MATHEMATICS AND THE NATURAL SCIENCES.

The Physical Singularity of Life.

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INTRODUCTION

This book aims to draw a possibly unified conceptual framework in reference to the current state of the sciences – mainly physics and biology. This framework will be put into close relationship, while not being subordinated, to the analyses of the foundations of mathematics. As integral part of this framework, we will propose some principles for a modern philosophy of nature and will develop a theoretical approach for certain aspects of biology. This approach, all the while being inspired by physico-mathematical practices and conceptualizations, will clearly distinguish itself from common physical theories in the specification of living phenomena.

Our analyses will greatly involve reflections pertaining to the epistemology of mathematics given that questions regarding "truth", "how reason functions", the role of "mathematical theorization/formalization" or "how knowledge is constructed", are highly correlated to questions pertaining to the foundations of mathematics. Referring to history only, this is the lesson we learn, as scientists, from Plato, Descartes, Kant, Husserl, Wittgenstein and many other philosophers whose theories of knowledge so often interact with reflections on mathematics. But why would mathematics be one of the pillars of the intelligibility of the world? Why would any philosophy of knowledge or of nature refer to it, while most foundational analyses (Platonism, formalism, logicism…) explicitly posit mathematics to be "outside of the physical and biological world"?

According to the analysis which we develop here, mathematics helps, on the one hand, to constitute the very objects and objectivity of the exact sciences, because it is within mathematics that "thought stabilizes itself". In this way, the foundation of mathematics "mingles" with that of other spheres of knowledge and with their constitutive dynamics: it is the very result of our practices of knowledge. On the other hand, the conceptual stability of mathematics, its relative simplicity (it can be profound all the while stemming from principles which are elementary, sometimes very simple) is at the center of the connection which we will draw with certain elementary cognitive processes. We will mainly refer to those which reflect regularities of the world through our active presence within this world as living beings (living also in intersubjectivity and history). Particularly, it will be a question of grasping the role of action in time and space, as well as the organization of these by means of "gestures", as we will put it, and by means of concepts, rich in history and language, both being, since their origin, eminently mathematical concepts. It is for these reasons, in our opinion, that all theories of knowledge have addressed, in a way or another, issues pertaining to the foundations of mathematics, a "purified" knowledge, which is both mysterious and simple, where the analysis of reasoning is performed with extreme clarity and the construction of concepts is rooted upon praxes originating with our humanity.

Symmetrically, a sound epistemology of mathematics must try to make a philosophy of nature explicit. In any case, such a philosophy is implicit to this epistemology, because the great choices concerning the foundations of mathematics, logicism, formalism, Platonism and various types of constructivism, including the "geometrical" perspective that we will adopt, contain an approach to the knowledge of nature which is, in turn, highly influenced. We will attempt to discern the consequences of this implicit philosophy for the analyses of human cognition.

In contrast to the very abstract paradigms which still dominate the foundations of mathematics, physics and biology still constitute themselves respectively around the concepts of *matter* and of *life*, which appear to be very "concrete" although being indefinable (if not negatively) within the internal framework of these disciplines. But they also present the difficulty of endlessly and essentially having recourse to the requirements of rational coherence, highly mathematized in physics, as well as to the necessities of adequacy, through experimentation, with an independent phenomenality, albeit conceptually constructed.

The specificity of living phenomena will be the object of a relatively new conceptualization, which we will present to the reader by means of a complex play of differentiation and synthesis of physics and biology. Theoretical delimitation is, in our view, a very first step in the construction of scientific knowledge, especially with regard to phenomenalities, so difficultly reducible to one another, which are the living and the inert. Quantum physics provides us with a very important paradigm for understanding this method: along its history, first were constituted the bases of an autonomous theory and objectivity, which were quite different from the well established causal structures of classical (and relativistic) physics and having their own analyses of determination (intrinsic indetermination of the simultaneous measurement of position and momentum; the non-locality and non-separability of certain quantum objects; the absence of trajectories as such within classical space-time… we will go back to this at length). Then, the problem of unification was brought forth, at least the problem of the construction of conceptual gateways, of relationships, of dualities and symmetries for objects and theories. Likewise, in our view, the theoretical autonomy of the analyses of living phenomena must precede any attempt, as important as it may be, at unification/ correlation to physical phenomenalities.

In this perspective of a highlighting of the theoretical differences as well as the strong correlations, or, of a desirable unity of epistemic nature between physics and biology, it is necessary to articulate and, if possible, to put into correspondence "law of thought" and law of object, abstract formalization and experience, in the specificity of their different roles within these disciplines. Now, it seems that is one of the first requirements of any "natural philosophy" to clarify and to interpret, in terms of theory of knowledge, this problematic articulation and, in order to do so, to identify the fundamental principles which ensure its synthesis within these sciences.

Our starting point will be a comparative analysis of the construction of objectivity in mathematics and in physics, two disciplines having been constituted with a strong intertwining with one another at the very origin of modern science. This reciprocal determination goes back at least to Galileo, but in fact, it has its origin within Greek science and extends to the geometrization of physics, which has marked the great turning point of the XXth century, with the movement corresponding to the physicalization of geometry. One will recognize, in this process, Riemannian geometry and relativity, the geometry of dynamic systems and, today, the geometrization of quantum physics.

To this end, our reflection introduces a distinction between "principles of proof" and "principles of construction" which we develop in parallel between the foundations of mathematics and of physics as well as within these disciplines. We will then understand the great theorems of incompleteness of formal systems as a discrepancy (a "gap") between proofs and constructions and we will discuss the so-called incompleteness of quantum physics in order to highlight its analogies and differences with mathematical incompleteness. Our approach lies beyond the debate having dominated the analyses of the foundations of mathematics for too long, a play between Platonism and formalism, and which has shifted away from any relationship to physics and our life-world. Particularly, we will refer to the cognitive foundations of mathematics, of which the analysis constitutes in our view a primary link to other forms of knowledge, via action within physical time and space and via living phenomena.

The reasons for a centrality of the relationship to space and to time, of the "active access to the world" which accompanies any form of knowledge, will often be emphasized. In this way, the constitution of scientific knowledge will, in our approach, be founded upon these relativizing forms specific to modern physics: the clear enunciation of a reference system and of a measurement, in the broadest sense, is in our view the starting point for any construction of scientific objectivity. Digital modelization, and thereby intelligibility by means of computerization, providing today an essential contribution but not being neutral with regard to this construction, will also be at the center of our analyses.

In this spirit, one of the viewpoints which we will defend here, from the standpoint of physics, consists in seeing in the "geodesic principle" (which concerns "action" within physical or conceptual spaces) one of these great constitutive principles of knowledge (a "construction principle", in our terms). These principles are not limited to the very strong determination of physical objectivities; they also seem to respond to profound cognitive requirements for any form of knowledge. The way by which the geodesic principle is called to operate within the great theories of physics will lead us to reflect, even more generally, on the roles played by symmetries, breakings of symmetry, invariances, and variabilities. In particular, we will closely examine the symmetries proposed and breakings operated by the computer as a discrete state machine: by these means, in our view, the latter's strength of conceptual framework and of calculus marks the intelligibility of the world, by imposing its own causal structure upon it.

Symmetries, invariances and their breakings not only manifest through physical phenomenality, but appear to effectively govern its objectivity, at an even deeper level of abstract determination and maybe even of cognitive regulation. It is precisely within this framework that we will examine the relationships that such principles, which contribute to construct and determine scientific knowledge, may bear with their cognitive sources. These relationships constitute modern science in a seemingly contradictory fashion: on the one hand, it could be an issue of mathematical retranscription, adapted to more or less formalized disciplines, of common mental structures of human cognition itself, in such way that the geodesics, or the symmetries of living matter and of action, could be correlated to their great mathematical formulations. On the other hand, and conversely, one could say, it is an issue of forced constructions which are most often counter-intuitive and which must break with spontaneous cognitive representations such as empiricity would tend to develop them within the limited framework of daily experience.

We will then attempt to address the relevance of these issues of general invariances for the field of biology and to see the way by which they seem to emerge given the current state of some of biology's sub-disciplines. The situation is actually particularly complex in biology, where the "reduction" to one or another of the common physico-mathematical theories is far from being accomplished (inasmuch as it would be possible or even desirable). In our view, one of the difficulties in doing this lies as much in the specificity of the causal regimes of the physical theories (which, moreover, differ amongst themselves), as in the richness specific to the dynamics of living phenomena. We are thinking here of the intertwining and coupling of levels of organization, of the phenomena of auto-organization and of causal correlation (global and local) which are to be found in biological phenomenality.

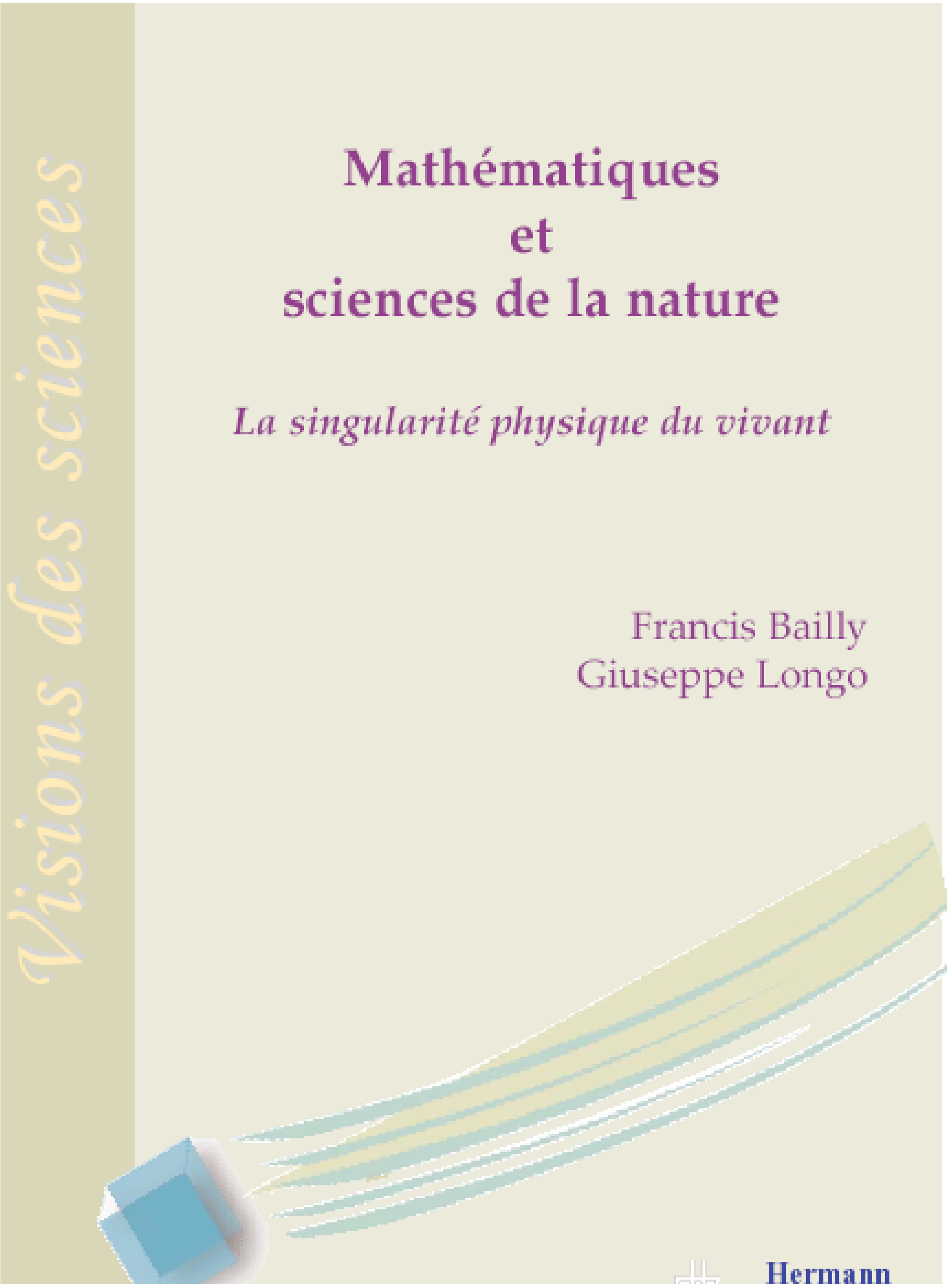
We will thus analyze these aspects as finding themselves in a "boundary" situation for physics. The "physical singularity of living phenomena" refers to the difficulties, which in our view are intrinsic for current physical theories, in grasping these phenomena which enable to maintain physical states such as those pertaining to life within an extended "criticality" in space and time, in a very specific physical sense, which we will discuss. It is thus physics that enables to speak of singularities, in a mathematical sense to be clarified (though informally). The latter are states which are not physical as such, or, better, which are not (yet?) addressed by existing physical theories. In short, for us, the biological is to be analyzed as a "limit physical situation", an "external limit" with regard to contemporary physical theories.

This discussion will be conducted all the while attempting to explain the theoretical and conceptual effects of our approaches upon the characterization – or recategorization – of concepts as fundamental as those of *space* and *time* such as they are addressed within the contemporary framework of the natural sciences.

To conclude and to summarize ourselves, in this book, we attempt to identify the organizing concepts of some physical and biological phenomena, by means of an analysis of the foundations of mathematics and of physics, in the aim of unifying "facts", of bringing different conceptual universes into dialog. The analysis of the "order" and of the symmetries in the foundations of mathematics will be linked to the great invariants and principles, among which the geodesic principle, which govern and confer unity to the various physical theories. Particularly, we will attempt to understand causal structures, a central element of physical intelligibility, in terms of symmetries and their breakings. The importance of the mathematical tool will also be highlighted, enabling us to grasp the differences in the models for physics and biology which are proposed by continuous and discrete mathematics, such as computational simulations.

In the case of biology, being particularly difficult and not as thoroughly examined at a theoretical level, we will propose a "unification by concepts", an attempt which should always precede mathematization. This will constitute an outline for unification also basing itself upon the highlighting of conceptual differences, of complex points of passage, of technical irreducibilities of one field to another. Indeed, a monist point of view such as ours should not make us blind: there is no doubt, in our view, that "physical matter" is unique and that there is nothing else in the universe. Nevertheless, the tools for knowledge which humanity has constructed, throughout history, in order to make intelligible natural phenomena, are not unified and for good reasons, reasons which are relevant to the very effectiveness of the construction of the scientific objectivity with regard to different phenomenalities. And we cannot claim to unify them by means of a forced methodological and technical monism, according to which such mathematical method or physical theory, constructed around a very specific phenomenal field, could make us understand everything. Even physics, as a theoretical construction, is far from being unified: Quantum Mechanics and General Relativity are not – yet – unified (the notions of quantum and relativistic field differ). And, as we have mentioned earlier, physicists aim for unification, not reduction, meaning that they aim for a new notion of field (either of physical object or of space-time) which will unify them, if necessary by putting each theory into perspective. Such is the case, for instance, with the theory of superstrings which proposes new quantum "objects", or with the radical changes in the very concepts of time and space, such as contemplated by non-commutative geometry. And, since Copernicus and Galileo, Leibniz and Newton – or since always in science –, these theories, for microphysics, construct their own revolutionary conceptual frameworks which are in fact counter-intuitive as well as constructing, afterwards, their own mathematical tools.

In short, we are monists in what concerns "matter" given that unity can be found, we presume, in this matter which enters into friction with and opposes our theorizing and experiences. However, unity is not necessarily to be found within the existing *theories* within which it must be constructed. The dialog of the disciplines aims to lead us towards new methodological ideas and syntheses: coherence in the intelligibility of the world is a difficult achievement. If it is possible, it is not to be found, we insist, within the claimed unicity of the theories currently available for addressing one or another of the historically given phenomenal fields. It is constructed, in our opinion, by means of conceptual bridges, sometimes of punctual reduction, capable of highlighting points of "contact" or of "friction", but also by means of theoretical differentiations or dualities and, if possible, of simultaneous changes in perspective within various fields. Conceptualizations which are at the boundary of widespread physical theories are at the center of our attempts in biology.



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