incomplete and far from fully theoretized). Causal relations are local in physics; they may be global only in the sense of a field, which relates physical entities by propagation of *local* interactions. That is, the global structure, and its correlation length, is obtained only by the transitivity of the local interactions. Thus, in either physical case (local/global), Mathematics, by fixing a space of phases, isolates a unique level of causality. In Biology, instead, local causality may differ radically from global correlations, yet it cannot be isolated from the latter: integration and regulation, typically, *causally affect* local interactions (local bio-chemical exchanges may be regulated by cascades of hormones or neural signals of an entirely different nature). In other words, the global causal regime may differ from the local ones, yet they may permanently interfere with each other: thus, the unity of a biolon is given by regulatory-integrative activity, which are of a different physical/biochemical nature w.r. to exchanges between and within its orgons, yet these exchanges are regulated by (and affect) the global. Note that it is the global regime that essentially contributes to the homeorhesis as maintenance of an extended critical state; its fluctuations, within the borders of criticality, corresponds to the pathologies a biolon may live with as well as to forms of "local death" (cellular apoptosis in an orgon). Of course, fluctuations exist in physics (and may be "tolerated"), but the underlying physical entities do not change along fluctuations. In this sense, in physical theories, there is nothing that resembles to phenomena such as pathologies and local apoptosis; by this, their mathematics has still to be invented, as much as their pre-formal conceptualization.

Thus, in view of the extended criticality of life and its peculiar causal regime, but just as a methodological proposal (no ontology), it may be sound to refer to the apparent teleonomy of life, as a phenomenon analyzed in terms of a contingent, material, finalism; as a matter of fact, this is preferred by many in Biology w.r. to the assumed theoretical reduction to one of the existing physical causalities, always promised, never fully spelled out. In other words, a teleonomic approach may be justified, when purely pragmatic, by the lack of a better account of the amazing ability of biolons to live in a state of maximal, physically unstable (we should even say: physically unsuitable), state of complexity, that we described as extended critical state and as a dynamical interference of global and local causality. In our views, though, a notion of biological causality can be better approximated by an understanding of the nesting and interaction of organization levels we tried to describe, since it depends on the vertical interactions, as schematized in our graph, a sort of geometric representation of the integration and regulation activities. It is this local/global interaction that organizes the “internal drive” and maintains the extended critical state, at least at the phenomenal level (see also [Stewart, 2002; Rosen, 1991]).

In summary*, integration is the (upward-causal) presence of the local into the global structure, while regulation is the global structure that causally affects (downwards) local ones*. In spite of the physical singularity of life phenomena, let's once more try to spell out our understanding in familiar physical terms. *Regulation* may play the role of the initial or boarder conditions on global behaviors of the solutions of equation systems (differential or at finite differences), that is of dynamical systems described by these equations. *Integration* may be understood, by a rough analogy, as the correlations of variables that give unity to a given system of equation; or, also, as the organizing role of singularities in their solutions; or, similarly, as the analytic extension of a locally defined solution. Once more, we are just trying to approximate by physico-mathematical concepts a phenomenology that, so far, goes largely beyond these descriptions, in the infinitary mathematical sense spelled out in §.4.

6. Stability and variability

We largely focused above on the stabilizing role of regulation and integration within the nesting and interaction of organization levels. Yet, phenomenal life is not only characterized by the preservation of some key invariants, as a result of stability, but also by variability. Typically, individual biolons all differ from each other: the individuals of a species do not need to be identical. Even cells have some form of "individuality". Indeed, variation is at the core of evolution and, jointly to individuation, underlies ontogenesis as well. Clearly, this is yet another singularity of phenomenal life w.r. to physical descriptions, where individual entities of the "same type" are all identical and their descriptions may be, conceptually and thus mathematically, perfectly stable.

6.1 Biolons as attractors and as individual trajectories

On one side, then, a species has a global structural stability, while its members may show individual variations in nature and behavior. Similarly, at its time scale, an individual is stable, while its cells evolve and die, each along a different path. In either case, the global structure is essentially (but not fully) preserved, while local variations take place. As a physical analogy, the larger *biolon* may be described by the geometry of an *attractor*, whose dynamics is essentially stable, in spite of the variations and even the instability of *individual trajectories*, which may fall within the same attractor (the *smaller biolons*, included in the larger). Then a biolon of the intermediate level (a metazoan) is a trajectory of a species, the latter being considered as an attractor; at the same time, that biolon behaves like a stabilizing attractor for its components cells.

Note that, in Physics, the dynamics of trajectories is given by geodetics in the intended phase space; thus, these are selected according to the highest stability and in a unique way. The individual biolons, instead, in our understanding, are represented by different simultaneous path, just *compatible* with the limit conditions (which in turn do not need to be stable and are modified by the ongoing process). Then, our approach is based on

- the interplay of global relative stability vs. individual variability (attractor vs. trajectories),
- compatibility instead of optimality (as staying within the evolving borders of the attractor),

where each individual trajectory starts on an instable state and actualizes different contextual potentialities (typically the first mitosis in embryogenesis, within the same genotype) and evolve along accessible potentialities, within the global structural stability.

Natural selection may reduce variability at the level of species, while increasing it at the level of individuals. More or less sustainable pathologies would concern instead cells within individuals. Of course, this conceptual model perfectly fits with the notion stressed above of "extended critical state", as the paradigmatic state of biolons. Moreover, it may also be seen as a specification of Waddington's notion of chreod [Waddington,1977].

Observe that this schema refers only to biolons at the various levels, since biolons only are concerned with identities (preserved identities, along change). Orgons would constitute instead, by energetic exchanges and functional activities, the material support for stability and variation: an orgon is not a trajectory, but it may support the ongoing variation of the individual represented by the trajectory.

As for objective and epistemic complexity, (Eqs 1) in 3.1 represent the quasi-invariance of objective complexity along the evolution of an attractor as well as the growth of epistemic complexity. As mentioned in that section, phylogenesis and ontogenesis do not modify greatly the basic objective processes and their complexity (energetic exchanges and reactions as well as the genotype), yet they affect the phenotype, the anatomic structure and the interactions, as epistemic complexity.

6.2 Stability as invariance. Absolute numbers.

In Physics, but in Mathematics as well, the "identity" of an object, of a structure, is phenomenally given by invariance or stability w.r. to certain transformations.

In Biology, classes and species are characterized by morphological, functional and genetic factors (for a recent account, see [Lecointre, Le Guyader, 2001]). These factors also include some numerical invariants, which allow correlating transversally different species. The cardiac and respiratory absolute numbers, a constant for all mammals, on the average, are the most familiar of course. The different rhythms have an allometric relation with mass or size W, as a scale factor. Some of the basic relations may be summarized as follows:

$R = \square_R W^{3/4}$ (metabolism, exponent 3/4)
$\square = \square_{\square} W^{1/4}$ (life time, exponent 1/4)

Some types of cells, share an absolute number of subdivisions, during their life span. The cases discussed in [Ameisen, 1999] concern fibroblasts whose telomers are shortened at each meiosis. The total number of splitting, an invariant in different contexts (living tissues or even "in vitro"), is approximatively 50. But, "in vivo", this gives several decades of life; while, in vitro, this invariant only yields a few weeks of life. Frequency depends then on control parameters, which refer to the overall mass of the biolon to which the cell belongs (itself or a complex organism).

In §. 3.1, we stressed that the different views on the changes in complexity, along evolution of species, could be understood in terms of the distinction we proposed between objective and epistemic complexity. Biochemical reactions and energetic exchanges where key factors of objective complexity. In some cases, the epistemic (thus descriptive) approaches may yield very different classifications of phenomena, which are objectively very close. A paradigmatic example is given by respiratory functions. These are based on biochemical exchanges of oxygen, which may take place both in air and in water. Of course, the corresponding important changes in organs are related to very different species, that is, in our description, to very different geometries of attractors.

On the other end, one may consider the underlying energetic exchange as equivalently given in anaerobic fashion: metabolism of glucose, in cells, may replace oxygen exchange (this is even possible, exceptionally or pathologically, in situations where the oxygen exchange is the norm). In these cases, the invariants are very low-level phenomena, which "go across" very different "objective" organization of life (the nature of the biochemical exchange), while the epistemic complexity (the physiological function within a cell) doesn't change much.

Finally, neural and vascular plasticity do not change either objective or epistemic complexity: they are internal invariants of an organism. However, the role of the first in the dynamics with the environment may be considered a variation of epistemic complexity as learning and behavioral change (an external component of complexity).

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