Efficient Smooth Projective Hash Functions and Applications

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Outline

Conditional Actions

An authority, or a server, may accept to process a request
under some conditions only:

- Certification of public key: if the associated secret key is known
- Transmission of private information:
  if the receiver owns a credential

Blind signature on a message:
if the user knows the message (for the security proof)

$\rightarrow$ Proof of validity/knowledge

Why should the authority learn the final status?

$\rightarrow$ Implicit proof of validity/knowledge?

Certification of Public Keys: ZKPoK

In the registered key setting, a user can ask for the certification of a public key *pk*, but only if he knows the associated secret key *sk*:

With an Interactive Zero-Knowledge Proof of Knowledge

- the user *U* sends his public key *pk*;
- *U* and the authority *A* run a ZK proof of knowledge of *sk*
- if convinced, *A* generates and sends the certificate Cert for *pk*

For extracting *sk* (required in some security proofs),
the reduction has to make a rewind
(that is not always allowed: *e.g.*, in the UC Framework)

And the authority learns the final status!

Certification of Public Keys: ZK and NIZK Proofs

In the registered key setting, a user can ask for the certification of a public key pk, but only if he knows the associated secret key sk:

With an Interactive Zero-Knowledge Proof of Membership

- the user U sends his public key pk, and an encryption C of sk;
- U and the authority A run a ZK proof
  that C contains the secret key sk associated to pk
- if convinced, A generates and sends the certificate Cert for pk

With a Non-Interactive Zero-Knowledge Proof of Membership

- the user U sends his public key pk, and an encryption C of sk
  together with a NIZK proof
  that C contains the secret key sk associated to pk
- if convinced, A generates and sends the certificate Cert for pk

Certification of Public Keys: SPHF

[Abdalla, Chevalier, Pointcheval, 2009]

In the registered key setting, a user can ask for the certification of a public key pk, but only if he knows the associated secret key sk:

With a Smooth Projective Hash Function

The user U and the authority A use a smooth projective hash system for L: pk and $C = \mathcal{E}_{pk'}(sk; r)$ are associated to the same sk

- the user U sends his public key pk, and an encryption C of sk;
- A generates the certificate Cert for pk, and sends it,
  masked by $\text{Hash} = \text{Hash}(\text{hk}; (pk, C))$
- U computes $\text{Hash} = \text{ProjHash}(\text{hp}; (pk, C), r))$, and gets Cert

Implicit proof of knowledge of sk
$\rightarrow$ the authority does not learn the final status!

Smooth Projective Hash Functions

[Cramer, Shoup, 2002]

Definition [Cramer, Shoup, 2002] [Gennaro, Lindell, 2003]

Let $\{H\}$ be a family of functions:

- X, domain of these functions
- L, subset (a language) of this domain

such that, for any point x in L, $H(x)$ can be computed by using

- either a *secret* hashing key hk: $H(x) = \text{Hash}_L(\text{hk}; x)$;
- or a *public* projected key hp: $H(x) = \text{ProjHash}_L(\text{hp}; x, w)$

While the former works for all points in the domain X,
the latter works for $x \in L$ only, and requires a witness w to this fact.

Public mapping $\text{hk} \mapsto \text{hp} = \text{ProjKG}_L(\text{hk}, x)$

Properties

For any $x \in X$, $H(x) = \text{Hash}_L(\text{hk}; x)$
For any $x \in L$, $H(x) = \text{ProjHash}_L(\text{hp}; x, w)$ $\quad$ w witness that $x \in L$

Smoothness

For any $x \notin L$, $H(x)$ and hp are independent

Pseudo-Randomness

For any $x \in L$, $H(x)$ is pseudo-random, without a witness w

The latter property requires L to be a hard-partitioned subset of X:

Hard-Partitioned Subset

L is a hard-partitioned subset of X if it is computationally hard to distinguish a random element in L from a random element in $X \setminus L$

Examples

DH Language [Cramer, Shoup, 2002]

$L_{g,h} = \{(u,v)\}$ where (g,h,u,v) is DH tuple:
there exists r such that $u = g^r$ and $v = h^r$

$\rightarrow$ Public-key Encryption with IND-CCA Security

Algorithms

- $\mathsf{HashKG}() = \mathsf{hk} = (\gamma_1, \gamma_3) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$
- $\mathsf{ProjKG}(\mathsf{hk}) = \mathsf{hp} = g^{\gamma_1} h^{\gamma_3}$

$\mathsf{Hash}(\mathsf{hk}, (u,v)) = u^{\gamma_1} v^{\gamma_3} = \mathsf{hp}^r = \mathsf{ProjHash}(\mathsf{hp}, (u,v); r)$

Examples (Con'd)

Commitment/Encryption [Gennaro, Lindell, 2003]

$L_{pk,m} = \{c\}$ where c is an encryption of m under pk:
there exists r such that $c = \mathcal{E}_{pk}(m; r)$

$\rightarrow$ Password-Authenticated Key Exchange in the Standard Model

Labeled Encryption [Canetti, Halevi, Katz, Lindell, MacKenzie, 2005]

$L_{pk,(\ell,m)} = \{c\}$ where c is an encryption of m under pk, with label ℓ

$\rightarrow$ PAKE in the UC Framework (passive corruptions)

Extractable/Equivocable Commitment [Abdalla, Chevalier, Pointcheval, 2009]

$L_{pk,m} = \{c\}$ where c is an equivocable/extractable commitment of m

$\rightarrow$ PAKE in the UC Framework with Adaptive Corruptions

Assumptions: *CDH* and *DLin*

$\mathbb{G}$ a cyclic group of prime order p (with or without bilinear map).

Definition (The Computational Diffie-Hellman problem (*CDH*))

For any generator $g \stackrel{\$}{\leftarrow} \mathbb{G}$, and any scalars $a, b \stackrel{\$}{\leftarrow} \mathbb{Z}_p^*$,
given (g, g^a, g^b), compute g^{ab}.

Decisional variant easy if a bilinear map is available.

Definition (Decision Linear Problem (*DLin*) [Boneh, Boyen, Shacham, 2004])

For any generator $g \stackrel{\$}{\leftarrow} \mathbb{G}$, and any scalars $a, b, x, y, c \stackrel{\$}{\leftarrow} \mathbb{Z}_p^*$,
given $(g, g^x, g^y, g^{xa}, g^{yb}, g^c)$, decide whether $c = a + b$ or not.

Equivalently, given a reference triple $(u = g^x, v = g^y, g)$
and a new triple $(U = u^a = g^{xa}, V = v^b = g^{yb}, T = g^c)$,
decide whether $T = g^{a+b}$ or not (that is $c = a + b$).
(U, V, T) is (or not) a linear tuple w.r.t. (u, v, g)

General Tools: Signature

Definition (Signature Scheme)

$\mathcal{S} = (\mathit{Setup}, \mathit{KeyGen}, \mathit{Sign}, \mathit{Verif})$:

- $\mathit{Setup}(1^k) \rightarrow$ global parameters param
- $\mathit{KeyGen}(\mathit{param}) \rightarrow$ pair of keys (sk, vk)
- $\mathit{Sign}(sk, m; s) \rightarrow$ signature σ, using the random coins s
- $\mathit{Verif}(vk, m, \sigma) \rightarrow$ validity of σ

Definition (Security: EF-CMA [Goldwasser, Micali, Rivest, 1984])

An adversary should not be able to generate a new valid message-signature pair for a new message (Existential Forgery) even when having access to any signature of its choice (Chosen-Message Attack).

Signature: Waters

$\mathbb{G} = \langle g \rangle = \langle h \rangle$ group of order p, and a bilinear map $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$

Waters Signature [Waters, 2005]

For a k-bit message $M = (M_i)$, we define $\mathcal{F}(M) = u_0 \prod_{i=1}^{k} u_i^{M_i}$

- Keys: $vk = Y = g^x$, $sk = X = h^x$, for $x \stackrel{\$}{\leftarrow} \mathbb{Z}_p$
- $Sign(sk = X, M; s)$, for $M \in \{0,1\}^k$ and $s \stackrel{\$}{\leftarrow} \mathbb{Z}_p$
  $\to \quad \sigma = (\sigma_1 = X \cdot \mathcal{F}(M)^s, \sigma_2 = g^{-s})$
- $Verif(vk = X, M, \sigma = (\sigma_1, \sigma_2))$ checks whether

$$e(g, \sigma_1) \cdot e(\mathcal{F}(M), \sigma_2) = e(Y, h)$$

Security

Waters signature reaches EF-CMA under the *CDH* assumption

General Tools: Encryption

Definition (Encryption Scheme)

$\mathcal{E} = (Setup, KeyGen, Encrypt, Decrypt)$:

- $Setup(1^k) \quad \to \quad$ global parameters *param*
- $KeyGen(param) \quad \to \quad$ pair of keys (pk, dk)
- $Encrypt(pk, m; r) \quad \to \quad$ ciphertext c, using the random coins r
- $Decrypt(dk, c) \quad \to \quad$ plaintext, or $\perp$ if the ciphertext is invalid

Definition (Security: IND-CPA [Goldwasser, Micali, 1984])

An adversary should not be able to distinguish the encrytion of m_0 from the encryption of m_1 (Indistinguishability) whereas it can encrypt any message of its choice (Chosen-Plaintext Attack).

Encryption: Linear

$\mathbb{G} = \langle g \rangle$ group of order p

Linear Encryption [Boneh, Boyen, Shacham, 2004]

- Keys: $dk = (x_1, x_2) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$, $pk = (X_1 = g^{x_1}, X_2 = g^{x_2})$
- $Encrypt(pk = (X_1, X_2), M; (r_1, r_2))$, for $M \in \mathbb{G}$ and $(r_1, r_2) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^2$
  $\to \quad C = (C_1 = X_1^{r_1}, C_2 = X_2^{r_2}, C_3 = g^{r_1+r_2} \cdot M)$
- $Decrypt(dk = (x_1, x_2), C = (C_1, C_2, C_3)) \to M = C_3 / C_1^{1/x_1} C_2^{1/x_2}$

Security

Linear encryption reaches IND-CPA under the *DLin* assumption

Encryption: Linear Cramer-Shoup

$\mathbb{G}$ group of order p, with three independent generators $g_1, g_2, g_3 \in \mathbb{G}$

Linear Cramer-Shoup Encryption [Shacham, 2007]

- Keys: $dk = (x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3) \stackrel{\$}{\leftarrow} \mathbb{Z}_p^9$,
  $$pk = \begin{pmatrix} g_1, & c_1 & = & g_1^{x_1} g_3^{x_3}, & c_2 & = & g_2^{x_2} g_3^{x_3} & \\ g_2, & d_1 & = & g_1^{y_1} g_3^{y_3}, & d_2 & = & g_2^{x_2} g_3^{y_3}, & \mathcal{H} \\ g_3, & h_1 & = & g_1^{z_1} g_3^{z_3}, & h_2 & = & g_2^{z_2} g_3^{z_3} & \end{pmatrix}$$
- $Encrypt(pk = (g_1, g_2, g_3, c_1, c_2, d_1, d_2, h_1, h_2, \mathcal{H}), m; (r, s))$, for $M \in \mathbb{G}$:
  $C = (\vec{u} = (u_1 = g_1^r, u_2 = g_2^s, u_3 = g_3^{r+s}), e = M \cdot h_1^r h_2^s, v = v_1^r v_2^s)$
  where $v_1 = c_1 d_1^{\xi}$, $v_2 = c_2 d_2^{\xi}$, and $\xi = \mathcal{H}(\vec{u}, e)$
- $Decrypt(dk = (x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3), C = (\vec{u}, e, v))$
  one checks $v \stackrel{?}{=} u_1^{x_1+\xi y_1} u_2^{x_2+\xi y_2} u_3^{x_3+\xi y_3} \quad \to \quad M = e / u_1^{z_1} u_2^{z_3} u_2^{z_3}$

Encryption: CCA Security

Definition (Security: IND-CCA [Rackoff, Simon, 1991]**)**

An adversary should not be able to distinguish
the encryption of m_0 from the encryption of m_1 (Indistinguishability)
whereas it can encrypt any message of its choice,
and ask any decryption of its choice (Chosen-Ciphertext Attack).

Security: Non-Malleability [Dolev,Dwork, Naor, 1991]

IND-CCA implies Non-Malleability [Bellare, Desai, Pointcheval, Rogaway, 1998]

Security of the Linear Cramer-Shoup [Shacham, 2007]

Linear Cramer-Shoup encryption reaches IND-CCA
under the *DLin* assumption

Groth-Sahai Proofs [Groth, Sahai, 2008]

For any pairing product equation of the form:

$$\prod e(A_i, X_i)^{\alpha_i} \prod e(X_i, X_j)^{\gamma_{i,j}} = e(A, B),$$

where the $A, B, A_i \in \mathbb{G}$ are constant group elements,
$\alpha_i \in \mathbb{Z}_p$, and $\gamma_{i,j} \in \mathbb{Z}_p$ are constant scalars, and X_i are unknowns

- either group elements in $\mathbb{G}$,
- or of the form g^{x_i},

one can make a proof of knowledge of values for the X_i's or x_i's so that the equation is satisfied:

- one first commits these secret values using random coins,
- and then provides proofs, that are group elements, using the above random coins,

$\rightarrow$ Under the *DLin* assumption: Efficient NIZK

Notations [Abdalla, Chevalier, Pointcheval, 2009]

We assume that G possesses a group structure, and we denote by $\oplus$ the commutative law of the group (and by $\ominus$ the opposite operation)
We assume to be given two smooth hash systems SHS_1 and SHS_2, on the sets G_1 and G_2 (included in G) corresponding to the languages L_1 and L_2 respectively:

$$\mathsf{SHS}_i = \{\mathsf{HashKG}_i, \mathsf{ProjKG}_i, \mathsf{Hash}_i, \mathsf{ProjHash}_i\}$$

Let $c \in X$, and r_1 and r_2 two random elements:

$$\begin{array}{lcllcl} \mathsf{hk}_1 & = & \mathsf{HashKG}_1(r_1) & \mathsf{hk}_2 & = & \mathsf{HashKG}_2(r_2) \\ \mathsf{hp}_1 & = & \mathsf{ProjKG}_1(\mathsf{hk}_1, c) & \mathsf{hp}_2 & = & \mathsf{ProjKG}_2(\mathsf{hk}_2, c) \end{array}$$

Conjunction of Languages

A hash system for the language $L = L_1 \cap L_2$ is defined as follows, if $c \in L_1 \cap L_2$ and w_i is a witness that $c \in L_i$, for $i = 1, 2$:

$$\begin{aligned} \mathsf{HashKG}_L(r = r_1 \| r_2) &= \mathsf{hk} = (\mathsf{hk}_1, \mathsf{hk}_2) \\ \mathsf{ProjKG}_L(\mathsf{hk}, c) &= \mathsf{hp} = (\mathsf{hp}_1, \mathsf{hp}_2) \\ \mathsf{Hash}_L(\mathsf{hk}, c) &= \mathsf{Hash}_1(\mathsf{hk}_1, c) \oplus \mathsf{Hash}_2(\mathsf{hk}_2, c) \\ \mathsf{ProjHash}_L(\mathsf{hp}, c; (w_1, w_2)) &= \mathsf{ProjHash}_1(\mathsf{hp}_1, c; w_1) \\ & \quad \oplus \mathsf{ProjHash}_2(\mathsf{hp}_2, c; w_2) \end{aligned}$$

- if c is not in one of the languages, then the corresponding hash value is perfectly random: smoothness
- without one of the witnesses, then the corresponding hash value is computationally unpredictable: pseudo-randomness

Disjunction of Languages

A hash system for the language $L = L_1 \cup L_2$ is defined as follows, if $c \in L_1 \cup L_2$ and w is a witness that $c \in L_i$ for $i \in \{1, 2\}$:

$$
\begin{array}{rcll}
\mathsf{HashKG}_L(r = r_1 \| r_2) & = & \mathsf{hk} = (\mathsf{hk}_1, \mathsf{hk}_2) & \\
\mathsf{ProjKG}_L(\mathsf{hk}, c) & = & \mathsf{hp} = (\mathsf{hp}_1, \mathsf{hp}_2, \mathsf{hp}_\Delta) & \\
\text{where } \mathsf{hp}_\Delta & = & \mathsf{Hash}_1(\mathsf{hk}_1, c) \oplus \mathsf{Hash}_2(\mathsf{hk}_2, c) & \\
\mathsf{Hash}_L(\mathsf{hk}, c) & = & \mathsf{Hash}_1(\mathsf{hk}_1, c) & \\
\mathsf{ProjHash}_L(\mathsf{hp}, c; w) & = & \mathsf{ProjHash}(\mathsf{hp}_1, c; w) & \text{if } c \in L_1 \\
 & \text{or} & \mathsf{hp}_\Delta \ominus \mathsf{ProjHash}_2(\mathsf{hp}_2, c; w) & \text{if } c \in L_2
\end{array}
$$

hp_Δ helps to compute the missing hash value, if and only if at least one can be computed

Pairing Product Equations

$A_i \in \mathbb{G}$ $(i = 1, \ldots, m)$, $\zeta_i \in \mathbb{Z}_p$ $(i = m+1, \ldots, n)$, and $B \in \mathbb{G}_T$ public.
One wants to show its knowledge of $X_i \in \mathbb{G}$ (for $i = 1, \ldots, m$) and $Z_i \in \mathbb{G}_T$ (for $i = m+1, \ldots, n$) that simultaneously satisfy

$$\left(\prod_{i=1}^{m} e(X_i, A_i)\right) \cdot \left(\prod_{i=m+1}^{n} Z_i^{\zeta_i}\right) = B$$

One thus commits X_i (linear encryption) in $\mathbb{G}$, into $\vec{c}_i$, for $i = 1, \ldots, m$,
encrypted under $pk = (g, u_1, u_2)$,
and Z_i (linear encryption) in $\mathbb{G}_T$, into $\vec{C}_i$, for $i = m+1, \ldots, n$,
encrypted under $PK_i = (G, U_1, U_2)$
where $G = e(g, g)$, $U_1 = e(u_1, g)$, $U_2 = e(u_2, g)$.

Commitments

$$
\begin{array}{rcll}
\vec{c}_i & = & (u_1^{r_i}, u_2^{s_i}, g^{r_i+s_i} \cdot X_i) & \text{for } i = 1, \ldots, m \\
\vec{C}_i & = & (U_1^{r_i}, U_2^{s_i}, G^{r_i+s_i} \cdot Z_i) & \text{for } i = m+1, \ldots, n
\end{array}
$$

The $\vec{c}_i$'s can be transposed into $\mathbb{G}_T$, for $i = 1, \ldots, m$:

$$\vec{C}_i = (U_{i,1}^{r_i}, U_{i,2}^{s_i}, G_i^{r_i+s_i} \cdot Z_i)$$

where $U_{i,1} = e(u_1, A_i)$, $U_{i,2} = e(u_2, A_i)$, $G_i = e(g, A_i)$,
but also, $Z_i = e(X_i, A_i)$, for $i = 1, \ldots, m$

We also denote $U_{i,1} = U_1$, $U_{i,2} = U_2$, $G_i = G$, for $i = m+1, \ldots, n$

Smooth Projective Hash Function

$(\lambda, (\eta_i, \theta_i)_{i=1,\ldots,n}) \xleftarrow{\$} \mathbb{Z}_p^{2n+1}$, one sets $\mathsf{hk}_i = (\eta_i, \theta_i, \lambda)$
and $\mathsf{hp}_i = (u_1^{\eta_i} g^{\zeta_i \lambda}, u_2^{\theta_i} g^{\zeta_i \lambda}) \in \mathbb{G}^2$
where $\zeta_i = 1$ for $i = 1, \ldots, m$.
The associated projection keys in $\mathbb{G}_T$ are
$\mathsf{HP}_i = (e(\mathsf{hp}_{i,1}, A_i), e(\mathsf{hp}_{i,2}, A_i))$, for $i = 1, \ldots, n$,
where $A_i = g$ for $i = m+1, \ldots, n$.
The hash value is

$$
\begin{array}{rcl}
H & = & \left(\prod_{i=1}^{n} C_{i,1}^{\eta_i} \cdot C_{i,2}^{\theta_i} \cdot C_{i,3}^{\zeta_i \lambda}\right) \times B^{-\lambda} \\
 & = & \left(\prod_{i=1}^{n} \mathsf{HP}_{i,1}^{r_i} \mathsf{HP}_{i,2}^{s_i}\right) \times \left(\prod_{i=1}^{m} e(X_i, A_i) \prod_{i=m+1}^{n} Z_i^{\zeta_i} / B\right)^{\lambda}
\end{array}
$$

Equality indeed holds if and only if the equation is satisfied

Multiple Equations

We have X_i committed in $\mathbb{G}$, in $\vec{c}_i$, for $i = 1, \ldots, m$
and Z_i committed in $\mathbb{G}_T$, in $\vec{C}_i$, for $i = m+1, \ldots, n$.
We want to show they simultaneously satisfy

$$\left(\prod_{i \in \mathcal{A}_k} e(X_i, A_{k,i})\right) \cdot \left(\prod_{i \in \mathcal{B}_k} Z_i^{\zeta_{k,i}}\right) = B_k, \text{ for } k = 1, \ldots, t$$

where $A_{k,i} \in \mathbb{G}$, $B_k \in \mathbb{G}_T$, and $\zeta_{k,i} \in \mathbb{Z}_p$ are public,
as well as $\mathcal{A}_k \subseteq \{1, \ldots, m\}$ and $\mathcal{B}_k \subseteq \{m+1, \ldots, n\}$

This is a conjunction of languages

$\rightarrow$ Similar Hash Proofs on Linear Cramer-Shoup Commitments

Blind RSA

[Chaum, 1981]

The easiest way for blind signatures, is to blind the message:
To get an RSA signature on m under public key (n, e),

- The user computes a blind version of the hash value: $M = H(m)$ and $M' = M \cdot r^e \bmod n$
- The signer signs M' into $\sigma' = M'^d \bmod n$
- The user unblinds the signature: $\sigma = \sigma'/r \bmod n$

Indeed,

$$\sigma = \sigma'/r = M'^d/r = (M \cdot r^e)^d/r = M^d \cdot r/r = M^d \bmod n$$

$\rightarrow$ Proven under the One-More RSA

[Bellare, Namprempre, Pointcheval, Semanko, 2001]

Blind Signatures

Randomizable Commutative Signature/Encryption

[Blazy, Fuchsbauer, Pointcheval, Vergnaud, 2011]

- The user "blinds" M into C, under random coins r
- The signer signs C into $\sigma(C)$, under random coins s
- The user "unblinds" the signature $\sigma(M)$, granted the coins r

Weakness

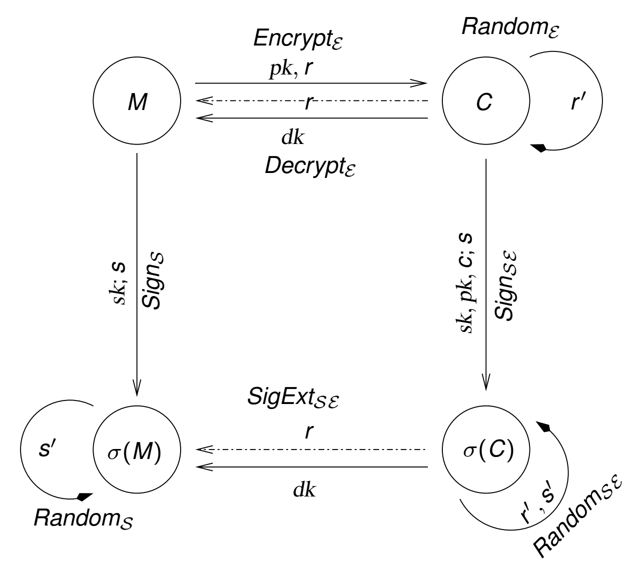
The signer can recognize his signature: the random coins s in $\sigma(M)$
$\rightarrow$ Randomizable Signature

Security

- Encryption hides M (blinding of the message)
- Re-randomization hides $\sigma(M)$ (blinding of the signature)

Randomizable Commutative Signature/Encryption

[Blazy, Fuchsbauer, Pointcheval, Vergnaud, 2011]



Blind Signatures

Such a primitive can be used for a Waters Blind Signature, by encrypting $\mathcal{F}(M)$:

- Unforgeability: one-more forgery would imply a forgery against the signature scheme (*CDH* assumption)
- Blindness: a distinguisher would break indistinguishability of the encryption scheme (*DLin* assumption)

Efficiency

One obtains a plain Waters Signature

Limitation

A proof of knowledge of M in $C = \mathcal{E}_{pk}(\mathcal{F}(M))$ has to be sent

Blind Signature

[Blazy, Fuchsbauer, Pointcheval, Vergnaud, 2011]

In order to get the ℓ-bit message $M = \{M_i\}$ blindly signed:

With Groth-Sahai NIZKP

- the user U encrypts M into C_1, and $\mathcal{F}(M)$ into C_2;
- U produces a Groth-Sahai NIZK Proof that C_1 and C_2 contain the same M (bit-by-bit proof)
- if convinced, A generates a signature on C_2
- granted the commutativity, U decrypts it into a Waters signature of M, and eventually re-randomizes the signature

$9\ell + 24$ group elements have to be sent:
$\rightarrow$ It was the most efficient blind signature up to 2011
Why NIZK, since there are already two flows?

Blind Signature

[Blazy, Pointcheval, Vergnaud, 2012]

In order to get the ℓ-bit message $M = \{M_i\}$ blindly signed:

With SPHF

The user U and the authority A use a smooth projective hash system for L: $C_1 = \mathcal{E}_{pk_1}(M; r)$ and $C_2 = \mathcal{E}_{pk_2}(\mathcal{F}(M); s)$ contain the same M

- U sends encryptions of M, into C_1, and $\mathcal{F}(M)$, into C_2;
- A generates
  - a signature σ on C_2,
  - masks it using $\mathsf{Hash} = \mathsf{Hash}(\mathsf{hk}; (C_1, C_2))$
- U computes $\mathsf{Hash} = \mathsf{ProjHash}(\mathsf{hp}; (C_1, C_2), (r, s))$, and gets σ. Granted the commutativity, U decrypts it into a Waters signature of M, and eventually re-randomizes it

Such a protocol requires $8\ell + 12$ group elements in total only!

Oblivious Transfers

Oblivious Transfer

[Rabin, 1981]

A sender S wants to send a message M to U such that

- U gets M with probability 1/2, or nothing
- S does not learn whereas U gets the message M or not

1-2 Oblivious Transfer

[Even, Goldreich, Lempel, 1985]

A sender S owns two messages m_0 and m_1, and U owns a bit b

- U gets m_b but nothing on the other message
- S does not learn anything about b

Oblivious Signature-Based Envelope [Li, Du, Boneh, 2003]

A sender S wants to send a message M to U such that

- U gets M if and only if it owns a signature σ on a message m valid under vk
- S does not learn whereas U gets the message M or not

Correctness: if U owns a valid signature, he learns M

Security Notions

- Oblivious: S does not know whether U owns a valid signature (and thus gets the message)
- Semantic Security: U does not learn any information about M if he does not own a valid signature

A Stronger Security Model

S wants to send a message M to U, if U owns/uses a valid signature.

Security Notions

- Oblivious w.r.t. the authority: the authority does not know whether U uses a valid signature (and thus gets the message);
- Semantic Security: U cannot distinguish multiple interactions with S sending M_0 from multiple interactions with S sending M_1 if he does not own/use a valid signature;
- Semantic Security w.r.t. the Authority: after the interaction, the authority does not learn any information about M.

A New OSBE [Blazy, Pointcheval, Vergnaud, 2012]

S wants to send a message M to U, if U owns a valid signature σ on m under vk:

With a Smooth Projective Hash Function

The user U and the sender S use a smooth projective hash system for L: $C = \mathcal{E}_{pk}(\sigma; r)$ contains a valid signature σ of m under vk

- the user U sends an encryption C of σ;
- S generates a hk and the associated hp, computes $\mathsf{Hash} = \mathsf{Hash}(\mathsf{hk}; C)$, and sends hp together with $c = M \oplus \mathsf{Hash}$;
- U computes $\mathsf{Hash} = \mathsf{ProjHash}(\mathsf{hp}; C, r)$, and gets M.

Security Properties

- Oblivious (even w.r.t. the Authority): IND-CPA of the encryption scheme (Hard-partitioned Subset of the SPHF);
- Semantic Security: Smoothness of the SPHF
- Semantic Security w.r.t. the Authority: Pseudo-randomness of the SPHF

Semantic Security w.r.t. the Authority requires one interaction
$\rightarrow$ round-optimal
Standard model with Waters Signature + Linear Encryption
$\rightarrow$ *CDH* and *DLin* assumptions

General Construction

- The user U sends a commitment C of a word w
- S generates a hk and the associated hp,
  computes $\text{Hash} = \text{Hash}(\text{hk}; C)$,
  and sends hp together with $c = M \oplus \text{Hash}$;
- U computes $\text{Hash} = \text{ProjHash}(\text{hp}; C, r)$, and gets M.

U gets M iff w is in the appropriate language:

- a signature on a public message: OSBE
- a signature on a private message: Anonymous Credential
- a private message (low entropy): Password

Password-based Authenticated Key Exchange

GL – Generic Approach [Gennaro, Lindell, 2003]

Additional tricks are required for the security!

Alice		Bob
$C_1 = Commit(pw; r_1)$	$\xrightarrow{C_1}$	$C_2 = Commit(pw; r_2)$
	$\xleftarrow{C_2, \text{hp}_1}$	hk_1, hp_1 on C_1
hk_2, hp_2 on C_2	$\xleftarrow{\text{hp}_2}$	

$$\text{ProjHash}(\text{hp}_1; C_1, r_1) = H_1 = \text{Hash}(\text{hk}_1; C_1)$$
$$\text{Hash}(\text{hk}_2; C_2) = H_2 = \text{ProjHash}(\text{hp}_2; C_2, r_2)$$

$$K = H_1 \cdot H_2$$

The language is: valid commitments of pw

Language-based Authenticated Key Exchange

Definition

- Alice owns a word w_1 is a language $L_1(Pub_1, Priv_1)$;
- Bob owns a word w_2 is a language $L_2(Pub_2, Priv_2)$;
- If Alice and Bob agree on the languages,
  and actually own valid words (implicit authentication),
  they will agree on a common session key (semantic security)

- $Pub = \emptyset$, $Priv = pw$ and $L(Pub, Priv) = \{Priv\}$: PAKE
- $Pub = M$, $Priv = vk$, $L(Pub, Priv) = \{\sigma, Verif(Priv, Pub, \sigma) = 1\}$:
  Secret Handshake
- $Pub = \emptyset$, $Priv = (vk, M)$, $L(Pub, Priv) = \{\sigma, Verif(Priv, \sigma) = 1\}$:
  CAKE – Credential-based AKE [Camenisch, Casati, Gross, Shoup, 2010]

Language-based Authenticated Key Exchange

Our Construction

- With a Linear Cramer-Shoup UC commitment [Lindell, 2011]
- Using the GL approach [Gennaro, Lindell, 2003]

$\rightarrow$ UC Secure LAKE

Languages

- Password: PAKE secure under $DLin$
- Waters Signature: Secret Handshake, Credentials
  secure under $DLin + CDH$

Any Linear Pairing Product Equation Systems in both $\mathbb{G}$ and $\mathbb{G}_T$

Conclusion

Smooth Projective Hash Functions
can be used as implicit proofs of knowledge or membership

Various Applications

- IND-CCA [Cramer, Shoup, 2002]
- PAKE [Gennaro, Lindell, 2003]
- Certification of Public Keys [Abdalla, Chevalier, Pointcheval, 2009]

Privacy-preserving protocols

- Blind signatures [Blazy, Pointcheval, Vergnaud, 2012]
- Oblivious Signature-Based Envelope

$\rightarrow$ Round optimal!

More general: Language-based Authenticated Key Exchange