"Physical Determination, Unpredictability and Undecidability in Critical Processes"

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Poincaré's Three Bodies Theorem opened the way to a new area of Physics and Scientific Knowledge by a "negative" result: we cannot predict, actually compute, the evolution of a simple planetary system of just three interacting bodies in their gravitational fields. The point is that "des nuances presque insensibles" in the reciprocal actions, as Laplace had feared (and hopped to disprove) may affect the long-term evolution. One may view this result, and the many that followed it, as an "undecidability" result for the intended formal system (the system expressing the set of equations – just nine in Poincaré's case). Twenty years later, one hundred after Laplace, Hilbert conjectured the decidability of a large class of formal systems, at Poincaré's dismay. The analogy is evident, yet there are several differences. First, unpredictability is "in finite time", while undecidability, a la Turing say, is "at infinity": for example, recent results show that we provably cannot predict the situation of the solar system in 1 million years (which is not too long, as astronomical time), while the halting problem is a question concerning infinite time (possible divergence). In a sense, thus, Poincaré's unpredictability is "stronger" than undecidability, where it applies.

A way to relate mathematically the two issues should explicitly refer to a crucial physical assumption: measure, in Physics, is always an interval, by (classical) principles (the thermal fluctuation, to say the least). And "les nuances presque insensibles" are meant to be perturbations or fluctuations below the measure interval, which may *cause* dramatic changes in a not too remote future; the approximated physical measure and the sensitivity to initial or border conditions are, jointly, *the reasons for* unpredictability. Thus, we hint to an analysis where the evolution of a dynamical system is seen, computationally, as the evolution of an interval, the intended physical measure. Of course, it is the non-linearity of most interesting dynamical systems that poses major problems: first, the interval may expand exponentially (this is part of what Turing calls the exponential drift, in his fundamental Morphogenesis paper); second, and more importantly, the dynamics is mixing (the extreme points of an interval are very soon no longer extremes, maxima and minima are not preserved).

We will finally mention some critical states, where divergence of observables, w.r. to the intended control parameter, makes the computational simulation extremely unfaithful, as minor fluctuations, well below the best round-off, may radically change the evolution or even lead to divergence. These states, often observed in phase transitions, may be seen as "limit" situations for physical theories, when extended beyond isolated points; indeed, as external limits, conceptually. We will hint to two reasons for this. In an interesting one-dimensional dynamic, the transition from countable infinity to continua suggest an informal link to cardinality issues of the continuum, which have no physical meaning, but may recall well known problems in Mathematical Logic. Moreover, extended criticality may be seen as our way out of physical theories in order to deal with the singularity of life phenomena.

Some references (*downloadable*: http://www.di.ens.fr/users/longo):

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