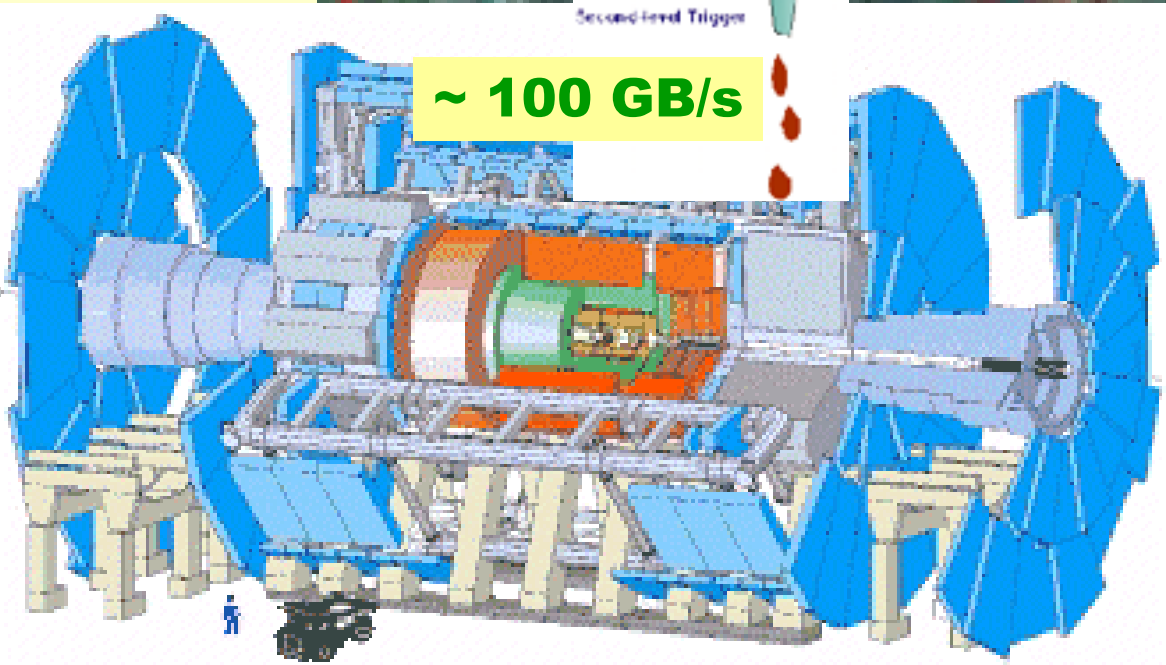
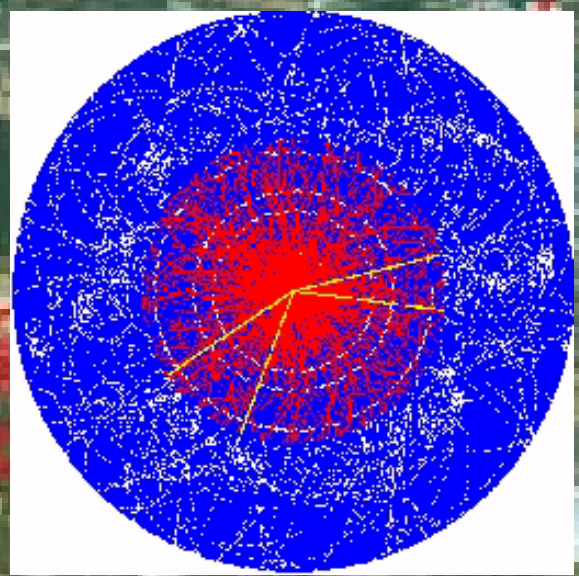
Large Hadron Collider

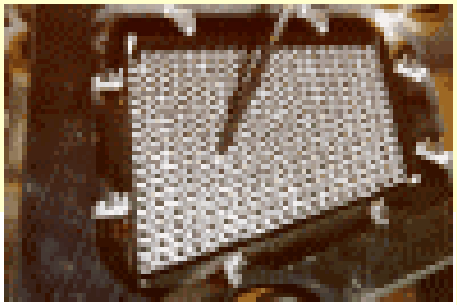
> 100 TB/s

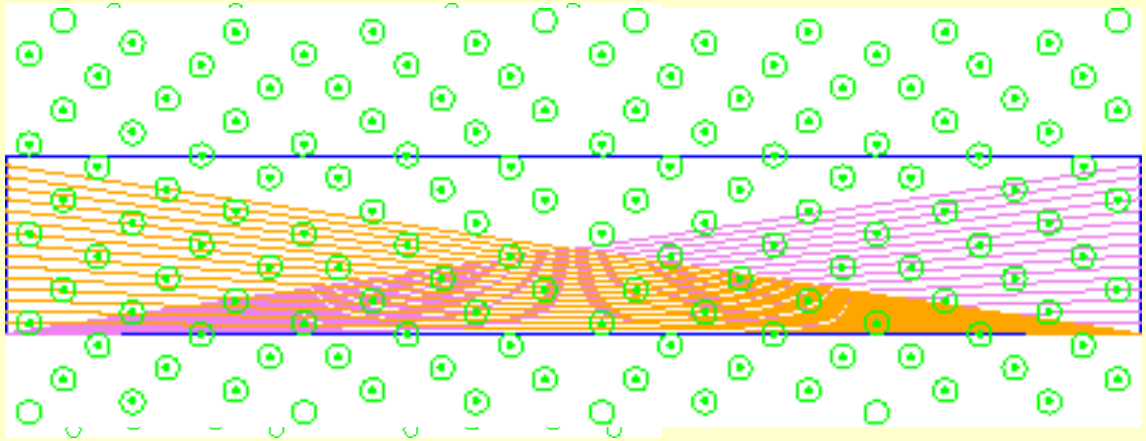
~ 100 GB/s

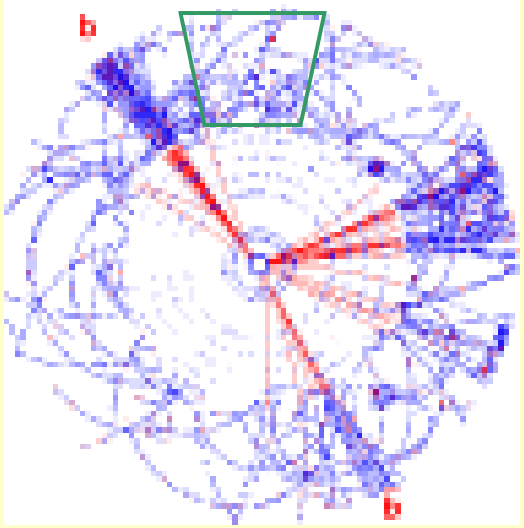


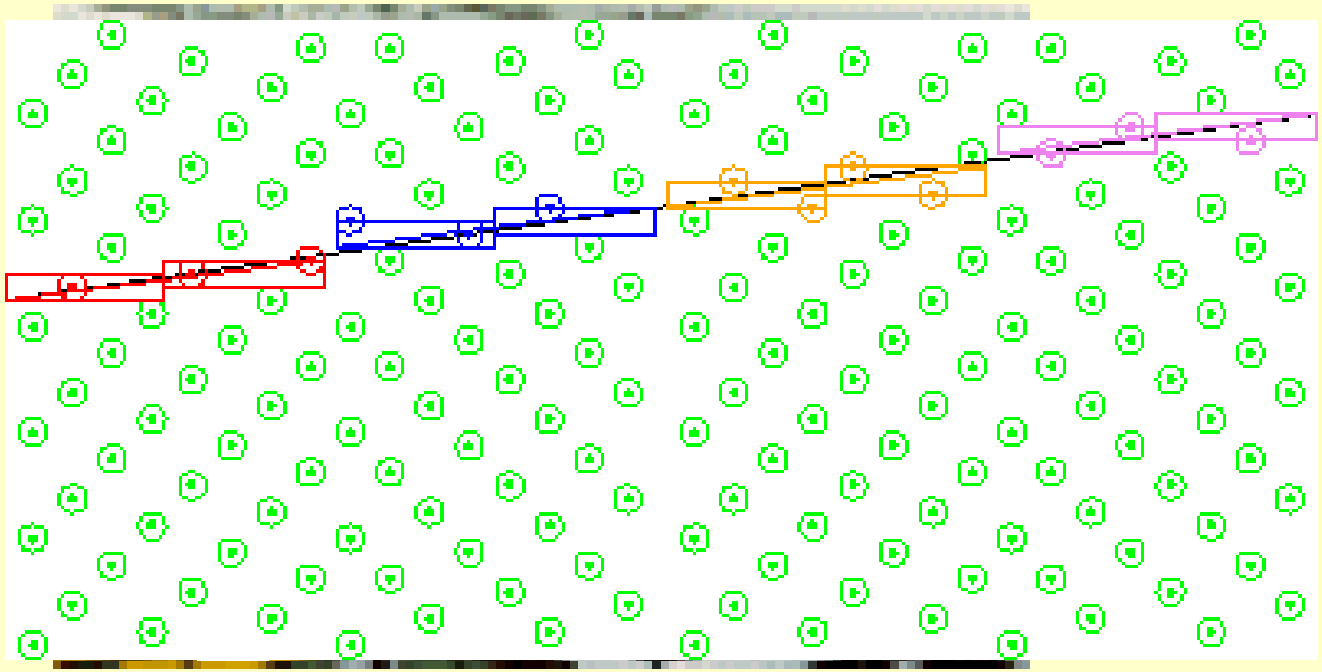


Transition Radiation Tracker

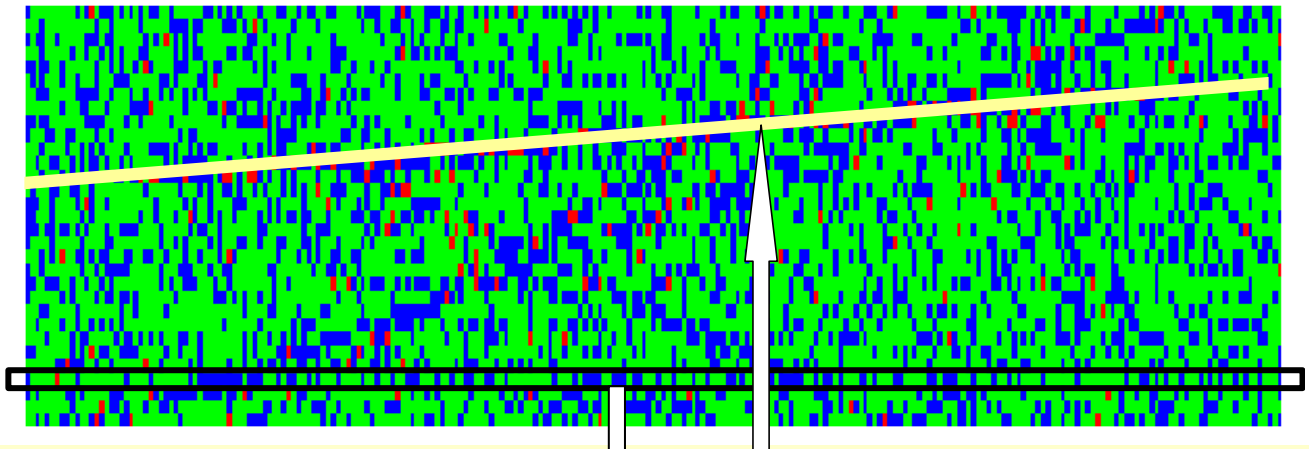






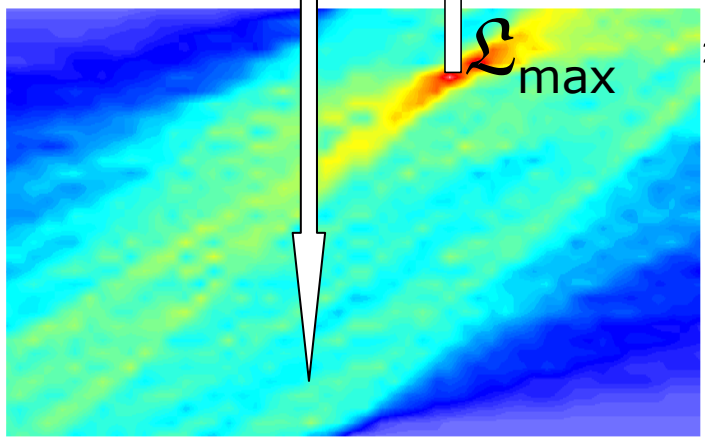


Region of interest



$\mathfrak{L}_{max}$

$\mathfrak{L}$



$\mathfrak{L}_{max} = \max_{\mathfrak{L}} \{h(\mathfrak{L})\}$

Linear Hough Transform

$$h(\mathfrak{L}) = \sum_{\mathfrak{p} \in \mathfrak{L}} \mathfrak{p}$$

Tomography

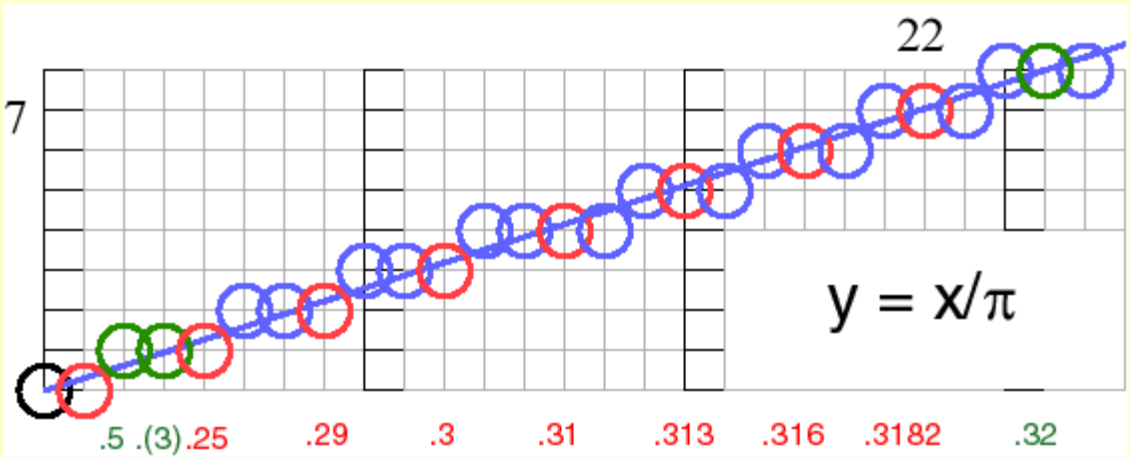
Discrete Straight Line

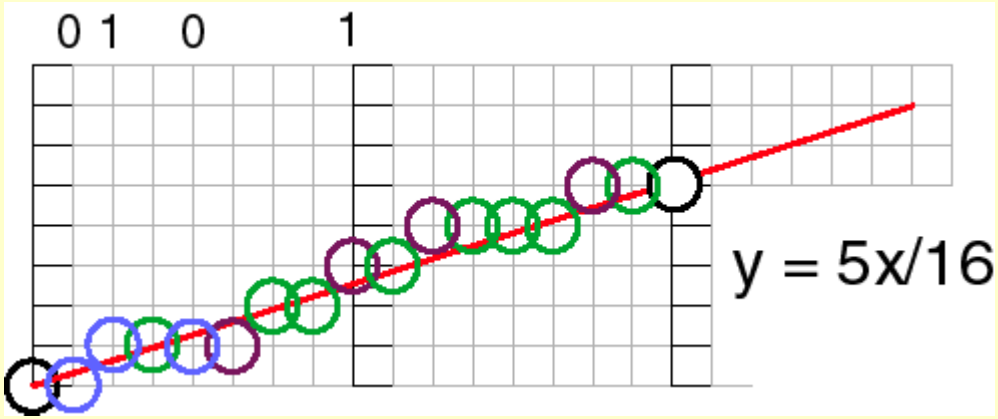
Draw a digital straight line?

Reduce to: y = s x and 0 ≤ s < 1.

Choose pixel nearest to the line.

$$\mathfrak{B}_s = \left\{ \left(N, \left\lfloor sN + \frac{1}{2} \right\rfloor\right) : N \in \mathbb{N} \right\}$$





Recursive ≠ Bresenham, is not relevant to CERN application.

Discrete Hough Transform

$$\mathfrak{L}_{i,h} = \left\{(x,y) : y = i + x\frac{h}{N}\right\} \qquad H_{i,h} = \sum_{(x,y)\in\mathfrak{L}_{i,h}} p_{x,y}$$

$H_{2,-1} = 0+1+2+0 = 3$

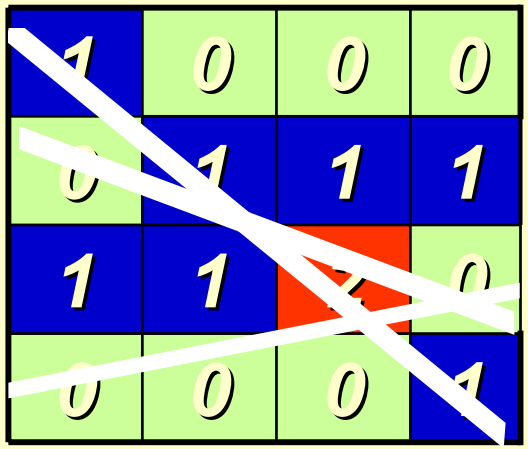
$H_{0,1} = 0+0+2+0 = 2$

-3	-2	-1	0	1	2	3

$\mathfrak{L}_{3,-3}$

$\mathfrak{L}_{2,-1}$

$\mathfrak{L}_{0,1}$

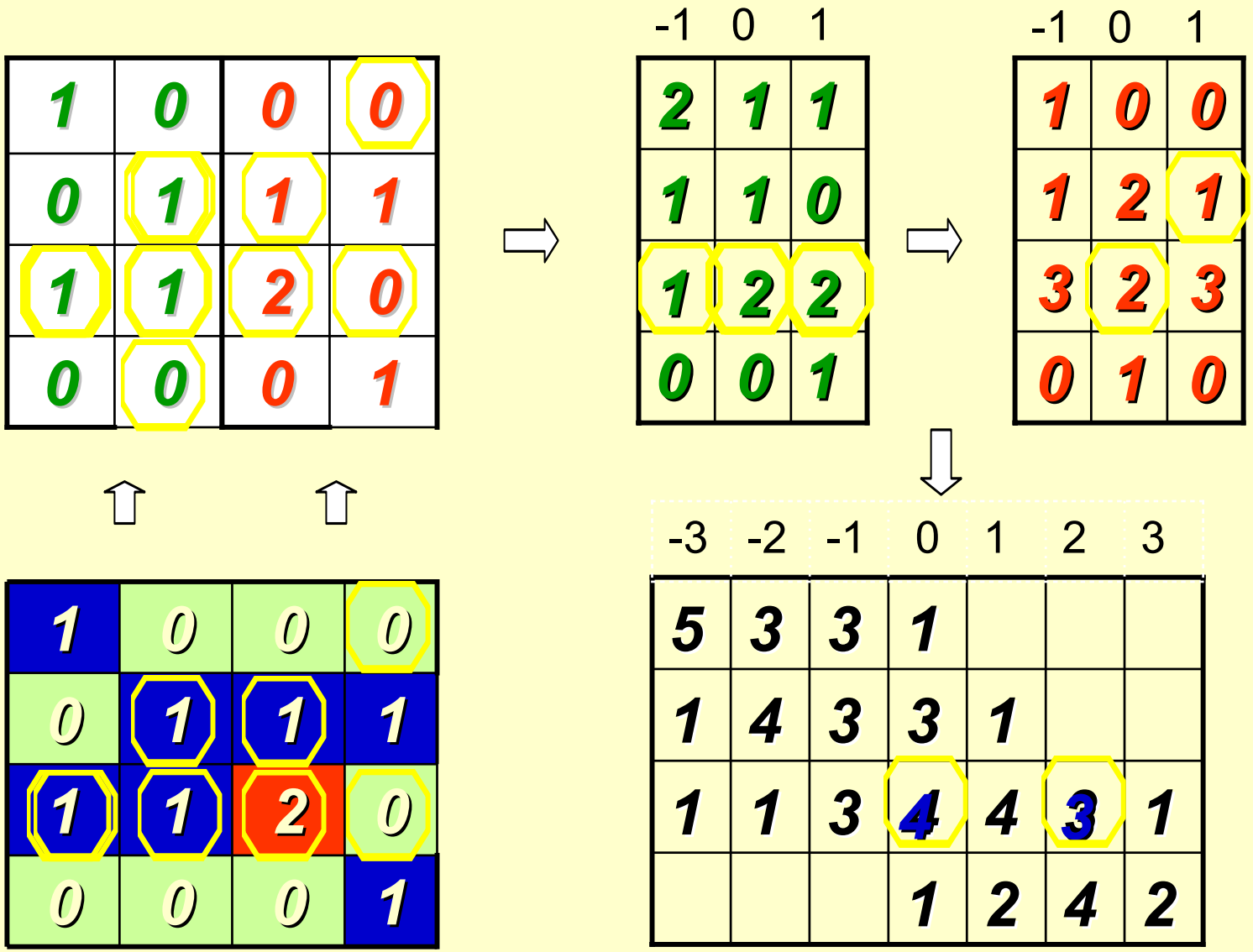


5	3	3	1			
1	4	3	3	1		
1	1	3	4	4	3	1
			1	2	4	2

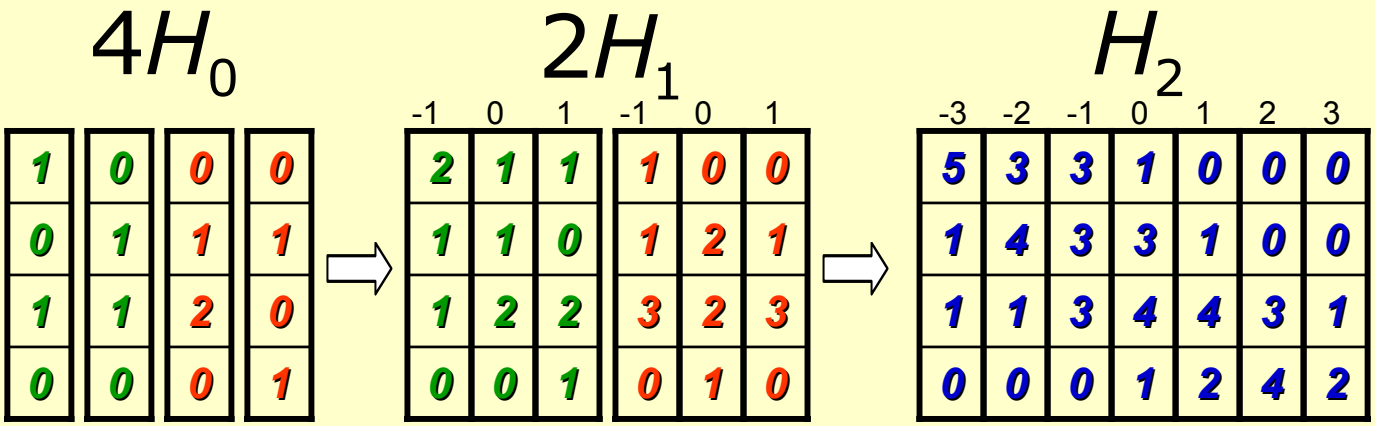
Naïve Algorithm

- **(2N-1)N lines**
- **(2N-1)N² additions, on p+l₂(N) bits**

Divide and Conquer!



Fast Hough Transform



$$
\begin{aligned}
H_0(0,i,j) &= pixel(i,j) \\
H_{p+1}(2h,i,j) &= H_p(h,i,j) + H_p(h,i+2^p,j+h) \\
H_{p+1}(2h+1,i,j) &= H_p(h,i,j) + H_p(h,i+2^p,j+h+1) \quad h>0 \\
H_{p+1}(2h-1,i,j) &= H_p(h,i,j) + H_p(h,i+2^p,j+h-1) \quad h<0
\end{aligned}
$$

For $N=2^n$, the complexity of the FHT is:

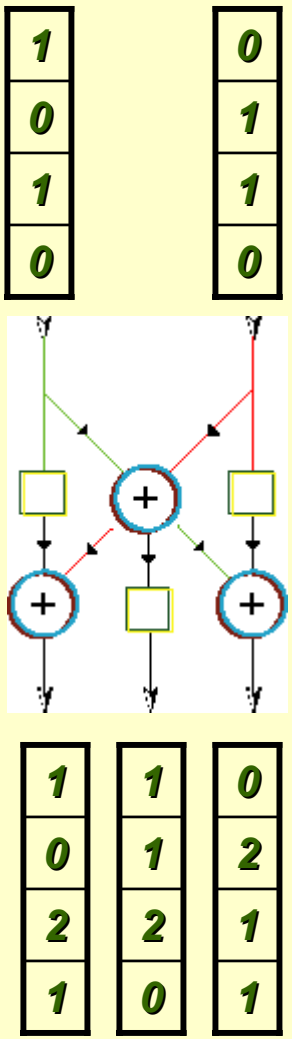
$(2n-1)N^2-N$

additions on n+p bits.

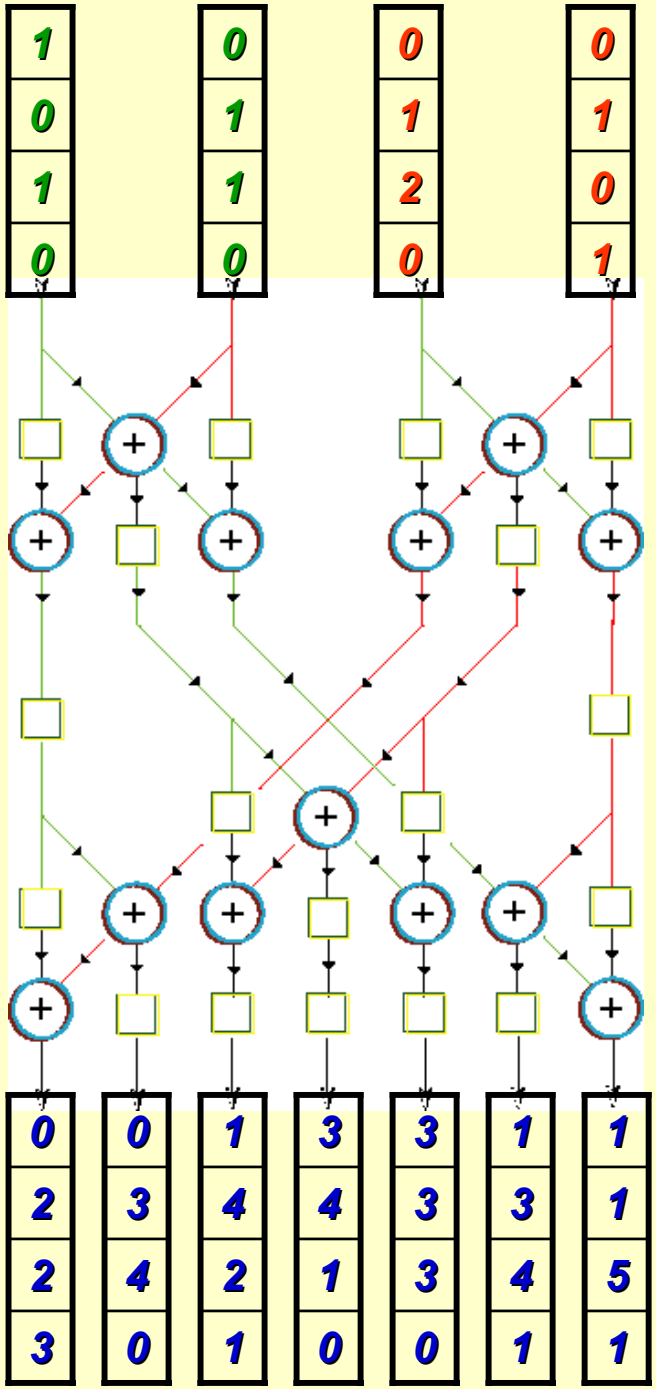
Additions, for N=32:

- **Naïve O(N^3):** **64512**
- **FHT O(N^2 log(N)):** **9248**
- **Software speed-up:** **x7**

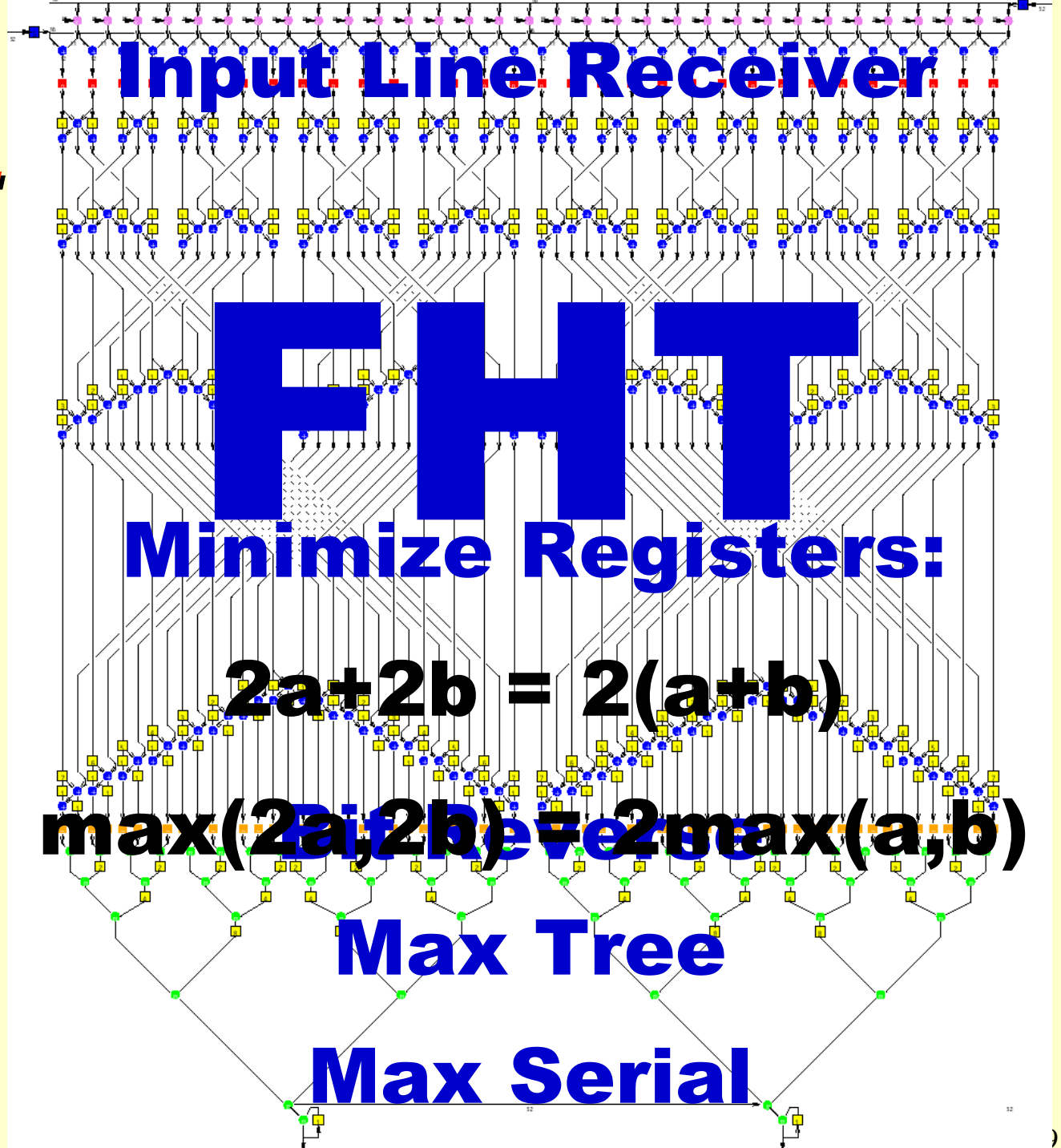
FHT Circuit



- **Parallel**
- **Serial**



TRT Circuit


Input Line Receiver
FHT
Minimize Registers:
2a+2b = 2(a+b)
max(2a,2b) = 2max(a,b)
Bit Reverse
Max Tree
Max Serial