# Programming hybrid systems with synchronous languages

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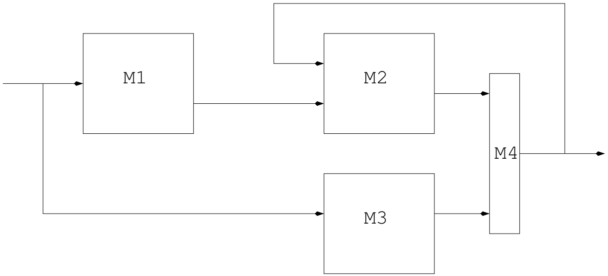
# Reactive systems

- They react continuously to the external environment.
- At the speed imposed by this environment.
- Statically bounded memory and response time.

## Conciliate three notions in the programming model:

- Parallelism, concurrency while preserving determinism.
  *e.g, control at the same time rolling and pitching*
  ↪ parallel description of the system
- Strong temporal constraints.
  *e.g, the physics does not wait!*
  ↪ temporal constraints should be expressed in the system
- Safety is important (critical systems).
  ↪ well founded languages, verification methods

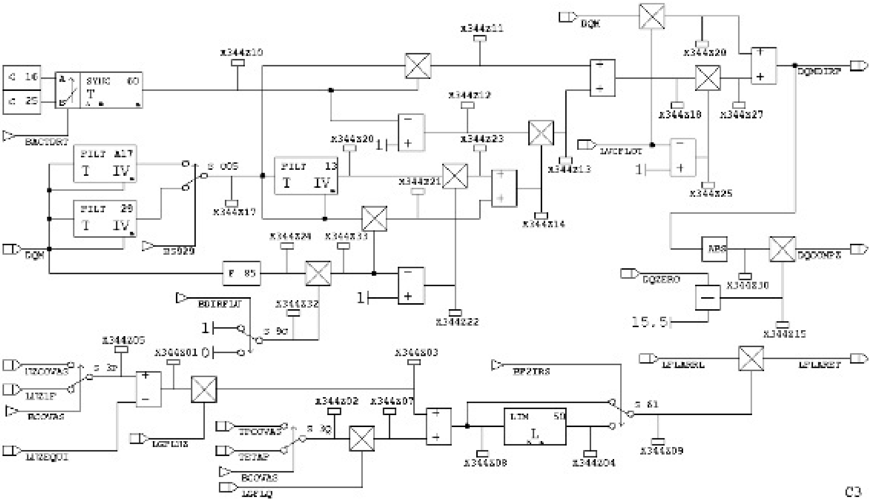
# Synchronous Kahn Networks




- parallel processes communicating through data-flows
- communication in zero time: data is available as soon as it is produced.
- a global logical time scale even though individual rhythms may differ
- these drawings are not so different from actual computer programs

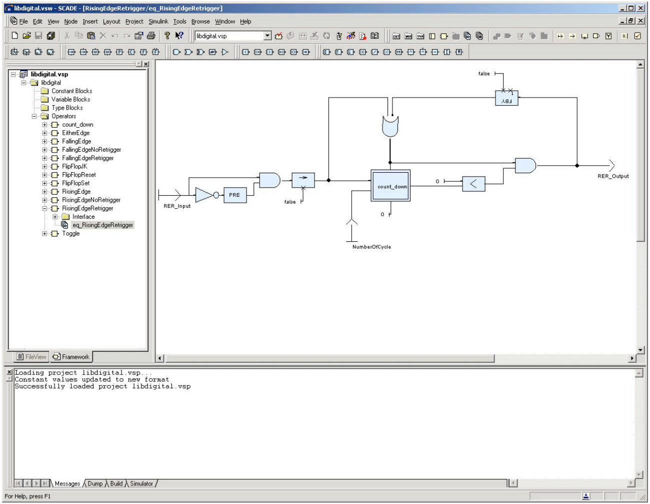
# SAO (Spécification Assistée par Ordinateur)—Airbus 80's

Describe the system as block diagrams (synchronous communicating machines)



# SCADE 4 (Safety Critical Application Development Env. – Esterel-Tech.)

From computer assisted drawings to executable (sequential/parallel) code!



# Lustre: a dataflow programming language

Caspi, Pilaud, Halbwachs, and Plaice. Lustre: A Declarative Language for Programming Synchronous Systems. 1987.

## Programming with streams

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constants $\quad 1 \quad = \quad 1 \quad 1 \quad 1 \quad 1 \quad \cdots$

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## Programming with streams

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|---|---|---|---|---|---|---|---|
| constants | 1 | = | $1$ | $1$ | $1$ | $1$ | $\cdots$ |
| operators | x + y | = | $x_0 + y_0$ | $x_1 + y_1$ | $x_2 + y_2$ | $x_3 + y_3$ | $\cdots$ |

(z = x + y means that at every instant $i$ : $z_i = x_i + y_i$)

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|---|---|---|---|---|---|---|---|
| unit delay | 0 **fby** (x + y) | = | $0$ | $x_0 + y_0$ | $x_1 + x_1$ | $x_2 + x_2$ | $\cdots$ |

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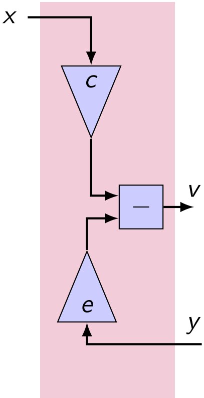


v = c * x − e * y

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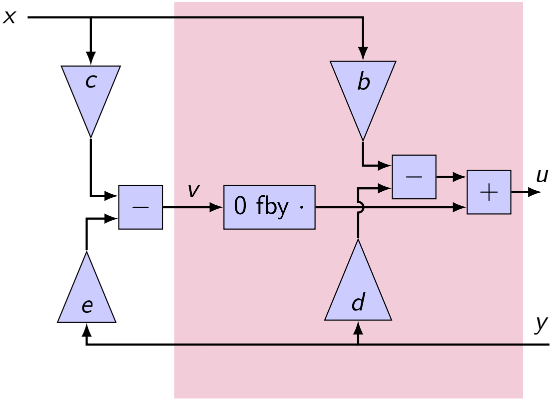


$$u = b * x - d * y + (0.0\ \textbf{fby}\ v)$$

$$\textbf{and}\ v = c * x - e * y$$

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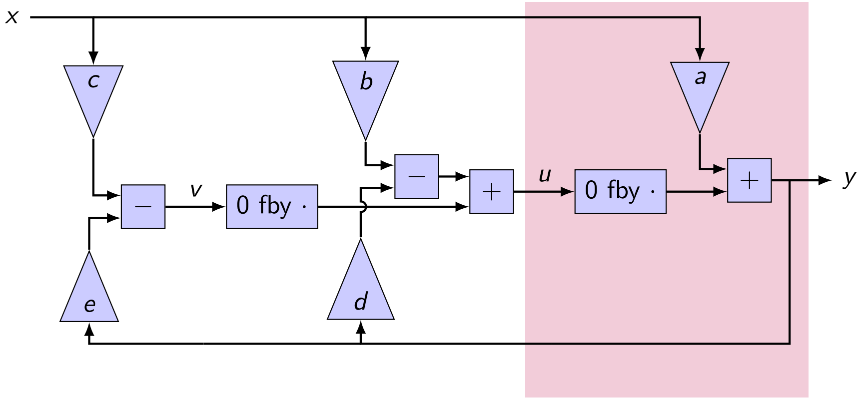


**rec** y = a ∗ x + (0.0 **fby** u)

**and** u = b ∗ x − d ∗ y + (0.0 **fby** v)

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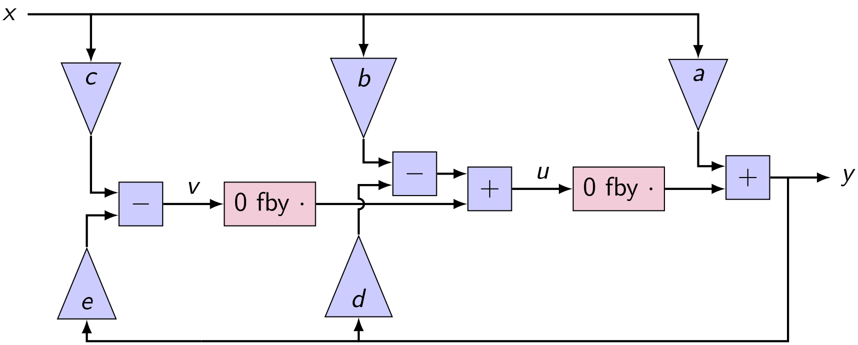


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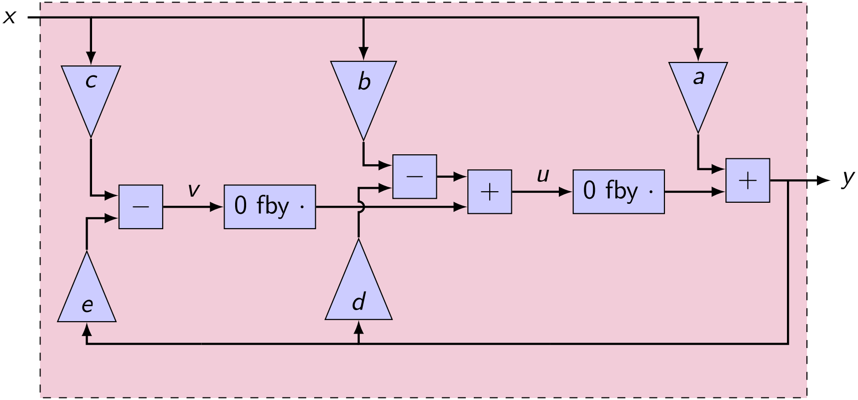


let node iir_filter_2 x = y where

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**and** u = b ∗ x − d ∗ y + (0.0 **fby** v)

**and** v = c ∗ x − e ∗ y

# Lustre: beautiful ideas

- A simple and pure notion of execution in discrete time
- Parallel composition is
  - well-defined
  - deterministic: very important in practice for reproducibility
- Parallelism is compiled: programs can be translated into efficient sequential
- The code executes in bounded memory and bounded time
- Programs are finite-state and can be verified by model-checking

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Lustre can be extended in several ways. . .

# Extended dataflow programming: automata

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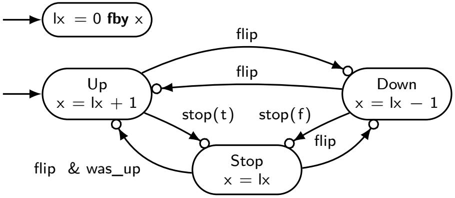
let node counter (flip, stop) = x
 where
 rec lx = 0 fby x
 and automaton
      | Up ->
        do
          x = lx + 1
        until flip then Down
            | stop then Stop(true)
        done

      | Down ->
        do
          x = lx - 1
        until flip then Up
            | stop then Stop(false)
        done

      | Stop(was_up) ->
        do
          x = lx
        until flip & was_up then Up
            | flip then Down
        done
     end
```




- Parallel composition of dataflow equations and automata
- x has a different definition in each mode
- But only a single definition in a reaction

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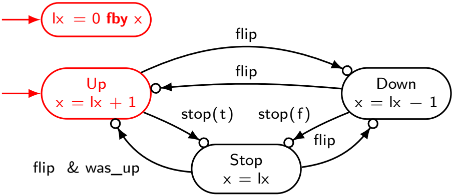


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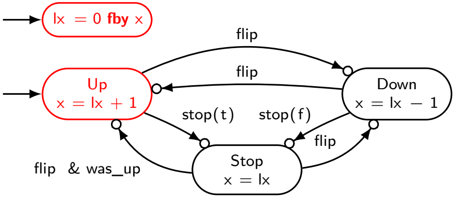


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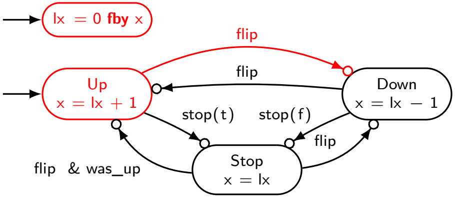


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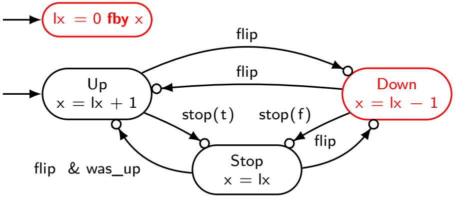


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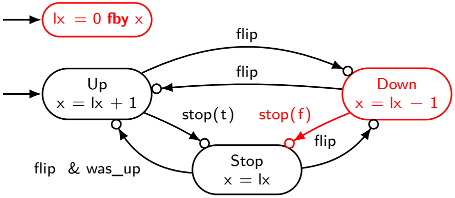


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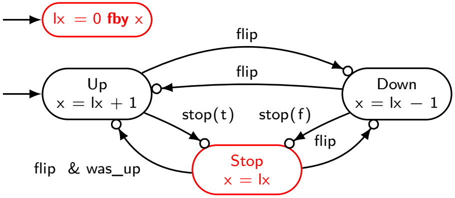


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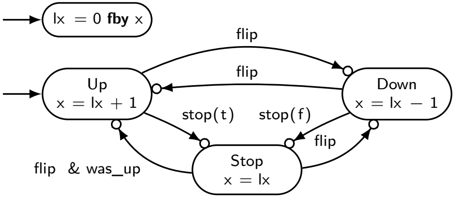


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- They can be reduced to discrete dataflow equations by a source-to-source transformation

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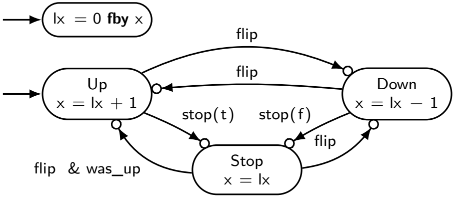


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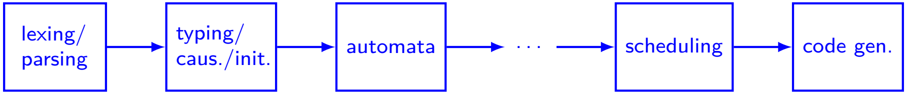


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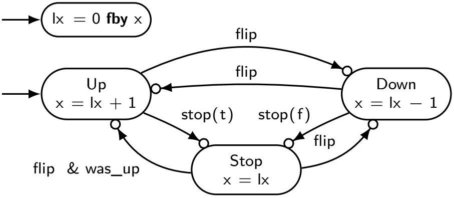


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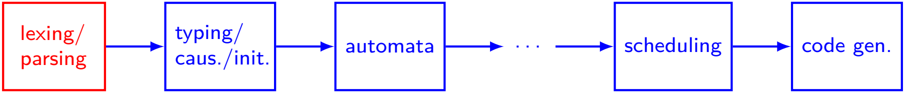


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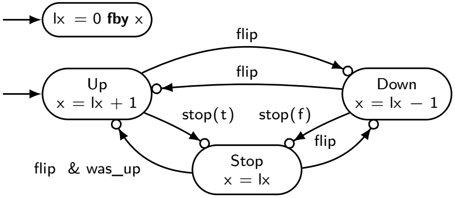


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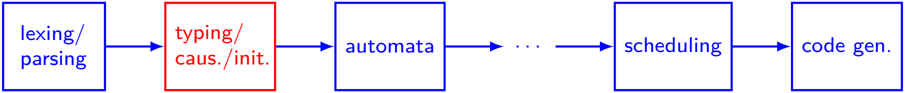


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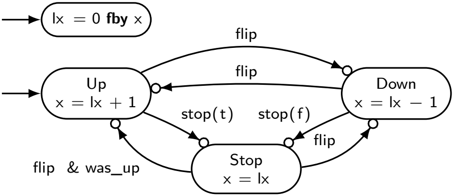


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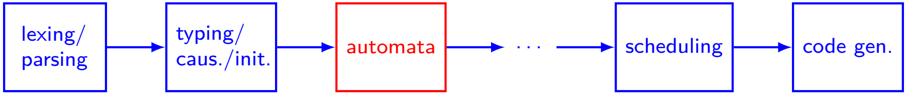


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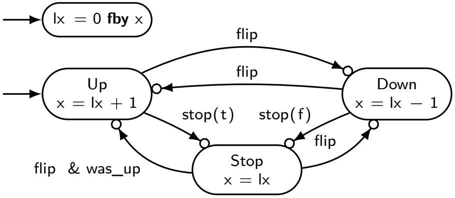


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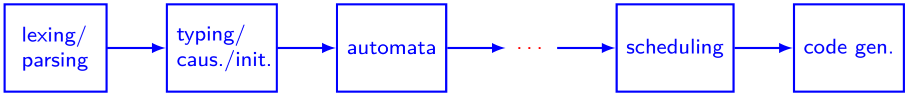


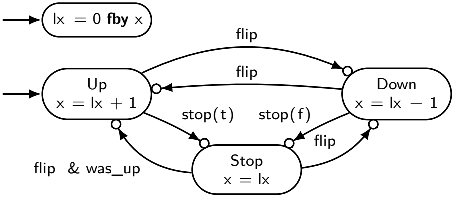

Pagano and Pouzet. A Conservative Extension of Synchronous Data-flow with State Machines. EMSOFT'2005.

# Extended dataflow programming: automata

```
let node counter (flip , stop) = x
 where
 rec lx = 0 fby x
 and automaton
     | Up →
       do
         x = lx + 1
       until flip then Down
           | stop then Stop(true)
       done

     | Down →
       do
         x = lx − 1
       until flip then Up
           | stop then Stop(false)
       done

     | Stop(was_up) →
       do
         x = lx
       until flip & was_up then Up
           | flip then Down
       done
     end
```

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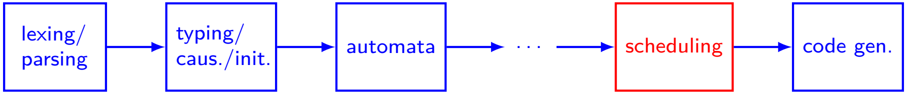


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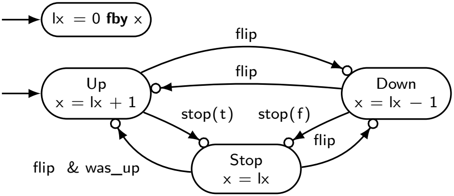


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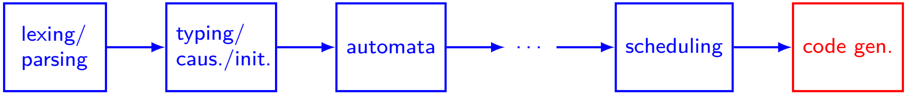
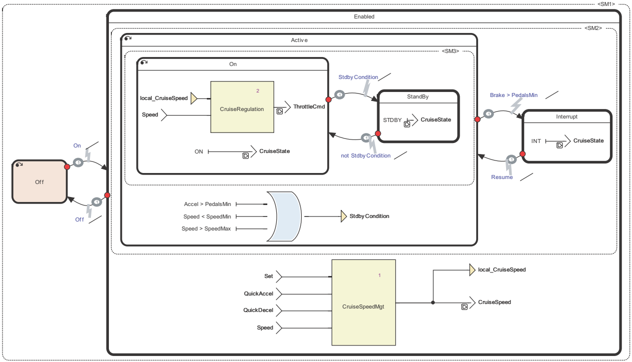



Pagano and Pouzet. A Conservative Extension of Synchronous Data-flow with State Machines. EMSOFT'2005.

# SCADE 6




- Industrial version of Lustre/Lucid Synchrone with automata
- Used in critical systems (DO-178B certified)
- Airbus flight control; Train (interlocking, on-board); Nuclear safety

# So, what's left to do?

- We want a language for programming **complex discrete systems** and modelling their **physical environments**
- (Also: embedded software that includes physical models)

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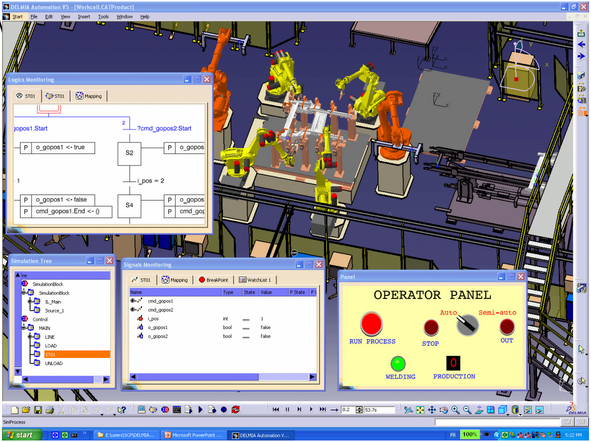


Dassault Systèmes Delmia and Catia http://www.3ds.com/products

# So, what's left to do?

- We want a language for programming **complex discrete systems** and modelling their **physical environments**
- (Also: embedded software that includes physical models)

- Something like Simulink/Stateflow, but
  - Simpler and more consistent semantics and compilation
  - Better understand interactions between discrete and continuous
  - Simpler treatment of automata
  - Certifiability for the discrete parts

  Understand and improve the design of such modelling tools


Lee and Zheng. Operational semantics of hybrid systems. HSCC 2005.
Lee and Zheng. Leveraging synchronous language principles for heterogeneous modeling and design of embedded systems. EMSOFT'07.




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## Ptolemy and HyVisual

- Programming languages perspective
- Numerical solvers as directors
- Working tool and examples


Lee and Zheng. Operational semantics of hybrid systems. HSCC 2005.
Lee and Zheng. Leveraging synchronous language principles for heterogeneous modeling and design of embedded systems. EMSOFT'07.

Carloni et al. Languages and tools for hybrid systems design. 2006.

## Simulink/Stateflow

- Simulation $\rightsquigarrow$ development
- two distinct simulation engines
- semantics & consistency: non-obvious

Lee and Zheng. Operational semantics of hybrid systems. HSCC 2005.
Lee and Zheng. Leveraging synchronous language principles for heterogeneous modeling and design of embedded systems. EMSOFT'07.

## Our approach

- Source-to-source compilation
- Automata $\rightsquigarrow$ data-flow
- Extend other languages (SCADE 6)

# Approach

- Add Ordinary Differential Equations to an existing synchronous language
- Two concrete reasons:
  - Increase modelling power (hybrid programming)
  - Exploit existing compiler (target for code generation)
- Simulate with an external off-the-shelf numerical solver (Sundials CVODE, Hindmarsh et al. SUNDIALS: Suite of nonlinear and differential/algebraic equation solvers. 2005. )
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discrete synchronous language: assume infinitely fast execution

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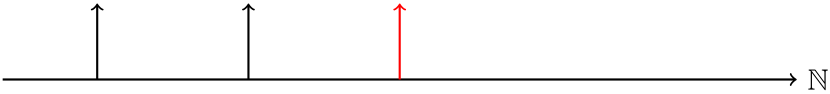

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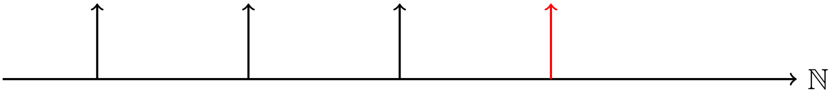

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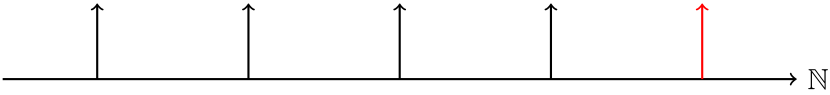

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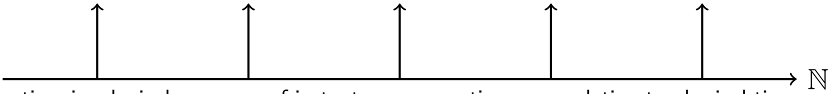


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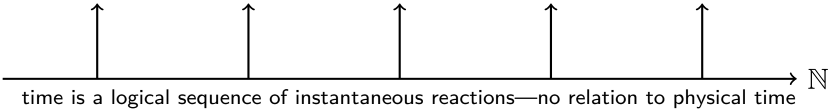


hybrid synchronous language: assume infinitely precise base clock

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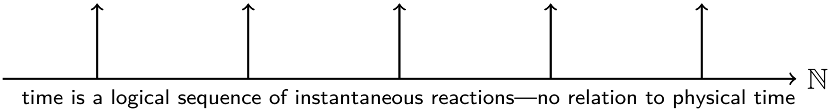


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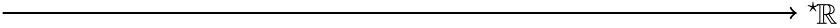

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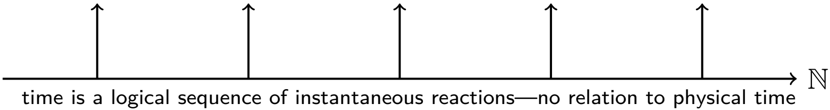


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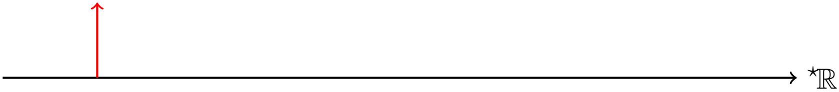

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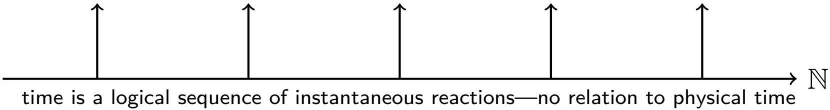


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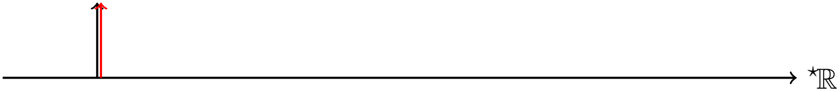

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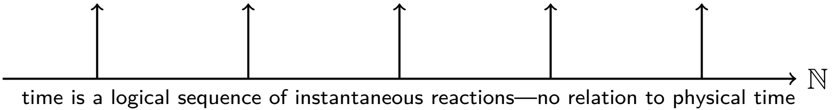


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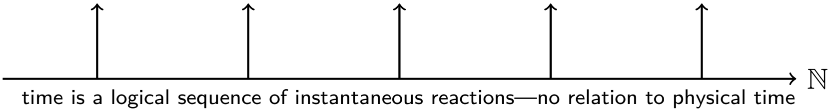


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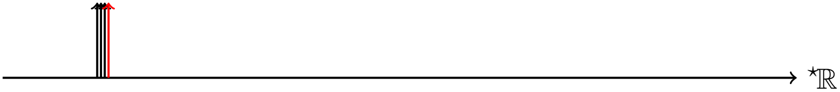

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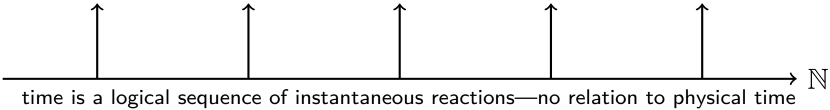


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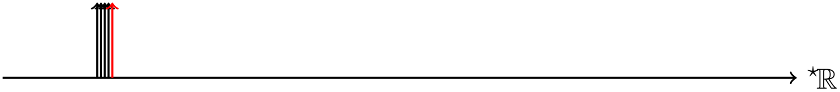

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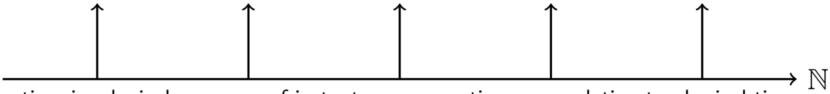


time is a logical sequence of instantaneous reactions—no relation to physical time

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assume an *infinitesimal* increment of the base clock—a **non-standard** semantics

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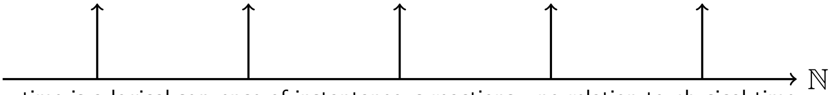


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**Q.** How to relate discrete and continuous time correctly?

hybrid synchronous language: assume infinitely precise base clock


assume an *infinitesimal* increment of the base clock—a **non-standard** semantics

**Q.** How to simulate effectively?

# Which programs make sense?

Given:

```
let node sum(x) = cpt where
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Interpretation:

- Option 1: $\mathbb{N} \subseteq \mathbb{R}$
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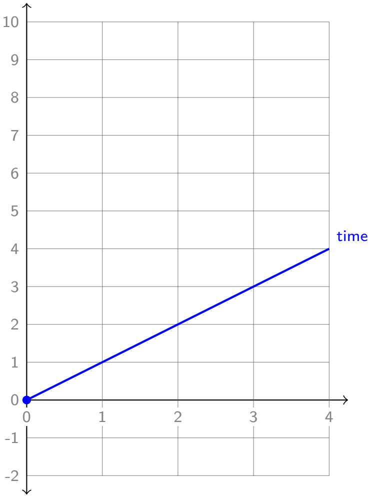

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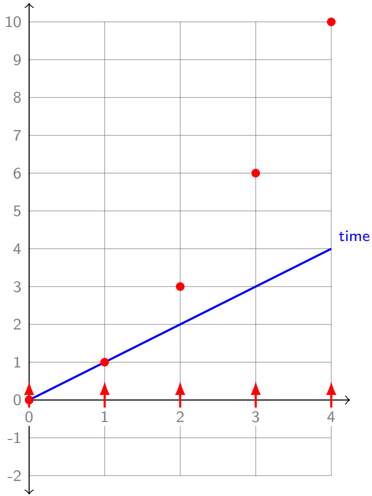

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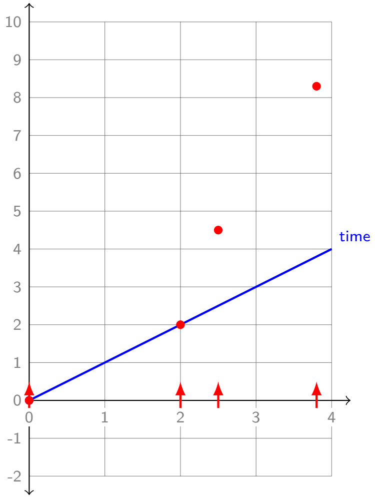

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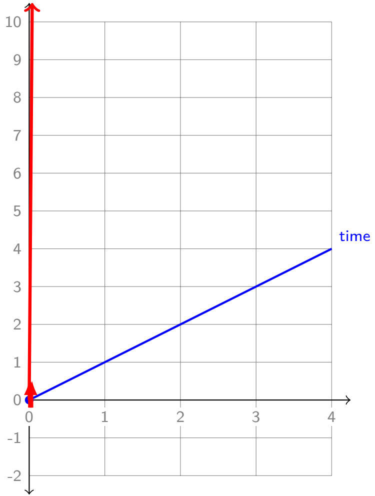

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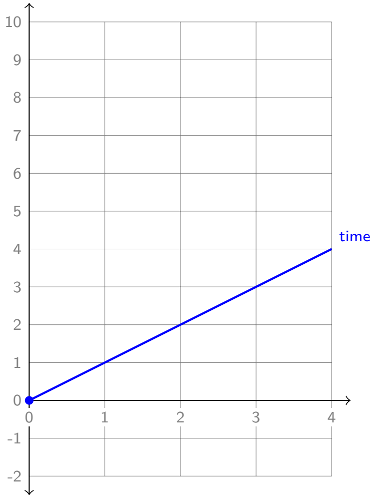

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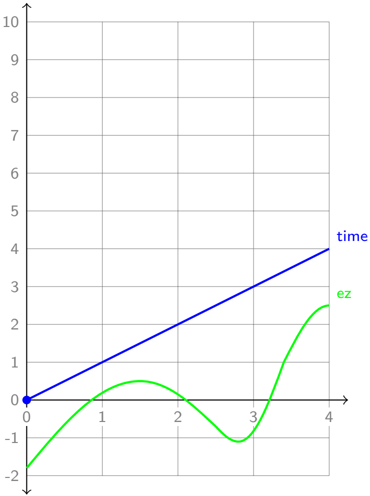

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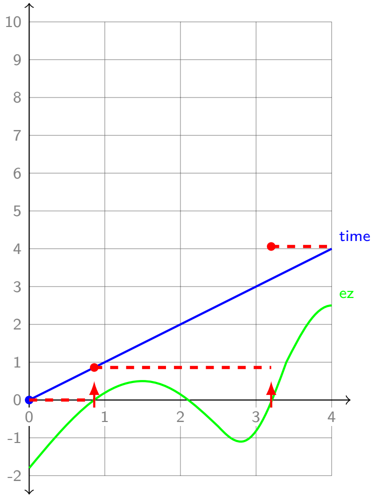


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**Explicitly relate simulation and logical time (using zero-crossings)**

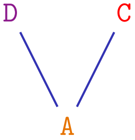
Try to minimize the effects of solver parameters and choices

# Basic typing

Milner-like type system

## The type language

$$
\begin{array}{lcl}
bt & ::= & \texttt{float} \mid \texttt{int} \mid \texttt{bool} \mid \texttt{zero} \\
t & ::= & bt \mid t \times t \mid \beta \\
\sigma & ::= & \forall \beta_1, ..., \beta_n . t \xrightarrow{k} t \\
k & ::= & \texttt{D} \mid \texttt{C} \mid \texttt{A}
\end{array}
$$

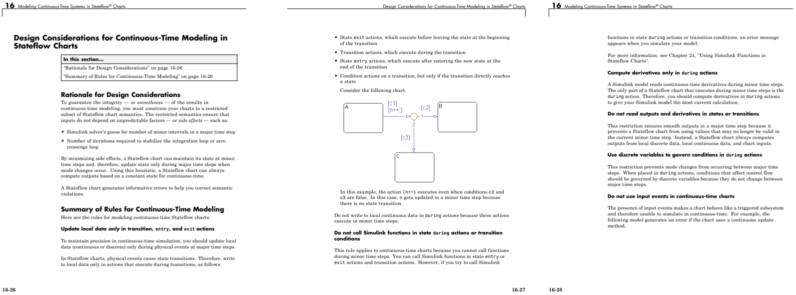



## Initial conditions

$$
\begin{array}{lcl}
(+) & : & \texttt{int} \times \texttt{int} \xrightarrow{\texttt{A}} \texttt{int} \\
(=) & : & \forall \beta . \beta \times \beta \xrightarrow{\texttt{A}} \texttt{bool} \\
\texttt{if} & : & \forall \beta . \texttt{bool} \times \beta \times \beta \xrightarrow{\texttt{A}} \beta \\
\cdot\, \texttt{fby} \,\cdot & : & \forall \beta . \beta \times \beta \xrightarrow{\texttt{D}} \beta \\
\texttt{up}(\cdot) & : & \texttt{float} \xrightarrow{\texttt{C}} \texttt{zero}
\end{array}
$$

# What about continuous automata?

## Stateflow User's Guide The Mathworks, pages 16-26 to 16-29, 2011.



16 Modeling Continuous-Time Systems in Stateflow® Charts

**Design Considerations for Continuous-Time Modeling in Stateflow Charts**

| In this section... |
| --- |
| "Rationale for Design Considerations" on page 16-26 |
| "Summary of Rules for Continuous-Time Modeling" on page 16-26 |

**Rationale for Design Considerations**

To guarantee the integrity — or *smoothness* — of the results in continuous-time modeling, you must constrain your charts to a restricted subset of Stateflow chart semantics. The restricted semantics ensure that inputs do not depend on unpredictable factors — or *side effects* — such as:

- Simulink solver's guess for number of minor intervals in a major time step
- Number of iterations required to stabilize the integration loop or zero crossings loop

By minimizing side effects, a Stateflow chart can maintain its state at minor time steps and, therefore, update state only during major time steps when mode changes occur. Using this heuristic, a Stateflow chart can always compute outputs based on a constant state for continuous-time.

A Stateflow chart generates informative errors to help you correct semantic violations.

**Summary of Rules for Continuous-Time Modeling**

Here are the rules for modeling continuous-time Stateflow charts:

**Update local data *only* in transition, `entry`, and `exit` actions**

To maintain precision in continuous-time simulation, you should update local data (continuous or discrete) only during physical events at major time steps.

In Stateflow charts, physical events cause state transitions. Therefore, write to local data only in actions that execute during transitions, as follows:

16-26

Design Considerations for Continuous-Time Modeling in Stateflow® Charts

- State `exit` actions, which execute before leaving the state at the beginning of the transition
- Transition actions, which execute during the transition
- State `entry` actions, which execute after entering the new state at the end of the transition
- Condition actions on a transition, but only if the transition directly reaches a state

Consider the following chart.



In this example, the action `{n++}` executes even when conditions `c2` and `c3` are false. In this case, `n` gets updated in a minor time step because there is no state transition.

Do not write to local continuous data in `during` actions because these actions execute in minor time steps.

**Do not call Simulink functions in state `during` actions or transition conditions**

This rule applies to continuous-time charts because you cannot call functions during minor time steps. You can call Simulink functions in state `entry` or `exit` actions and transition actions. However, if you try to call Simulink

16-27

16 Modeling Continuous-Time Systems in Stateflow® Charts

functions in state `during` actions or transition conditions, an error message appears when you simulate your model.

For more information, see Chapter 24, "Using Simulink Functions in Stateflow Charts".

**Compute derivatives only in `during` actions**

A Simulink model reads continuous-time derivatives during minor time steps. The only part of a Stateflow chart that executes during minor time steps is the `during` action. Therefore, you should compute derivatives in `during` actions to give your Simulink model the most current calculation.

**Do not read outputs and derivatives in states or transitions**

This restriction ensures smooth outputs in a major time step because it prevents a Stateflow chart from using values that may no longer be valid in the current minor time step. Instead, a Stateflow chart always computes outputs from local discrete data, local continuous data, and chart inputs.

**Use discrete variables to govern conditions in `during` actions**

This restriction prevents mode changes from occurring between major time steps. When placed in `during` actions, conditions that affect control flow should be governed by discrete variables because they do not change between major time steps.

**Do not use input events in continuous-time charts**

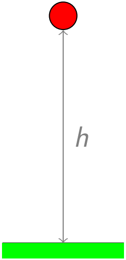
The presence of input events makes a chart behave like a triggered subsystem and therefore unable to simulate in continuous-time. For example, the following model generates an error if the chart uses a continuous update method.

16-28

. . .

- 'Restricted subset of Stateflow chart semantics'
  - restricts side-effects to major time steps
  - supported by warnings and errors in tool **(mostly)**
- Our `D`/`C`/`A`/`zero` system extends naturally for the same effect
- For both discrete (synchronous) and continuous (hybrid) contexts

# Compilation: source-to-source transformation




```
let hybrid ball () =
  let
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                  reset (-. 0.8 *. last v) every up(-. h)
  and der h =     v init h0
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```

```
let node ball (z1, (lh, lv), ()) =
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  and v = if i then v0
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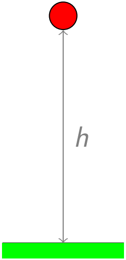
  in ((v, h), upz1, (h, v), (dh, dv))
```

# Compilation: source-to-source transformation

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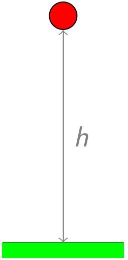


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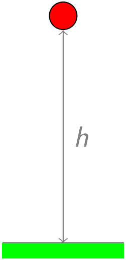
```

transform into discrete subset

```
, upz1, (h, v), (dh, dv))
```

# Compilation: source-to-source transformation

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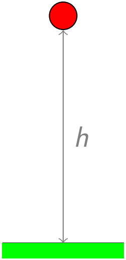


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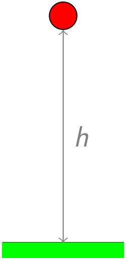


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# Demonstrations

- Bouncing ball (standard)
- Bang-bang temperature controller (Simulink/Stateflow)
- Sticky Masses (Ptolemy)

# Conclusion

- Synchronous languages **should** and **can** properly treat hybrid systems
- There are three good reasons for doing so:
  1. To exploit existing compilers and techniques
  2. For programming the discrete subcomponents
  3. To clarify underlying principles and guide language design/semantics
- Our approach
  1. Synchronous data-flow language with automata and ODEs
  2. Static type system to separate discrete from continuous behaviors
  3. Relate discrete to continuous via zero-crossings
  4. Compilation via source-to-source transformations
  5. Simulation using off-the-shelf numerical solvers
- Prototype compiler in OCaml using Sundials CVODE solver

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# Future work

## Short term

- Compiler implementation
- Language
- Examples / case studies: more!

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## Longer term

- Clock calculus: an finer analysis of piece-wise continuous/continuous/discrete
- Causality analysis: (partial) detection of discrete Zeno-behavior
- Semantics: how abstract is the solver?
- Real-time simulation (trade-off accuracy and execution time)

Bibliography at: `www.di.ens.fr/ pouzet`