The goal of this exercise session is to review basic algorithms in computational geometry. We only consider 2-dimensional objects (2D points, segments, rays, polygons) for simplicity. We also ignore errors that may result of numerical approximations, and assume all arithmetic operations have constant cost.

1. Given two points $p_1 \neq (0, 0)$ and $p_2 \neq (0, 0)$, give a simple algorithm, not using any trigonometric function, to determine whether the quantity $(\alpha_2 - \alpha_1 \mod 2\pi)$ (where $\alpha_1$ and $\alpha_2$ are the polar angles of $p_1$ and $p_2$) is $0$, $\pi$, in $(0; \pi)$ or in $(\pi; 2\pi)$.

2. Given three points $p_1$, $p_2$, and $p_3$ all distinct, give an algorithm to determine whether the succession of the segments $p_1p_2$ and $p_2p_3$ is a right turn, a left turn, a straight forward move, or a straight backward move.

3. Given four points $p_1$, $p_2$, $p_3$, $p_4$ all distinct, and using the previous questions, propose an algorithm to determine whether two segments $p_1p_2$ and $p_3p_4$ intersect. You can first assume that no three points are aligned, and then treat this special case.

4. Given a set $S$ of $n$ points, propose a $O(n^2 \log n)$ algorithm to determine whether there exist three aligned points in $S$.

5. Given an $n$-sided polygon $P$ described by its successive vertices $p_1, p_2, \ldots, p_n$, propose a $O(n)$ algorithm to determine whether $P$ is convex.

6. Given an $n$-sided convex polygon $P$ and a point $q$, propose a $O(n)$ algorithm to determine whether $q$ is the interior of $P$.

7. Given four points $p_1, p_2, p_3, p_4$ all distinct, propose an algorithm to determine whether the segment $p_1p_2$ and the half-line (ray) starting in $p_3$ and going through $p_4$ intersect.

8. Using the previous question, given an $n$-sided polygon $P$ and a point $q$, propose a $O(n)$ algorithm to determine whether $q$ is the interior of $P$. Make sure the algorithm works even in degenerate cases.

9. The convex hull of a set of $n$ points $S$ is the smallest convex polygon whose interior or boundary contains all points in $S$. Show that one of the point of the convex hull can be determined in $O(n)$. 

10. Propose a $O(n \log n)$ algorithm to compute the convex hull of a set of $n$ points. One can use the characterization of question 5 of convex polygons, keeping in mind the characterization of question 6 of points within convex polygons.

11. Propose a $O(nh)$ algorithm to compute the convex hull of a set of $n$ points, where $h$ is the size of the convex hull.

12. Give examples where each of the previous two algorithms is better than the other.