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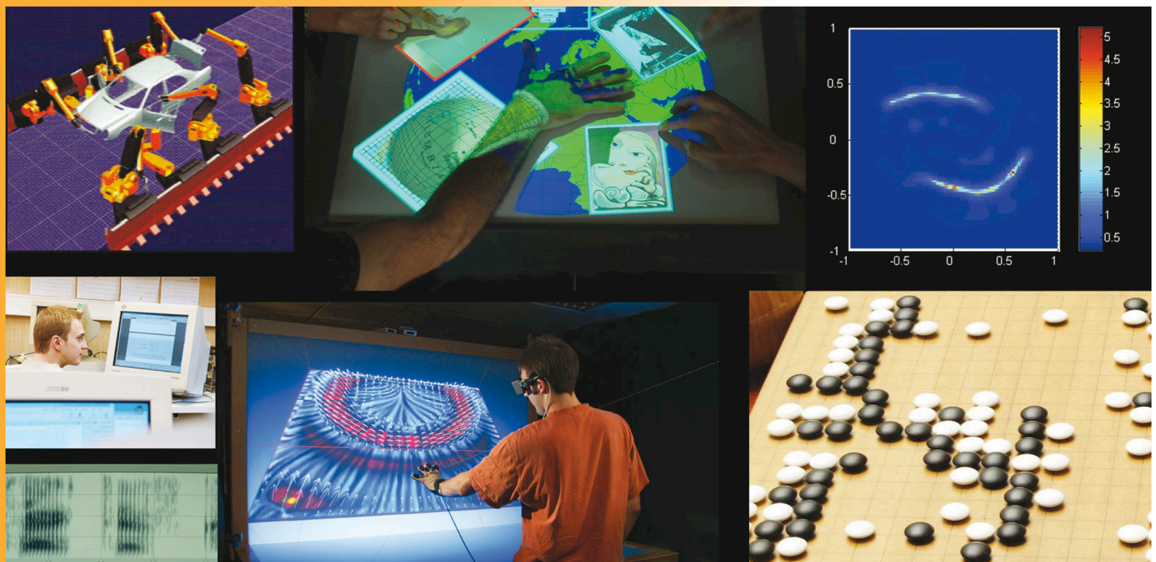
Synchronous Functional Programming

Marc Pouzet



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LRI & INRIA & IUF

Digiteo Meeting

October 2008, 2nd

Typed Functional Programming

Program in a **mathematical language** as an attempt to achieve code with zero defect. High-level languages abstracting some details to focus on *what* computes a system.

A computation is a sequence of reductions :

$$fact(3) \rightarrow 3 \times fact(2) \rightarrow 3 \times 2 \times fact(1) \rightarrow 3 \times 2 \times 1 \rightarrow 3 \times 2 \rightarrow 6$$

Follow few principles :

– Function composition

– Types as specifications/properties of these functions

– A method to check that a function agrees with its type

A rich collection of languages :

– Lazy languages restricted to *pure* functions (without side-effect) : Haskell, etc.

– Strict languages : Objective Caml, StandardML, etc.

– Proof assistants (e.g., Coq) to write **total** functions (which always terminate)

An **important vehicule of ideas** for other languages and the use of formal methods in industry (Esterel-Tech., Microsoft, etc.)

Real-time Systems

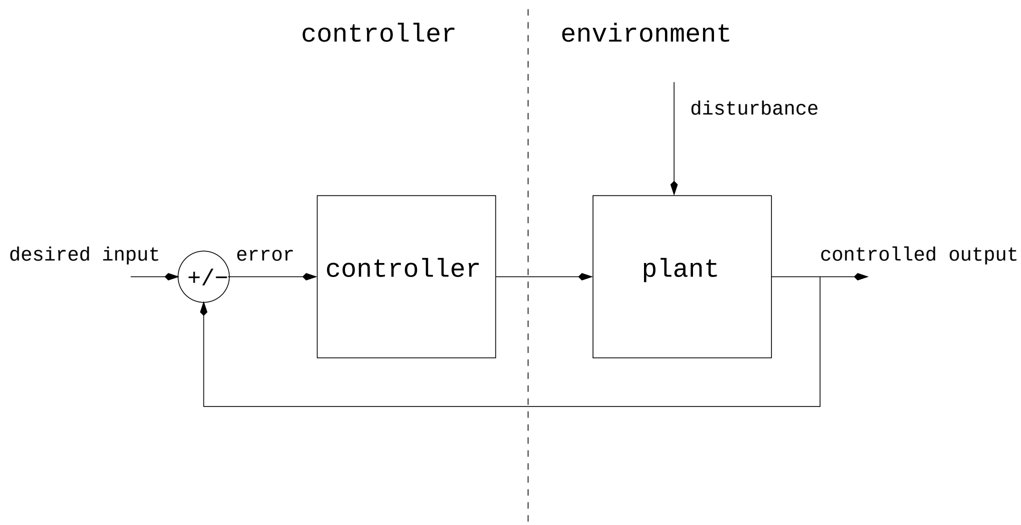
Focus on systems which continuously interact with each others.

– with a **physical environment** (e.g., fly-by-wire command, control-engine)

– or **other digital devices** (e.g., phone, TV boxes)

Real time is always **related to the environment** and is not an absolute notion. To ensure safety, think of **“what is the worst case”** ?

The environment is often not precisely known : most systems run in ***closed-loop***



How can we program those systems, focusing on the **functionality** and abstracting from subtle implementation details ?

What is new, why do we need mathematical languages ?

Conciliate three notions :

- a **formal** (and computable) model of time
  - express deadlines, simultaneous events, etc.
- **parallelism** to describe complex systems from simpler ones
  - control *at the same time* rolling and pitching
  - *closed-loop* systems (the controller and the plant run in parallel)
- **statically guaranty safety properties** (both functional and non functional)
  - determinism, dead-lock freedom
  - execution in bounded time and memory

Safety is important :

- critical systems : fly-by-wire, braking, airbags, etc.
- some systems do not have a stable position (plane ?)
  - properties must be guaranteed statically : **“dynamic” = “too late”**

A bit of History

In the 80's, several team invented (at about the same time) languages dedicated to the design/implementation of control-systems.

– Lustre (Caspi & Halbwachs, Grenoble) : data-flow (block-diagrams), functional model (deterministic) ;
– Signal (Benveniste & Le Guernic, Rennes) : data-flow but relational (to define also non-deterministic systems) ;
– Esterel (Berry & Gonthier, Sophia) : hierarchical automata and process algebra

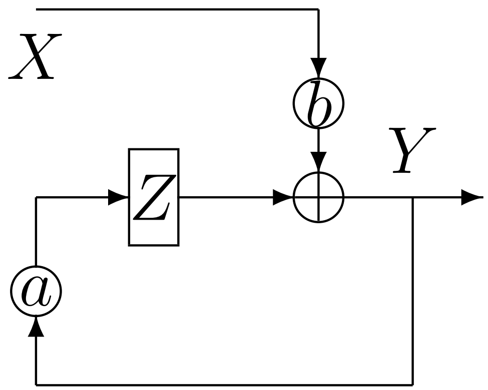
Base it on the **mathematical culture and models** of the field of embedded control-systems

Quite a successful story : security systems in nuclear plants (Schneider Electric), fly-by-wire (Airbus A340-380), automotive, trains, etc.

Control-theory, Signal Processing, Electronic

- A discrete signal/event is a stream
  - $\hookrightarrow$ *stream equations, Z transforms*
  - $\hookrightarrow$ *graphical formalisms (block-diagrams) to represent these networks*
- manual transcription of these equations into imperative code
- hard and error-prone

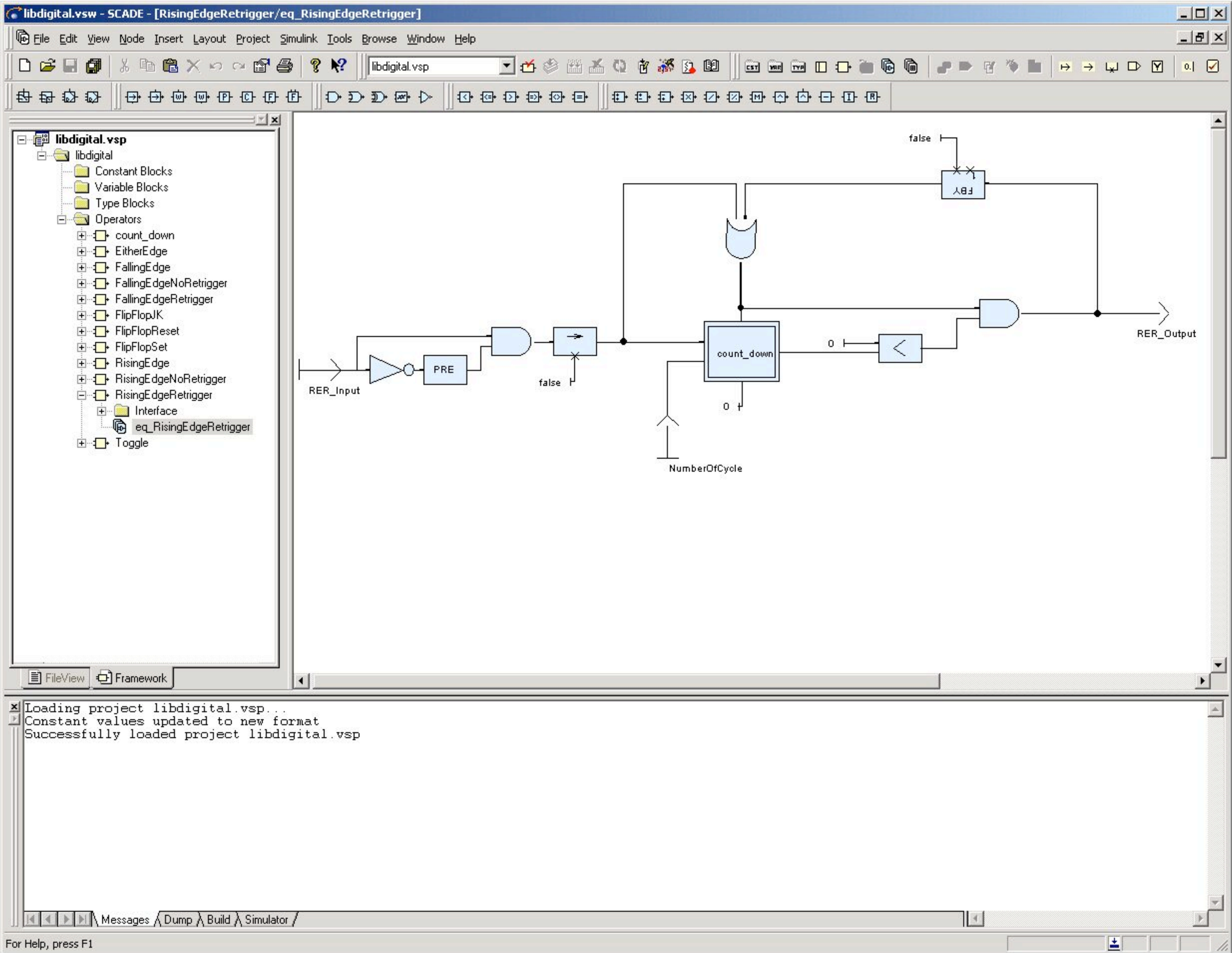
$$Y_0 = bX_0 \ , \ \forall n \ Y_{n+1} = aY_n + bX_{n+1}$$



The idea of Lustre :

- write stream equations as **executable specifications**
- provide **static analysis/verification** tools and a compiler to produce code
- the generated code is **correct-by-construction**

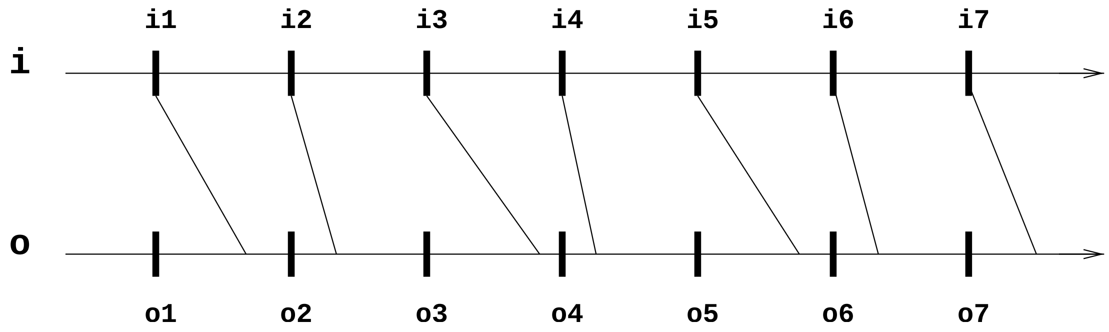
SCADE V5



libdigital.vsw - SCADE - [RisingEdgeRetrigger/eq_RisingEdgeRetrigger]
File Edit View Node Insert Layout Project Simulink Tools Browse Window Help
libdigital.vsp
libdigital.vsp
libdigital
Constant Blocks
Variable Blocks
Type Blocks
Operators
count_down
EitherEdge
FallingEdge
FallingEdgeNoRetrigger
FallingEdgeRetrigger
FlipFlopJK
FlipFlopReset
FlipFlopSet
RisingEdge
RisingEdgeNoRetrigger
RisingEdgeRetrigger
Interface
eq_RisingEdgeRetrigger
Toggle
FileView
Framework
false
FBY
RER_Input
PRE
false
count_down
0
0
NumberOfCycle
RER_Output
Loading project libdigital.vsp...
Constant values updated to new format
Successfully loaded project libdigital.vsp
Messages
Dump
Build
Simulator
For Help, press F1

The Synchronous Model of Time

Separate the **functionality** of the system from its **implementation**.



– Time is a **logical notion** as a sequence of atomic reactions of the system

  think as if the machine was infinitely fast and react instantaneously

– Is my abstraction reasonable ? Is the **machine fast enough ?**

  Worst Case Execution Time analysis

This coincide with the **zero-model** of synchronous circuits but for software

Important consequences :

– Specifications become mathematical objects which can be statically verified, transformed and compiled

– They become portable and can be reused

Synchronous Functional Programming

Lustre is a (first-order) functional language : a system is a stream function and we write **invariants**

```
y = a * pre(y) + z;
z = b * x
```

for $\forall t \in D.\{y_t = (a * pre(y) + z)_t = a * y_{t-1} + z \wedge z_t = b * x_t\}$

Questions :

– increase **modularity/expressiveness**, re-usability (software components) : E.g., type synthesis, polymorphism, higher-order, etc.

– **mix data and control-dominated systems** in a unified model

– more **dedicated static analysis** to ensure safety properties
E.g., does the program behave synchronously ? is-it causal ? Is-it deterministic ?

– (mathematically) **certify** code-generation ? Improve compilation techniques ?

What can we learn from the relationships with typed functional languages ?

Lucid Synchrone

How to extend Lustre in a conservative way (without breaking it) ?

Build a "laboratory" language

- study and prototype extensions of Lustre
- experiment things, language extensions/static analysis and manage all the compilation chain

Follow a few principles :

- be conservative *wrt* the Lustre semantics
- formulate the synchronous data-flow model into the typed lambda-calculus
- functional composition, static properties as (special) types
- modularity everywhere (type analysis, separate compilation)

Clocks as Types

How to mix several time scales in a safe manner ?

- **multi-sampled** systems (software), **multi-clock** (hardware)
- what does it mean to communicate between two software components which do dot **agree on a common time scale** ?
- **statically detect** possible synchronisation issues

x	$=$	x_0	$*$	$*$	$*$	x_1	$*$	x_2	$*$	$*$	x_3	x_4	$*$	x_5	$\ldots$
y	$=$	$*$	y_0	y_1	y_2	$*$	y_3	$*$	$*$	y_4	$*$	y_5	$*$	$*$	$\ldots$

- Express the **clock** information as a **type** which express the instants where a value is defined
- Express the verification as a (classical) type system. Can I prove ?

$$H \vdash e : ct$$

- Safety property : every well-clocked program can be executed synchronously

Unifying Data-dominated and Control-dominated Systems

Data-dominated systems : typically from periodic sampling of continuous systems

– simulation tools (e.g., Simulink), programming tool (e.g., SCADE)

Control-dominated systems : discrete control, transition systems, automata

– StateFlow, StateCharts or synchronous models (e.g., SSM, Esterel)

Real systems rarely fall in one of these category

– systems have **modes** : take-off, landing, full-flight

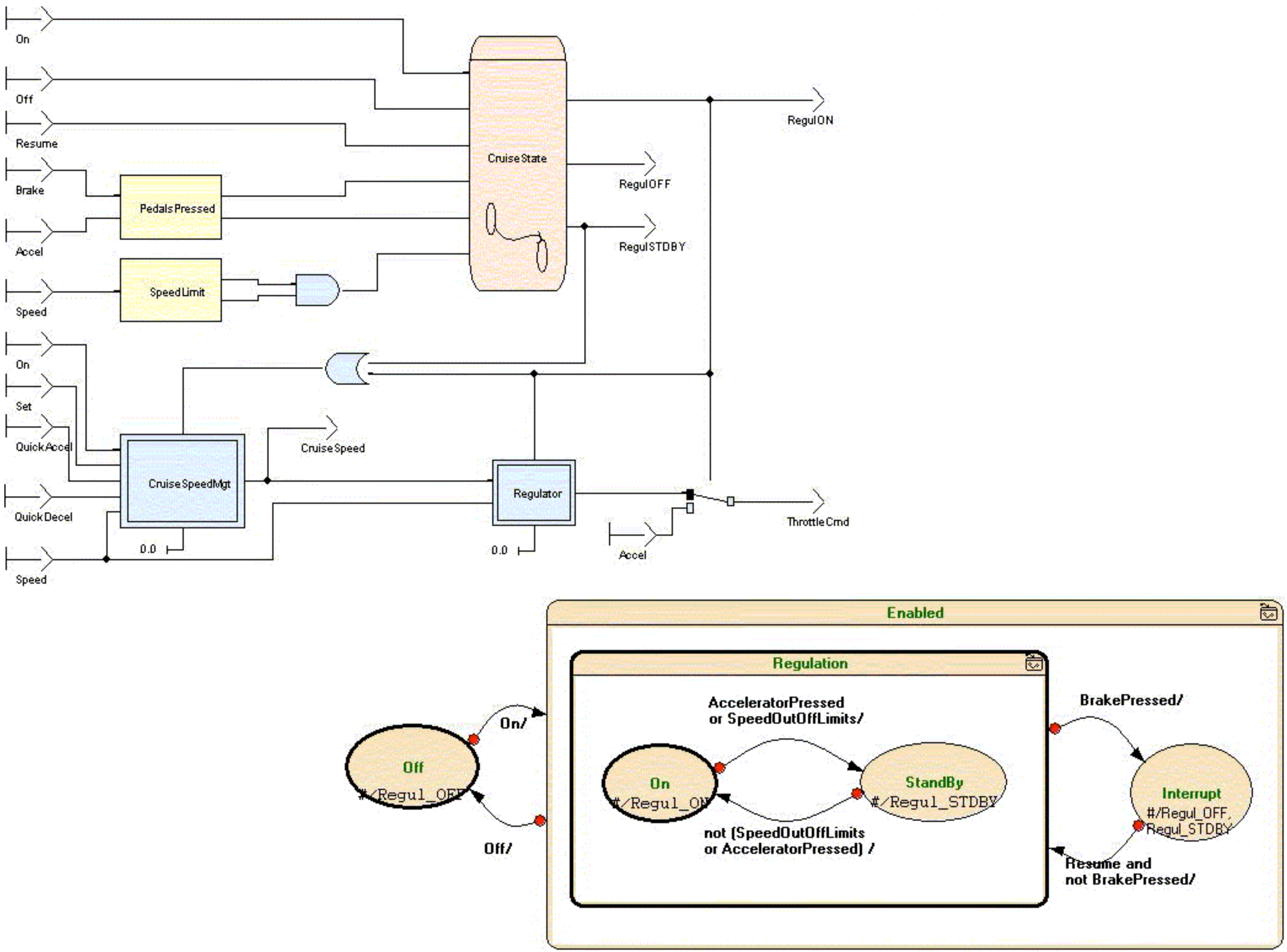
– each mode is defined by its control-law, naturally described by stream equations

– the control part is made as an automaton which activate some of the modes

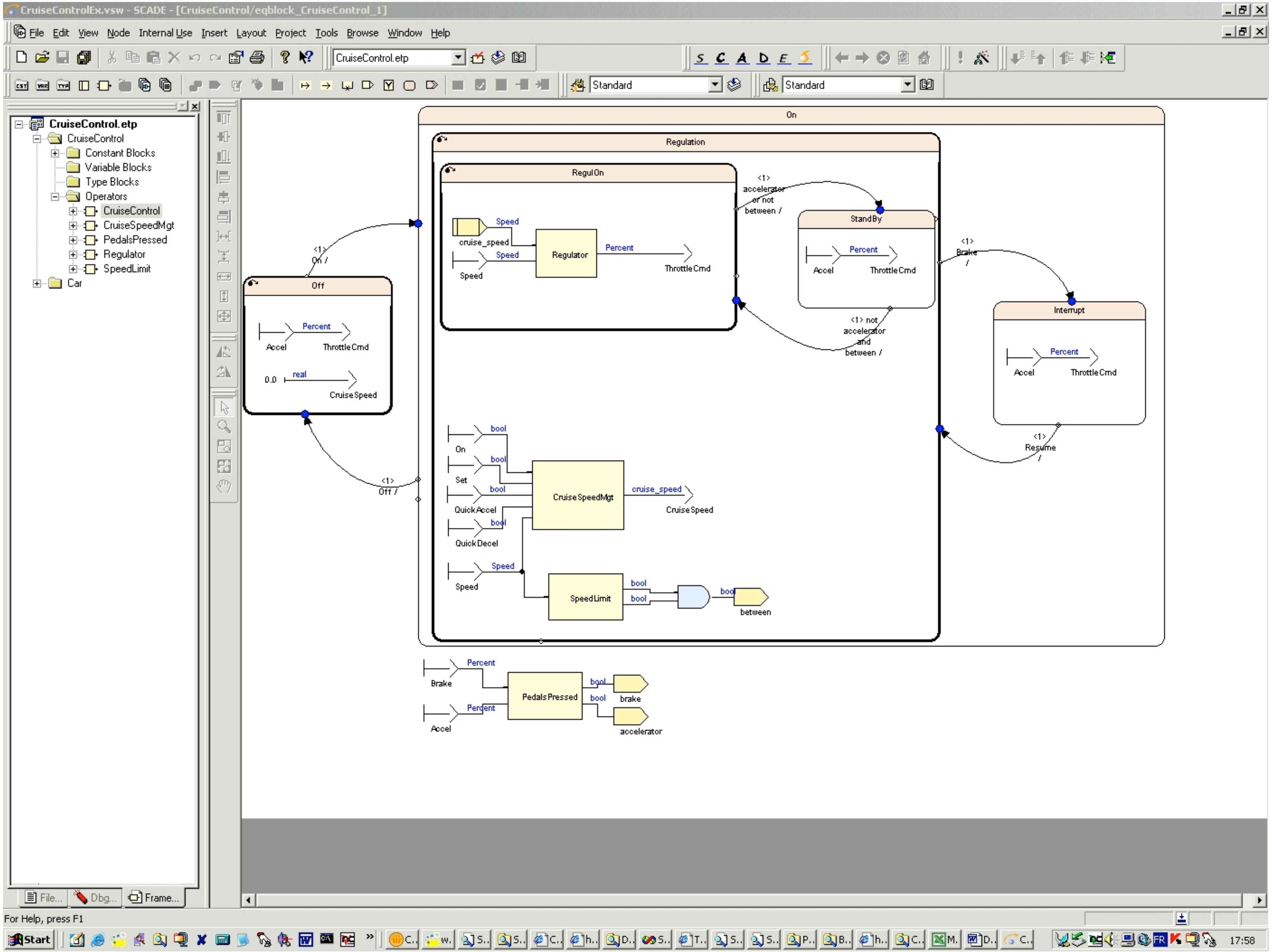
Most tools provide means to combine both : Simulink/StateFlow, SCADE/Esterel SSM, Ptolemy Mocs, etc.

This is rather ad-hoc and there is no real unified semantics of the whole.

Cruise-control in SCADE+SSM (V5)



Scade V6 (dec. 2007)



Unifying Data-flow and Control-flow

Follow a **clock-based approach**. Take a basis synchronous data-flow with clocks (e.g., Lustre)

- **efficient code generators** exist
- in the case of SCADE, the code generator is **certified** (DO 178B)
- clocks play a central role in synchronous compilers (semantics/optimisation)
- clocks are about "when" a data is ready

Extending the basic language with rich automata constructs and define a **translation semantics**

- a clock-preserving program transformation into the basic language

$$H \vdash e : ct \Rightarrow H \vdash T(e) : ct$$

- reuse the existing code generation techniques
- the final code appeared to be as good than dedicated techniques

Practical Applications

This embedding of synchronous data-flow into a functional setting is fruitful.

Several features originally introduced in **Lucid Synchrone** are now integrated in industrial tools.

- the ReLuC compiler of SCADE is based (and improves) techniques introduced in Lucid Synchrone
- same philosophy : types everywhere, modularity, etc.
- program constructs (e.g., `merge`), control-structures
- static analysis (initialisation, clock calculus)
- design/semantics of SCADE 6

Now new applications for CAD tools, mixing both programming and simulation.

What Else ?

Can we relax the synchronous model to address simulation problems and general reactive systems (e.g., graphical interfaces, games) ?

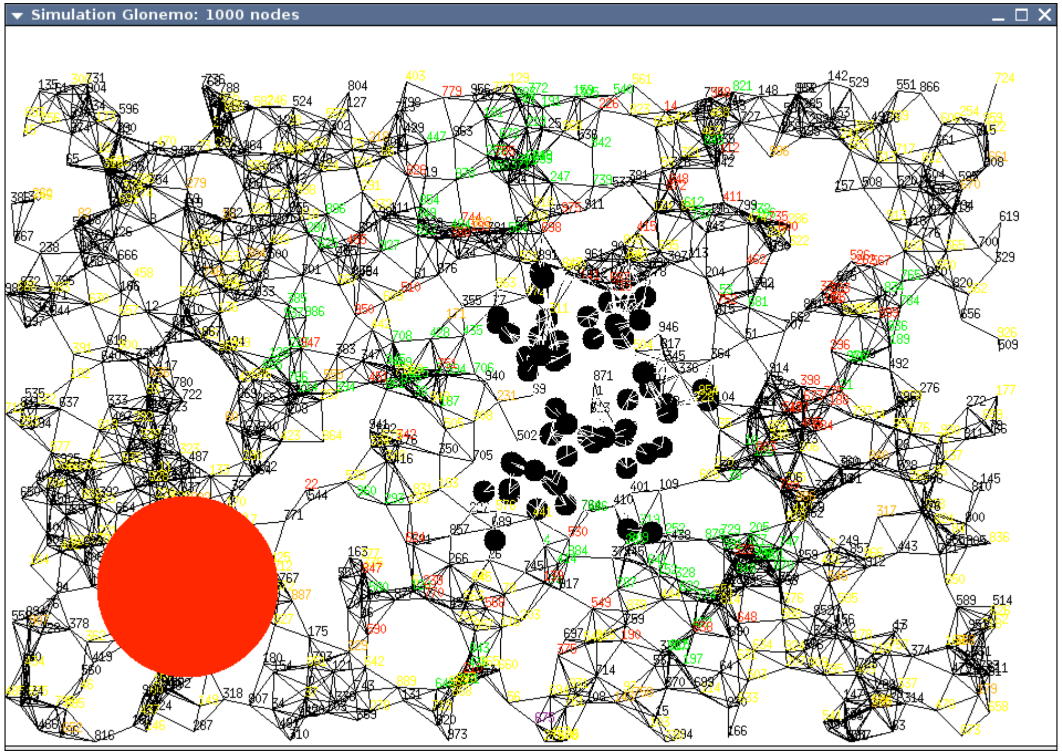
Reactive Programming

Fundational work by Boussinot (Reactive C, Loft, etc.). ReactiveML (by Mandel)

Simulation of sensor networks (VERIMAG and FT, 2006-2008)

– The system is both real-time and dynamical

– Global simulation : each node, the interaction between nodes and its environment, simulation aspects (display, computing metrics, etc)...



Example : Simulation of the power consumption in a sensor network

Conclusion and Perspectives

Mixing discrete/continuous time

– can we have more faithful models by integrating programming and numerical simulation techniques (for continuous laws) ?

– a unified model/language for both discrete and continuous time with clean semantics ?

Video Intensive Computation

– Kahn Process Networks widely used (e.g., NXP)

– A synchronous model to program parallel architectures ?

Certified compilation

– do not program into C anymore (or rarely) embedded software

– only prove the specification, not the code

– can we certify a synchronous compiler using a proof assistant (e.g., Coq) ?

References

- Synchronous Kahn Networks [ICFP'96]
- Clocks as Dependent types [ICFP'96]
- Modular compilation of higher-order stream functions [CMCS'98]
- ML-like clock calculus [EMSOFT'03]
- Type-based causality analysis [ESOP'01]
- Initialisation analysis [SLAP'02, STTT'04]
- Type-system and higher-order [EMSOFT'04]
- Mixing data-flow and hierarchical automata [EMSOFT'05, EMSOFT'06]
- N-Synchronous Kahn Networks [EMSOFT'05, POPL'06, APLAS'08]
- Formalisation of a synchronous compiler [LCTES'08]
- ReactiveML [PPDP'05,SLAP'08]