

Communication Channel

Discrete

Continuous

Error-less

With errors

Distortion : deterministic; reversible or not

Noise: random

Error Control:

$$Pr\{communication_error\} \sim Pr\{computing_error\}$$

Error-free Channel Capacity

A VLSI wire (error-less & digital) has capacity: 1 Sh/cycle

A teletype, with 32 keys: 5 Sh/cycle

Let $C(N-1)$ be the number of code words of length $<N$:

Capacity of the error-free discrete channel:

$$C = \lim_{N \rightarrow \infty} \frac{1}{N} \log_2 C(N) \quad \text{Sh/cycle.}$$

Telegraph Capacity

Telegraph punctuation: $\{D,L,S,W\} = \{10, 1110, 000, 000000\}$

Each code ends with a word in: $\{D,L,SD,SL,WD,WP\}$

Let $C(N)$ be the number of code words of length $\leq N$:

- $C(N)=0$ for $N<2$, and $C(2)=1$;
- $C(N) = C(N-2) + C(N-4) + C(N-5) + C(N-7) + C(N-8) + C(N-10)$.

Let $\rho = 2^{-0.539\dots}$ be the largest root of: $1 = x^2 + x^4 + x^5 + x^7 + x^8 + x^{10}$;

We have:

$$C(N) \sim \rho^N.$$

The capacity of the telegraph is:

$$C = -\log_2 \rho = 0.539\dots$$

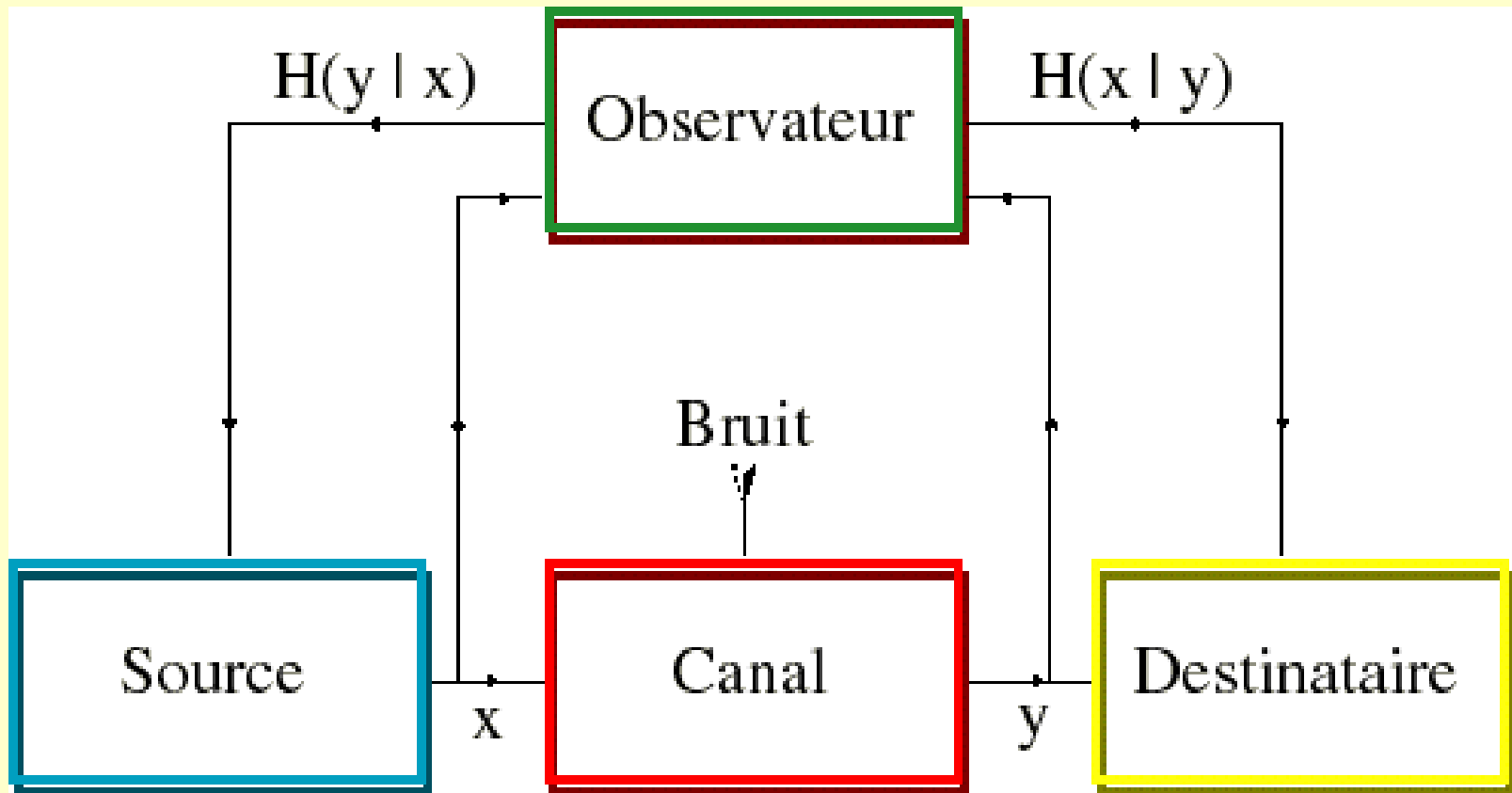
Communicating a source of entropy H through a channel of capacity C is:

- Possible when $H < C$.
- Impossible when $H > C$.

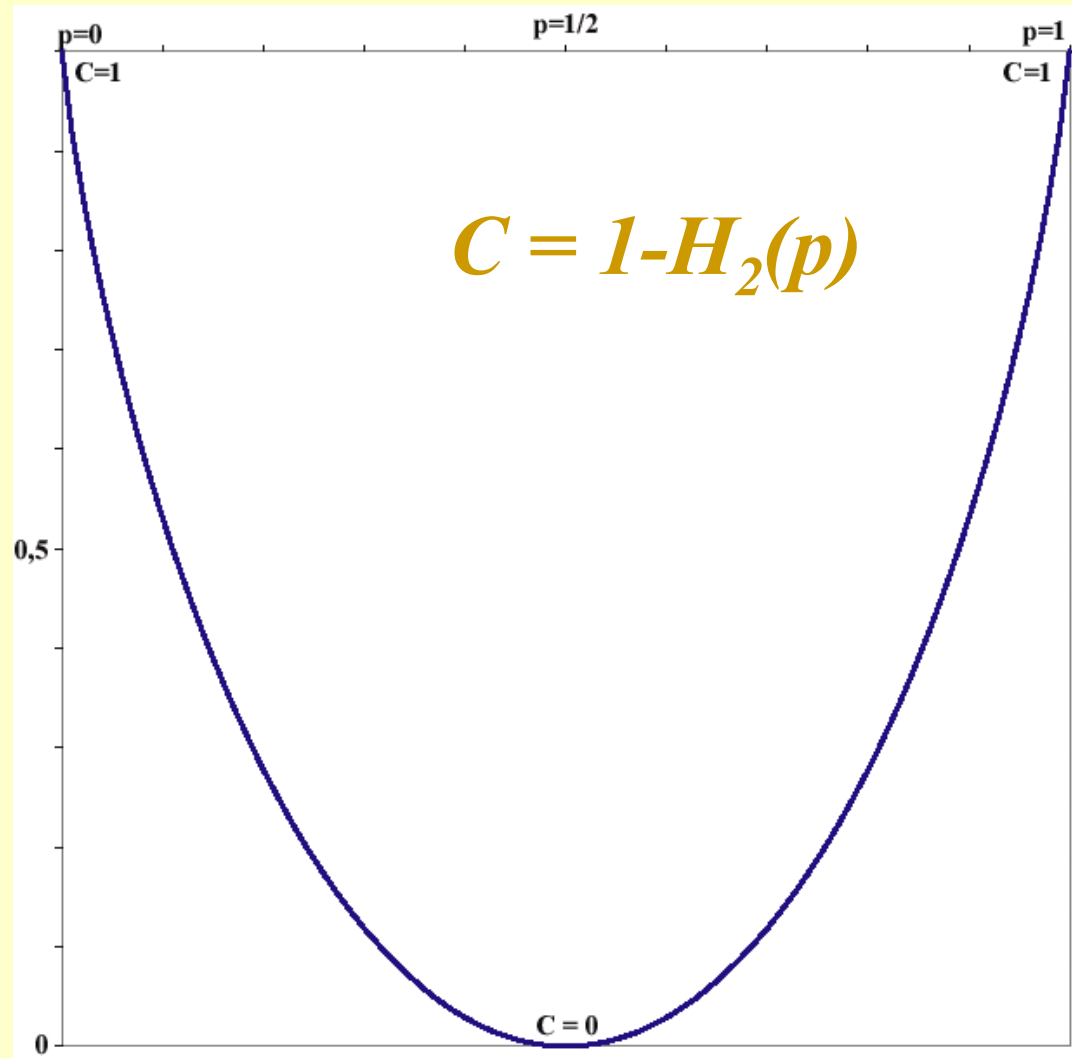
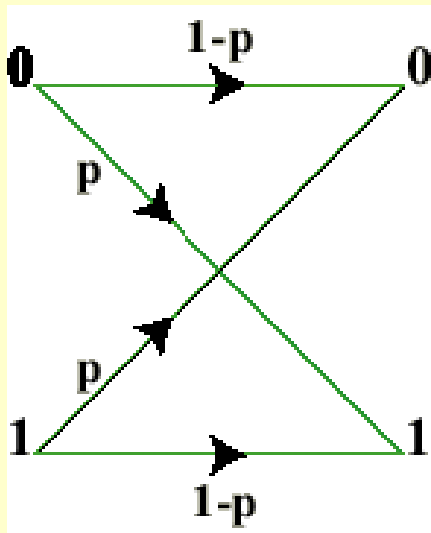
Shannon's
Channel
Theorem

Channel Capacity with errors

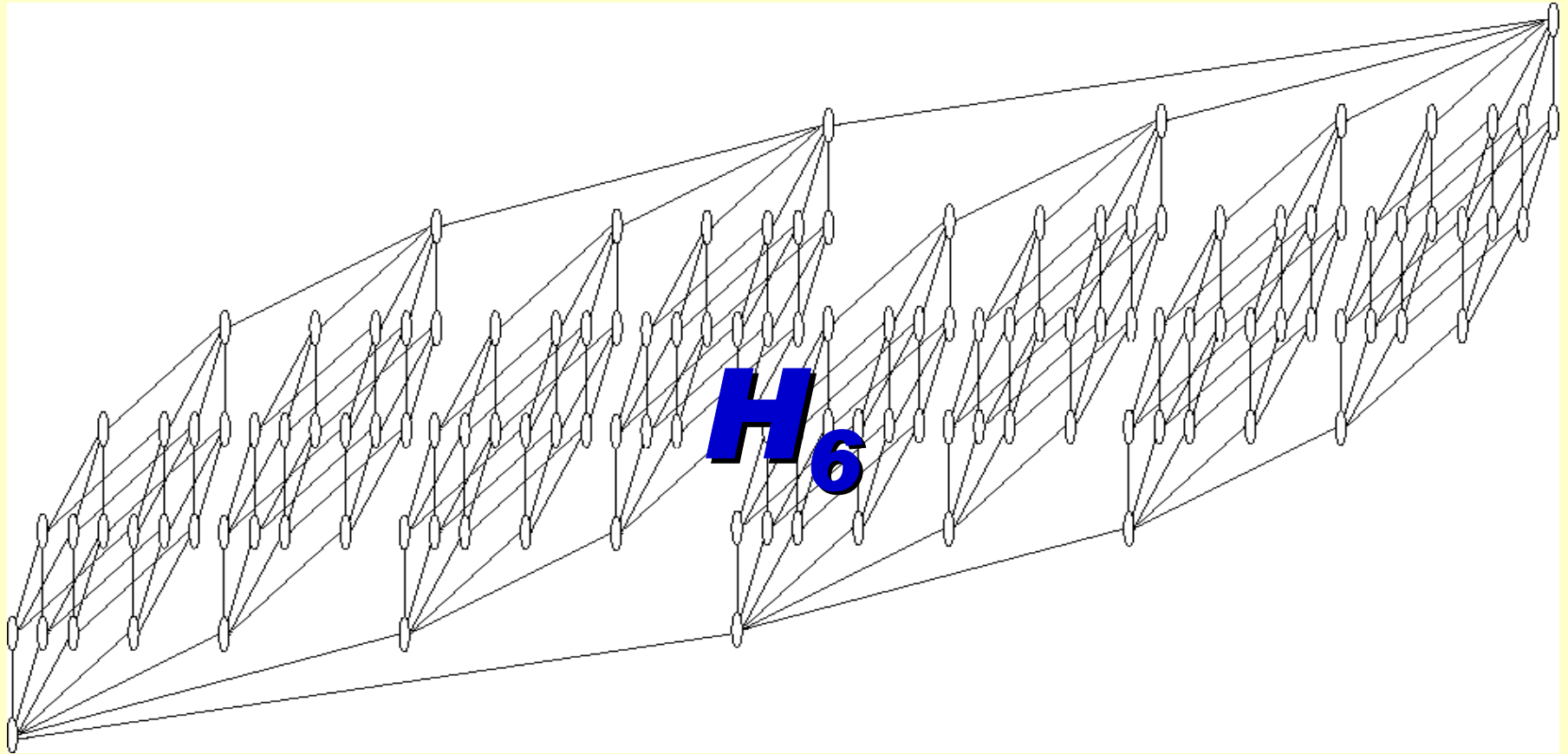
$$C = C_{ef} - H(error) \quad \text{Sh/cycle.}$$



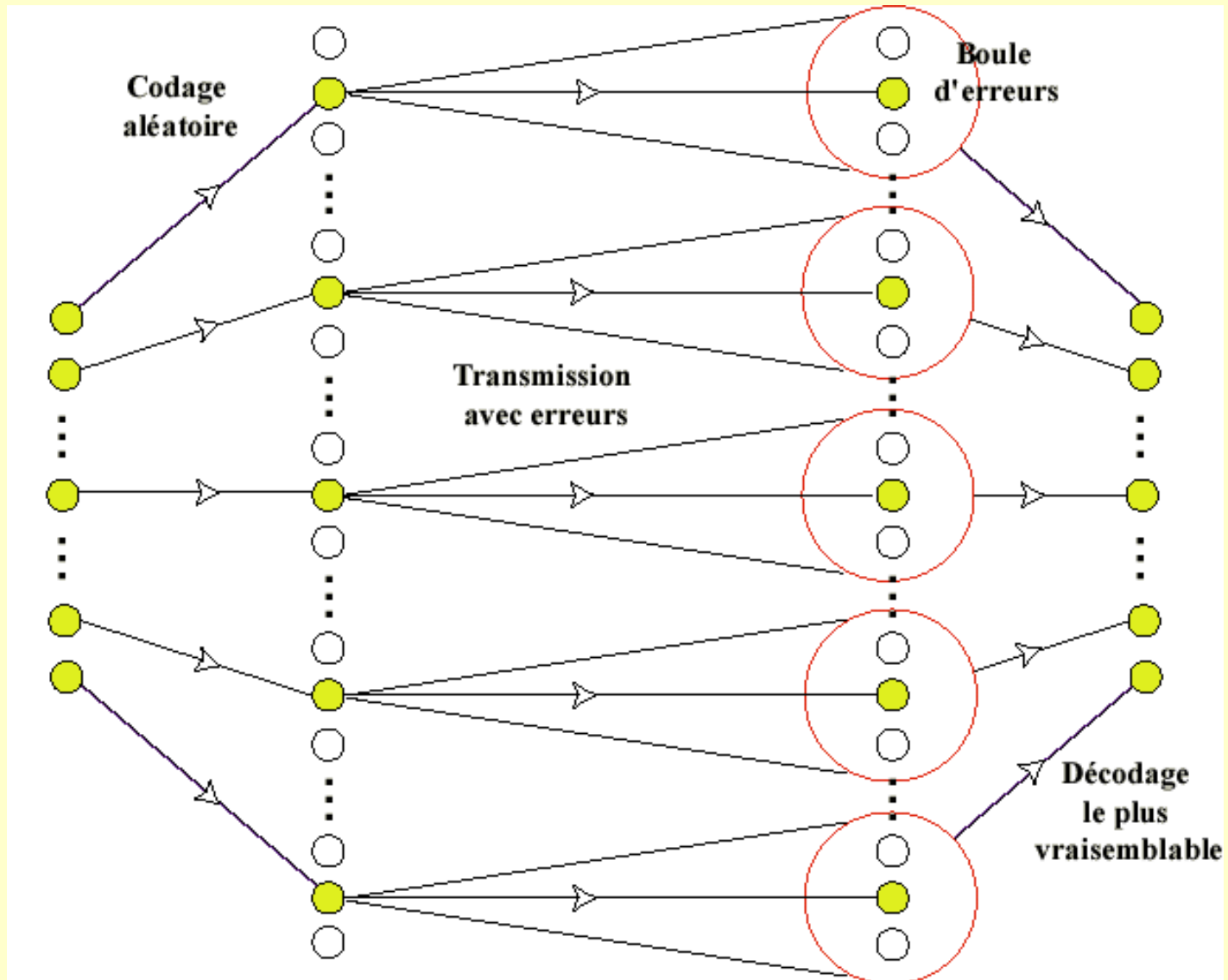
Binary Channel with Error

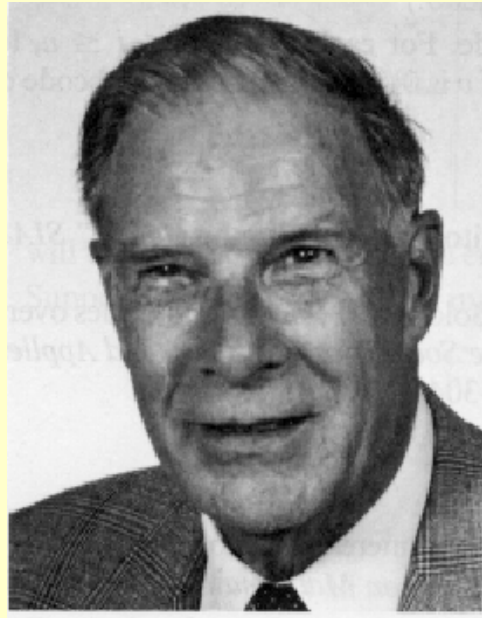


Hypercube

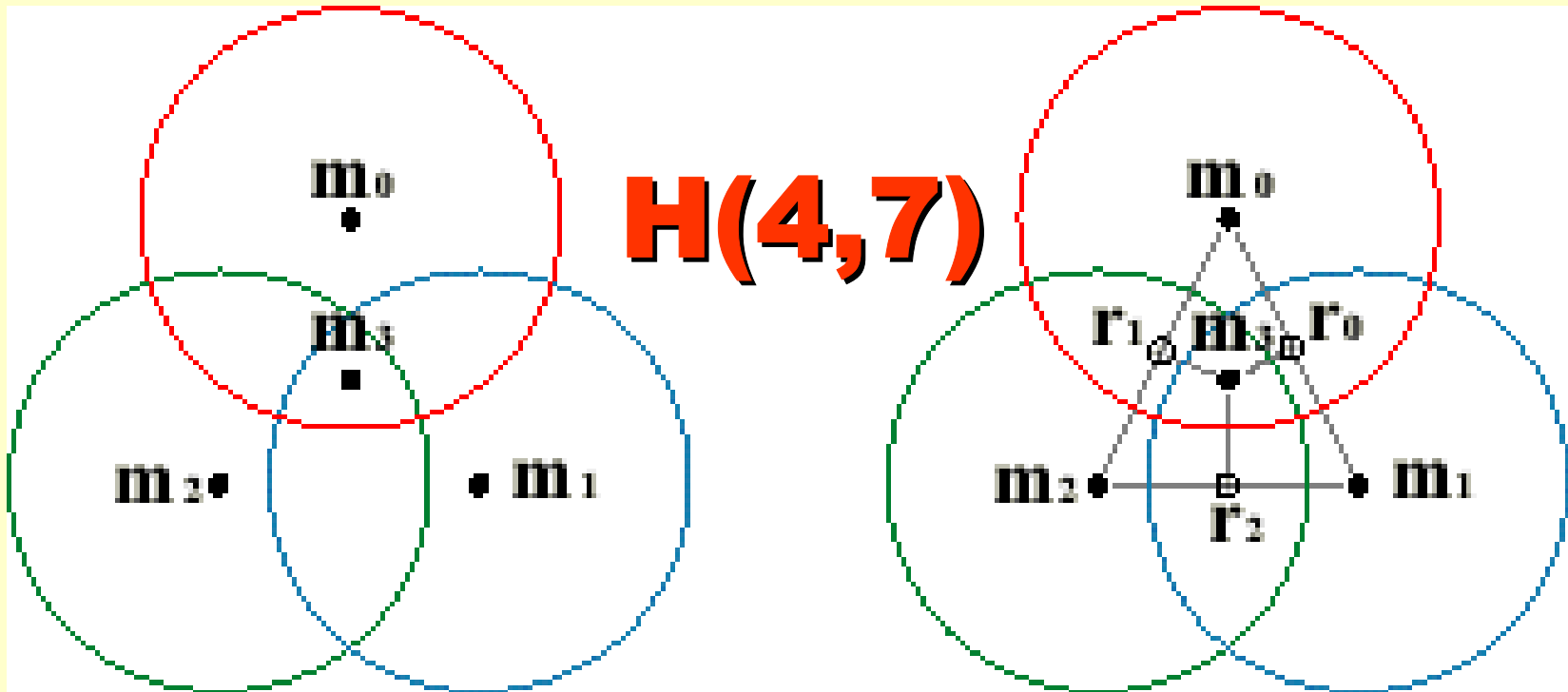


Random Coding





Hamming Code





Finite Fields

F_2 F_4 F_8

$$z^8 + z = z(z+1)(1+z+z^3)(1+z^2+z^3) \pmod{2}$$

k	z^k	z^{2k}	$P(z)=0$
0	100	100	$z+1$
1	010	001	$1+z+z^3$ a
2	001	011	$1+z^2+z^{2 \times 3}$ b
3	110	101	$1+(z^3)^2+(z^3)^3$ c'
4	011	010	$1+z^4+z^{4 \times 3}$ c
5	111	110	$1+(z^5)^2+(z^5)^3$ b'
6	101	111	$1+(z^6)^2+(z^6)^3$ a'

k	z	z^2	$P(z)=0$
0	100	100	$z+1$ 1
1	010	001	$1+z+z^3$ a
2	001	011	$1+z^2+z^{2 \times 3}$ a ²
3	110	101	$1+(z^3)^2+(z^3)^3$ a ³
4	011	010	$1+z^4+z^{4 \times 3}$ a ⁴
5	111	110	$1+(z^5)^2+(z^5)^3$ a ⁵
6	101	111	$1+(z^6)^2+(z^6)^3$ a ⁶

+	0	1	2	4	3	7	5	6
0	0	1	2	4	3	7	5	6
1	1	0	4	2	7	3	6	5
a	a	a ³	0	1	5	6	3	7
a ³	a ³	a	1	0	a ⁵	5	7	3
a ²	a ²	a ⁶	a ⁴	6	0	1	2	4
a ⁶	a ⁶	a ²	a ⁵	a ⁵	1	0	a ⁶	a ³
a ⁴	a ⁴	a ⁵	a ²	a ⁶	a	4	0	1
a ⁵	a ⁵	a ⁴	a ⁶	a ²	a ³	2	a	0

x	0	1	2	4	3	7	5	6
0	0	0	0	0	0	0	0	0
1	0	1	2	4	3	7	5	6
a	0	a	3	5	4	1	6	7
a ³	0	a ³	a ⁴	7	a ⁵	3	1	2
a ²	0	a ²	a ³	6	5	2	7	1
a ⁶	0	a ⁶	1	a ²	a	6	a ³	a ⁴
a ⁴	0	a ⁴	a ⁵	1	a ⁶	4	2	3
a ⁵	0	a ⁵	a ⁶	a	1	5	a ²	4

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	0	4	7	2	6	5	3
a	a	a ³	0	5	1	3	7	6
a ²	a ²	a ⁶	a ⁴	0	6	2	4	1
a ³	a ³	a	1	a ⁵	0	7	3	5
a ⁴	a ⁴	a ⁵	a ²	a	a ⁶	0	1	4
a ⁵	a ⁵	a ⁴	a ⁶	a ³	a ²	a	0	2
a ⁶	a ⁶	a ²	a ⁵	1	a ⁵	a ⁶	a ³	0

x	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
a	0	a	3	4	5	6	7	1
a ²	0	a ²	a ³	5	6	7	1	2
a ³	0	a ³	a ⁴	a ⁵	7	1	2	3
a ⁴	0	a ⁴	a ⁵	a ⁶	1	2	3	4
a ⁵	0	a ⁵	a ⁶	1	a	a ²	4	5
a ⁶	0	a ⁶	1	a	a ²	a ³	a ⁴	6

Error Control

Detect (parity,...) & Re-transmit Bloc Codes

- ***Hamming Distance***
- ***Finite Fields***

Convolution Codes

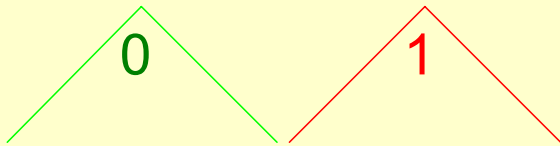
- ***Viterbi***
- ***Trellis Decoding***

Code Interleave

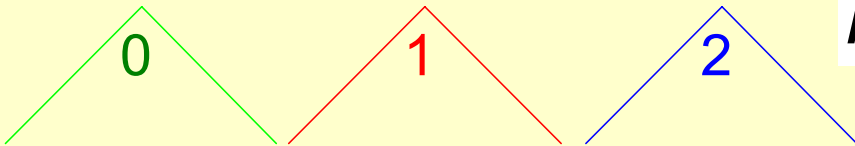
Continuous Channel

Capacity of the continuous channel:

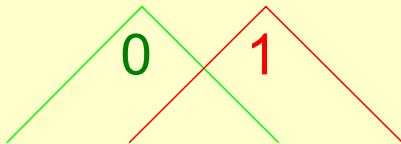
$$C = \frac{1}{2} \log_2 \left(1 + \frac{P}{N} \right) \quad \text{Sh/cycle.}$$



$$P = 3N, \quad C = 1 \quad \text{Sh/cycle.}$$

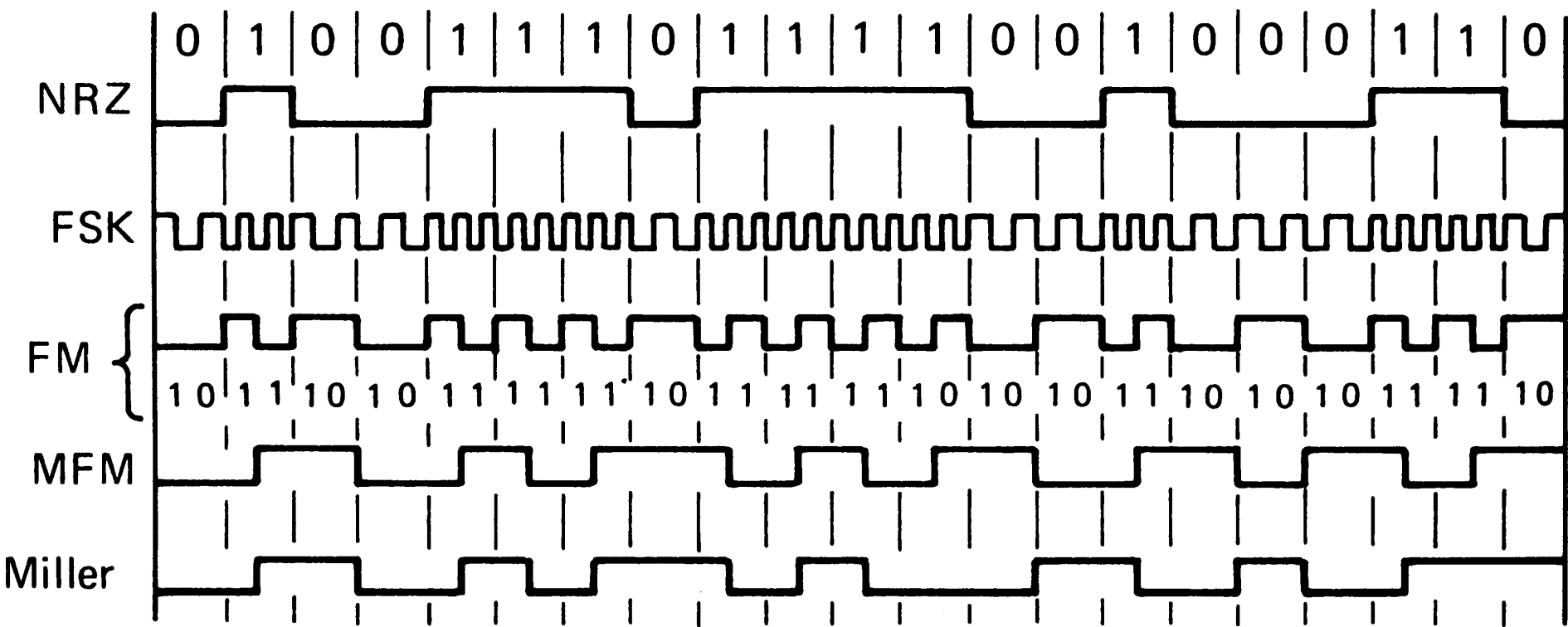


$$P = 8N, \quad C = \log_2(3) \quad \text{Sh/cycle.}$$



$$P = N, \quad C = \frac{1}{2} \quad \text{Sh/cycle.}$$

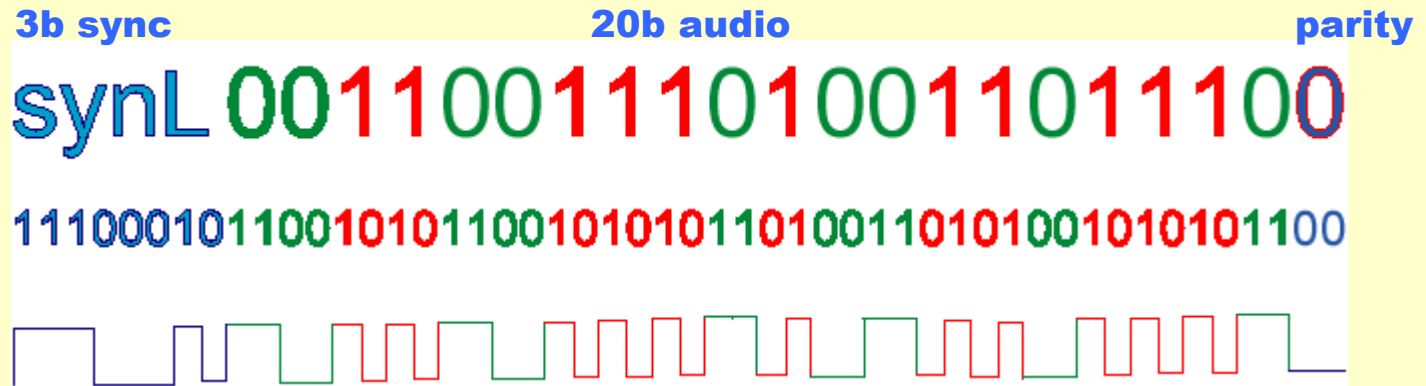
Base Band Modulation



Audio: AES/EBU

Goals:

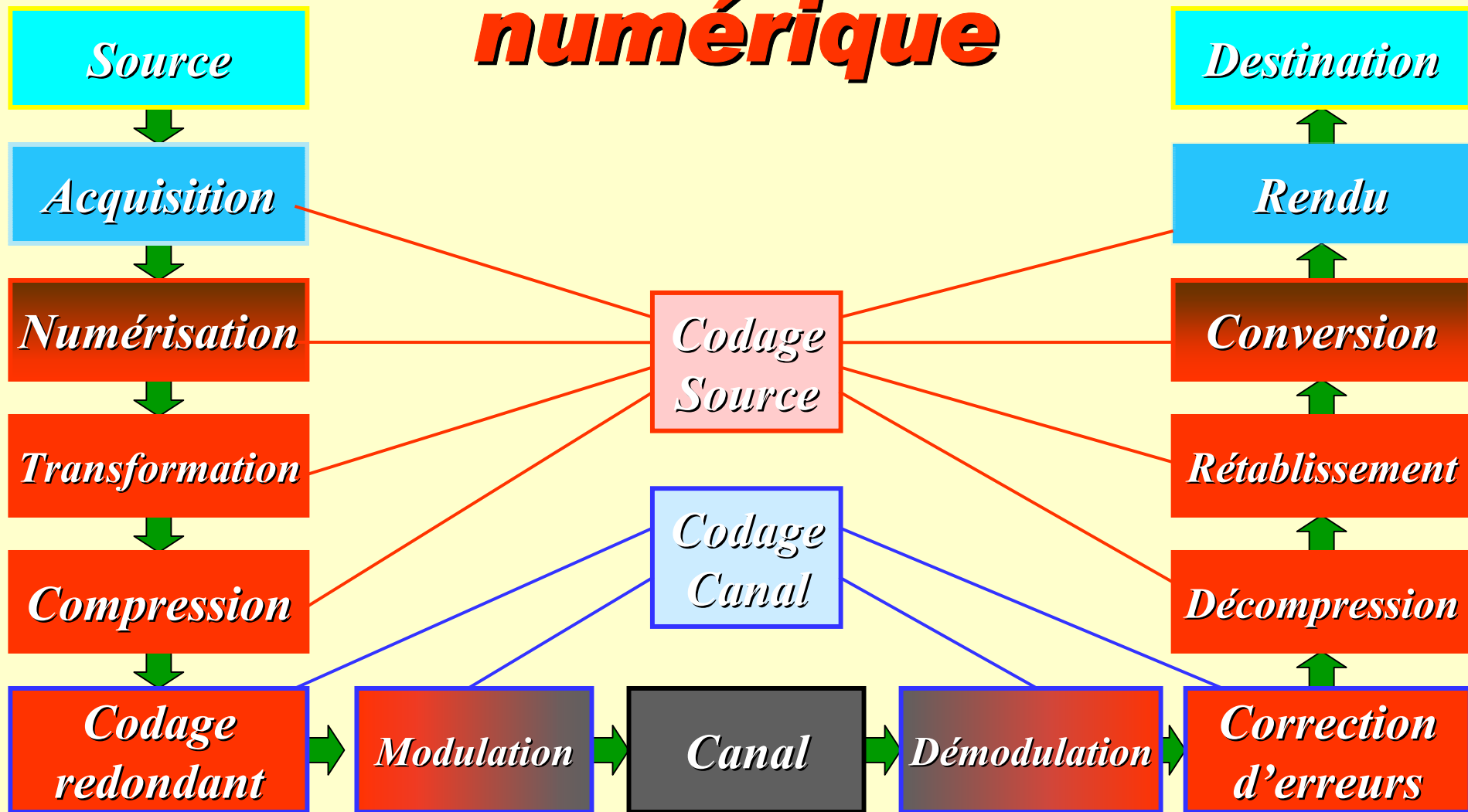
- Conventional twisted-pair cable: 0-100 meters
- Self-clocking and synchronizing for audio sampling frequencies: 32kHz - 64kHz



Code:

- Interleaved: Left/Right stereo samples, numerical signaling
- Sample = audio signal (20 Sh) + numeric (1 Sh)
- Sample is modulated over 48 cycles: sync3-audio20-parity1
- Clock frequency
 1. Emitter: $96 \times \text{audio sampling frequency} = 3 \text{ to } 6 \text{ MHz}$
 2. Receiver: $96 \times 2 \text{ (Nyquist)} \times 64 \text{ (max frequency)} > 12 \text{ MHz}$

Communication numérique



Bandwidth

Source

PCM Voice: 64 kb/s
Audio CD: 1.5 Mb/s
MPEG-1: 1.5 Mb/s
PAL/NTSC: 160 Mb/s
HDTV: 580 Mb/s

Channel

GSM: 6.5 or 13 kb/s
RTC: 64kb/s
T1: 1.5 Mb/s,
T3: 45 Mb/s
Ethernet: 10, 100, 1000 Mb/s
ATM: 155, 650 Mb/s