Zélus: a synchronous language with ODEs

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Hybrid Systems Modelers

Program complex discrete systems and
their physical environments in a single language

Many tools exist

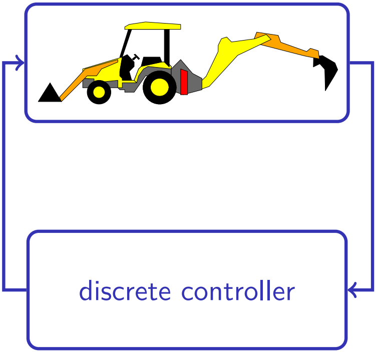
- Simulink/Stateflow, LabVIEW, Modelica, Ptolemy, ...

Focus on programming language issues to improve safety

Our proposal

- Build a hybrid modeler on top of a synchronous language
- Recycle existing techniques and tools
- Clarify underlying principles and guide language design/semantics

Typical system

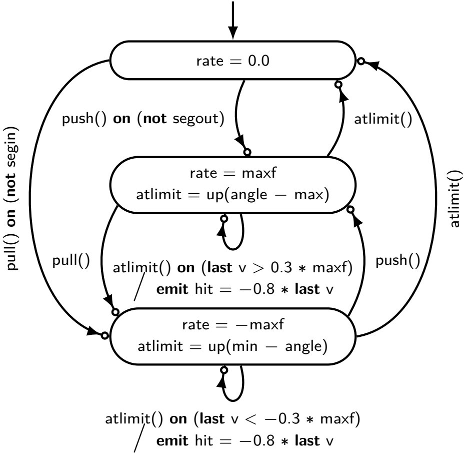


Physical environment

- ODEs with reset

  der v = (0.7 /. maxf) ∗. error **init** 0.0 **reset** hit(v0) → v0

- Hierarchical hybrid automata



Discrete controller

- Dataflow equations
- Hierarchical automata

Reuse existing tools and techniques

Synchronous languages (SCADE/Lustre)

- Widely used for critical systems design and implementation
  - mathematically sound semantics
  - certified compilation (DO178C)
- Expressive language for both discrete controllers and mode changes

Off-the-shelf ODEs numeric solvers

- Sundials CVODE (LLNL) among others, treated as black boxes
- Exploit existing techniques and (variable step) solvers

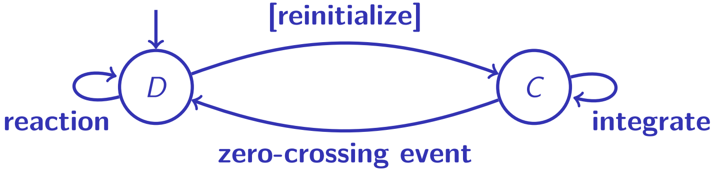
A conservative extension:
Any synchronous program must be compiled, optimized, and executed as per usual

Type systems to separate continuous from discrete

What is a discrete step?

- Reject unreasonable parallel compositions
- Ensure by static typing that discrete changes occur on zero-crossings
- Statically detect causality loops, initialization issues

Simulation engine



$$\sigma' = d_\sigma(t, y) \qquad upz = g_\sigma(t, y) \qquad \dot{y} = f_\sigma(t, y)$$

Compiler architecture

lexing/ parsing → typing → causality/ initialization → inlining → automata → normalize let/in → periods → discrete zero-crossing → present/ signals → variable completion → ODEs zero-crossings → last/fby/$\rightarrow$ → optimization → scheduling → code generation

Built on an existing synchronous compiler

- Source-to-source and traceable transformations
- Resulting program is synchronous and translated to sequential code

Comparison with existing tools

Simulink/Stateflow (Mathworks)

- Integrated treatment of automata *vs* two distinct languages
- More rigid separation of discrete and continuous behaviors

Modelica

- Do not handle DAEs
- Our proposal for automata will be integrated into new version 3.4

Ptolemy (E.A. Lee et al., Berkeley)

- A unique computational model: synchronous
- Everything is compiled to sequential code (not interpreted)

Zélus: A Synchronous Language with ODEs

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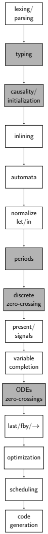
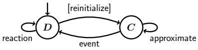


Compiler architecture: source-to-source and traceable transformations

Programming embedded systems and their environments in the same language

- A Lustre-like language with ODEs.
- Dedicated type systems to separate discrete time from continuous time behaviors.
- A compiler architecture based on checkable source-to-source transformations.
- Simulate with an off-the-shelf numeric solver.

Hybrid simulation run-time



The Type system

$(+)\ : \mathtt{int} \times \mathtt{int} \xrightarrow{\mathtt{A}} \mathtt{int}$

$(=)\ : \forall \beta . \beta \times \beta \xrightarrow{\mathtt{A}} \mathtt{bool}$

$\mathtt{if}\ : \forall \beta . \mathtt{bool} \times \beta \times \beta \xrightarrow{\mathtt{A}} \beta$

$\mathtt{pre}(\cdot)\ : \forall \beta . \beta \xrightarrow{\mathtt{D}} \beta$

$\cdot\,\mathtt{fby}\,\cdot\ : \forall \beta . \beta \times \beta \xrightarrow{\mathtt{D}} \beta$

$\mathtt{up}(\cdot)\ : \mathtt{float} \xrightarrow{\mathtt{C}} \mathtt{zero}$

$\cdot\,\mathtt{on}\,\cdot\ : \mathtt{zero} \times \mathtt{bool} \xrightarrow{\mathtt{A}} \mathtt{zero}$

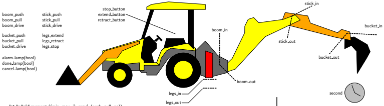
$bt ::= \mathtt{float} \mid \mathtt{int} \mid \mathtt{bool} \mid \mathtt{zero}$

$t ::= bt \mid t \times t \mid \beta$

$\sigma ::= \forall \beta_1, ..., \beta_n . t \xrightarrow{k} t$

$k ::= \mathtt{D} \mid \mathtt{C} \mid \mathtt{A}$

Example system with (hierarchical) Hybrid Automaton



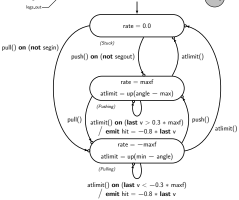

```
let hybrid segment ((min, max, i), maxf, (push, pull, go))
    = ((segin, segout), angle) where

 rec der angle = v init i
 and error = v_r −. v
 and der v = (0.7 /. maxf) ∗. error +. 0.3 ∗. z init 0.0
                  reset hit(v0) → v0
 and der z = error init 0.0 reset hit(_) → 0.0
 and v_r   = if go then rate else 0.0
 and (segin, segout) = (angle <= min, angle >= max)

 and automaton
   | Stuck →
     do
       rate = 0.0
     until push() on (not segout) then Pushing
     else  pull() on (not segin) then Pulling

   | Pushing → local atlimit in
     do
       rate = maxf
       and atlimit = up(angle −. max)
     until atlimit() on (last v > 0.3 ∗. maxf) then
           do emit hit = −0.8 ∗. last v in Pushing
     else atlimit() then Stuck
     else pull() then Pulling

   | Pulling → local atlimit in
     do
       rate = −. maxf
       and atlimit = up(min −. angle)
     until atlimit() on (last v < −0.3 ∗. maxf) then
           do emit hit = −0.8 ∗. last v in Pulling
     else atlimit() then Stuck
     else push() then Pushing
```



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