Interactive Diffie-Hellman Assumptions With Applications to Password-Based Authentication

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Password-Based Authenticated Key Exchange

The Three-Party Case

A Provably Secure Construction

Outline

Password-Based Authenticated Key Exchange

- Authenticated Key Exchange
- Security Model
- Password-Based Authentication

The Three-Party Case

- Generic Construction
- More Efficient Constructions

A Provably Secure Construction

- The New Scheme
- Computational Assumptions

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Authenticated Key Exchange

Key Exchange

Goal

Two parties want to agree on a common secret key *sk*, in order to establish a private/authenticated channel.

Example (Diffie-Hellman)

- Alice sends $X = g^x$ to Bob
- Bob sends $Y = g^{y}$ to Alice
- They can both compute $sk = X^y = Y^x = g^{xy}$

Man-in-the-middle attack

- Charlie can sit in between Alice and Bob
- He impersonates Alice to Bob, and Bob to Alice

Authentication is required!

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Authenticated Key Exchange

Authentication

Asymmetric Authentication

Flows can be signed

Symmetric Authentication

Entropy

- high-entropy secret: Message Authentication Codes
- Iow-entropy secret: Password

Shared secrets

- 2-party: the secret is shared by Alice and Bob
- 3-party: the secrets are shared between the users and an authentication server

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Security Model

Ability of the Adversary

The adversary is able to distinguishes the actual session key from a random one

Model

- *Test*-query: it tests one session key, and receives either the actual key *sk* if (*b* = 0), or a random key if (*b* = 1).
- The adversary ends the game by answering its guess b'
- It wins if b' = b

Security

Adv(A) = 2Pr[b' = b] - 1 must be negligible.

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Adversaries

Security Model

Passive Adversary

Eavesdrops all the network: transcripts and bad uses of the keys

Model

- Execute-queries: transcript of an execution of the protocol
- Reveal-queries: key agreed on by the players

Active Adversary

Controls all the network: intercepts, forwards, forges messages

Model

• Send[Client/Server]-queries: it sends any message of its choice to any player, who answers according to the protocol

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Password-Based Authentication

Dictionary Attacks

Password: low-entropy

4 digits: exhaustive search is possible!

Basic attack: on-line exhaustive search

- choose a password and try it
- In case of failure, erase it from the list
- \implies 5000 trials are enough: cannot be avoided!

Dictionary attack: off-line exhaustive search

- play a few active attacks
- eavesdrop many transcripts
- ⇒ find the good password: should be prevented

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Encrypted Key Exchange

Example

Password-Based Authentication

A Diffie-Hellman key exchange encrypted by the password

EKE

- Alice computes $X = g^x$ and sends $X' = \mathcal{E}_{pw}(X)$ to Bob
- Bob computes $Y = g^y$ and sends $Y' = \mathcal{E}_{pw}(Y)$ to Alice
- They can both compute sk = H(K), where $X^y = Y^x = q^{xy}$

Security

- Security against passive attacks: under the CDH problem
- Security against active attacks:

$$\mathsf{Adv}(t) \leq rac{2q_s}{N} + \mathcal{O}(\mathsf{Succ}^{\mathsf{cdh}}(t)) + e^{-it}$$

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Generic Construction

An Example

Scenario

- Alice shares a password pw₄ with the server
- and Bob shares a password pw_B with the server
- Alice and Bob want to establish a secure channel

3-GPAKE-Weak

Alice	Server			Bob
sk _A	$\xleftarrow{EKE(pw_A)}$	sk _A sk _B	$\xleftarrow{EKE(pw_B)}$	sk _B
sk	$\stackrel{\mathcal{E}_{sk_{\mathcal{A}}}(sk)}{\longleftarrow}$	sk	$\xrightarrow{\mathcal{E}_{sk_B}(sk)}$	sk

Key-Privacy

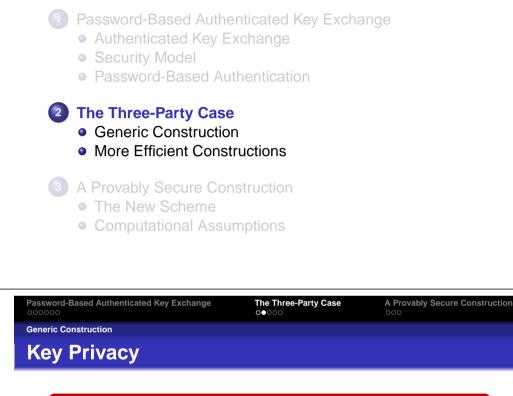
The server knows the key sk distributed to Alice and Bob

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Outline



Security Model

3-G

If Alice and Bob indeed agree on a key, it is hidden to the server

PAKE							
A	lice		Ser	/er		Bob	
8	sk _A		sk _A	sk _B	<i>∈EKE</i>	sk _B	
	sk	$\leftarrow \mathcal{E}_{sk_{\mathcal{A}}}(sk)$	sk	($\xrightarrow{\mathcal{E}_{sk_B}(sk)}$	sk	
3	SK	МА	AC-base	d Ak	★E	SK	

Efficiency

This protocol requires 4 exponentiations per player

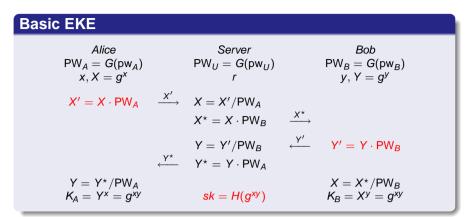
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More Efficient Constructions

A First Scheme



Efficiency

More Efficient Constructions

This protocol requires only 2 exponentiations per player

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A Second Scheme

Randomized EKE

	Server r	$\begin{matrix} \textit{Bob} \\ \textit{y}, \textit{Y} = \textit{g}^{\textit{y}} \end{matrix}$
$\xrightarrow{X'}$	$X = X' / PW_A$	
	$Y = Y' / PW_B$	$\stackrel{Y'}{\longleftarrow} Y' = Y \cdot PW_B$
,Υ*	$\overline{\mathbf{Y}} = \mathbf{Y}^r$ $\mathbf{Y}^* = \overline{\mathbf{Y}} \mathbf{P} \mathbf{W}$	
·	$r = r \cdot Fvv_A$	$\bar{X} = X^{\star} / PW_B$
	$sk = H(g^{xyr})$	$ar{X}=X^{\star}/PW_{B}$ $\mathcal{K}_{B}=ar{X}^{y}=g^{xyr}$
		$\begin{array}{ccc} & & r \\ & \xrightarrow{X'} & X = X' / PW_A \\ & \overline{X} = X' \\ & X^* = \overline{X} \cdot PW_B \\ & & Y = Y' / PW_B \\ & & \overline{Y} = Y' \\ & & & Y^* = \overline{Y} \cdot PW_A \end{array}$

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Security Proof Problem

With a fixed and unique password PW_A: no security proof

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More Efficient Constructions

Insider Attack

Insider Adversary

Bob may try to learn Alice's password

Example



Attack

- From Y' and Y*: One immediately gets PW_A/PW_B
- From Y and Y^{*}: Bob immediately gets PW_{Δ}

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The New Scheme

Our Scheme

Randomized EKE with Variable Passwords

$$\begin{array}{cccccccc} Alice & Server & Bob \\ x, X = g^{x} & r & y, Y = g^{y} \\ X' = X \cdot PW_{A} & \stackrel{X'}{\longrightarrow} & \stackrel{Y'}{\longleftarrow} & Y' = Y \cdot PW_{B} \\ & & X = X'/PW_{A} \\ & & \bar{X} = X' \\ & & X^{\star} = \bar{X} \cdot PW_{B}(Y') & \stackrel{X^{\star}}{\longrightarrow} \\ & & Y = Y'/PW_{B} \\ & & \bar{Y} = Y' \\ & & Y^{\star} = \bar{Y} \cdot PW_{A}(X') \\ \bar{Y} = Y^{\star}/PW_{A}(X') & & \bar{X} = X^{\star}/PW_{B}(Y') \\ \bar{Y} = Y^{\star}/PW_{A}(X') & & \bar{X} = X^{\star}/PW_{B}(Y') \\ K_{A} = \bar{Y}^{x} = g^{xyr} & sk = H(g^{xyr}) & K_{B} = \bar{X}^{y} = g^{xyr} \end{array}$$

Example

 $\mathsf{PW}_{\mathcal{A}} = G(\mathsf{pw}_{\mathcal{A}}) \qquad \mathsf{PW}_{\mathcal{A}}(X') = G(\mathsf{pw}_{\mathcal{A}},X')$

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Computational Assumptions

The Chosen-Basis Diffie-Hellman Problem

Formal Definition: CDDH(U, V)

- A outputs X and Y
- One chooses two random exponents r₀ and r₁, as well as two random bits b and b₀, and sets b₁ = b ⊕ b
- One sets $Y' = Y^{r_0}$ and $X_0 = (X/U)^{r_{b_0}}$, $X_1 = (X/V)^{r_{b_1}}$
- A is given Y' and X_0 , X_1 , it outputs b' (its guess for b)

Idea

Given *U* and *V*, no adversary can find *X* and *Y* so that given Y^r , it can compute $CDH_Y(X/U, Y^r)$ and $CDH_Y(X/V, Y^r)$

Either he can compute the former, with $X = Y^{\alpha}U$, or the latter, with $X = Y^{\alpha}V$. Password-Based Authenticated Key Exchange

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The New Scheme

Properties

Efficiency

This protocol requires only two exponentiations per player

Scenario

In the three-party setting, but for non-concurrent executions

Security

In the random-oracle model:

$$\begin{array}{ll} \mathsf{Adv}(t) & \leq & \displaystyle \frac{2q_{s}}{N} + q_{e} \times \mathsf{Adv}^{\mathsf{ddh}}(t) + \mathsf{poly}(\mathsf{Q}) \times \mathsf{Succ}^{\mathsf{cdh}}(t) \\ & + 2q_{s} \times \mathsf{Adv}^{\mathsf{pcddh}}(t) + \epsilon \end{array}$$

Summary

Summary

- A new password-based key exchange protocol
 - in the three-party setting
 - twice as much efficient as the generic scheme
 - provably secure in the random-oracle model

New computational assumptions

- Chosen-basis Diffie-Hellman problems
 - intuitively hard to solve
- Password-Based Chosen-basis Diffie-Hellman problems
 - formally related to the above ones
 - used in the security analysis