Digital Synchronous System

Binary values: $\forall t \in \mathbb{R} \geq 0 : v_t \in \mathbb{B} = \{0,1\}$

Synchronous: $\forall t \in \mathbb{R} : v_t = v_n, \; n = \lfloor t \rfloor$

Digital Signal:

$$v_t = \int_{t-1}^{t} v'_x \, dx$$

$$v'_t = \sum_{n \in \mathbb{N}} v_n \partial(t-n)$$

Binary sequence: $(v_{\mathbb{N}}) = v_0 v_1 v_2 \cdots$

Digital Function $\mathbb{D} \to \mathbb{D}$

Digital Number $d = [\![d_{0\ldots} \quad \{d\} \quad d(z) \quad d(2)]\!]$

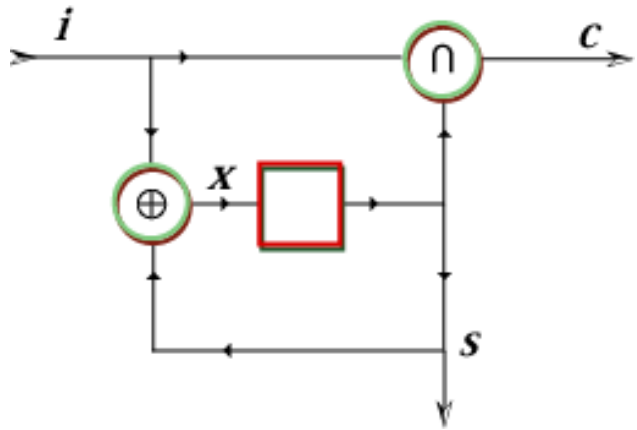
1. ***Continuous***: each output depends upon finitely many input.
2. ***Computable*** : presented by a finite program.
3. ***Causal*** : present output depends upon past input.
4. ***Sequential*** : causal + finite ⇔ circuit function.

Counter Modulo 2

$$s = z\,x$$

$$x = i \oplus s$$

$$c = i \cap s$$



$$s_0 = 0 \qquad s_N = x_{N-1}$$

$$x_0 = i_0 \qquad x_N = i_N \oplus s_N$$

$$c_0 = 0 \qquad c_N = i_N s_N$$

Causal

$$s_N = g_N(i_0 \cdots i_{N-1})$$

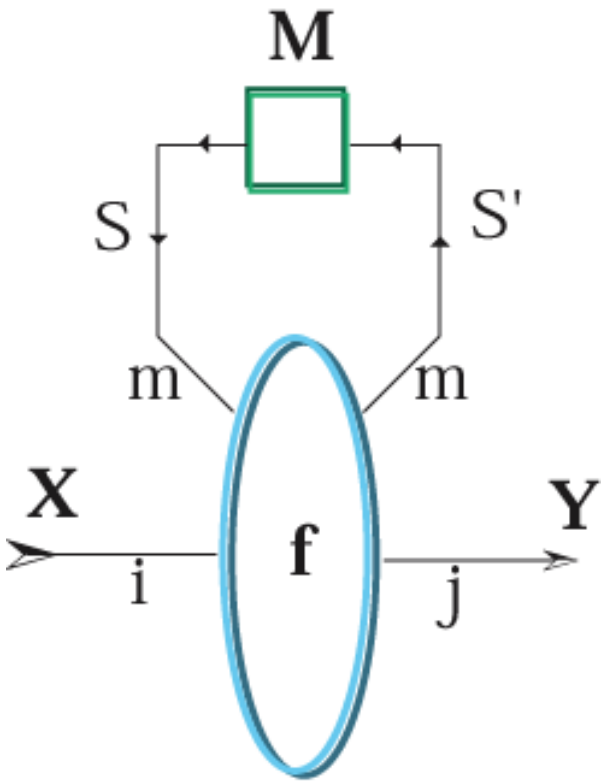
$$c_N = h_{N+1}(i_0 \cdots i_N)$$

Finite

$$s_{N+1} = f_s(i_N, s_N)$$

$$c_N = f_y(i_N, s_N)$$

Sequential Function



Causal

$$y_N = f_N(x_0 \cdots x_N)$$

$$\forall n \in \mathbb{N}:$$

$$f_n \in \mathbb{B}^{i(n+1)} \rightarrow \mathbb{B}^j$$

Finite Memory

$$s_{N+1} = f^s(x_N, s_N)$$

$$y_N = f_y(x_N, s_N)$$

$$f^s \in \mathbb{B}^{i+m} \rightarrow \mathbb{B}^m$$

$$f_y \in \mathbb{B}^{i+m} \rightarrow \mathbb{B}^j$$

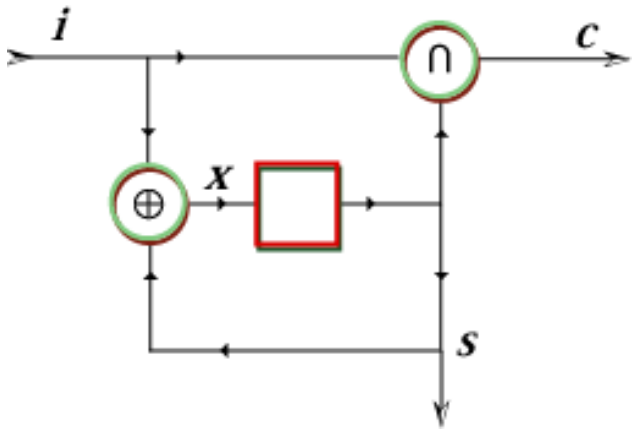
State Encoding:

$$s_N = \sum_{1 \le k \le m} m_N[k] 2^{k-1}$$

Reachable States: rs

$$|regs| \ge \log_2 |rs| \qquad |states| \le 2^{|regs|}$$

CM 2



$$s = z\, x$$

$$x = i \oplus s$$

$$c = i \cap s$$

$$s_N = 0$$

$$s_{N+1} = i_N \oplus s_N$$

$$c_N = i_N s_N$$

$$s_N = i_0 \oplus i_1 \oplus \cdots \oplus i_{N-1}$$

$$s_N = \sum_{k<N} i_k \pmod 2$$

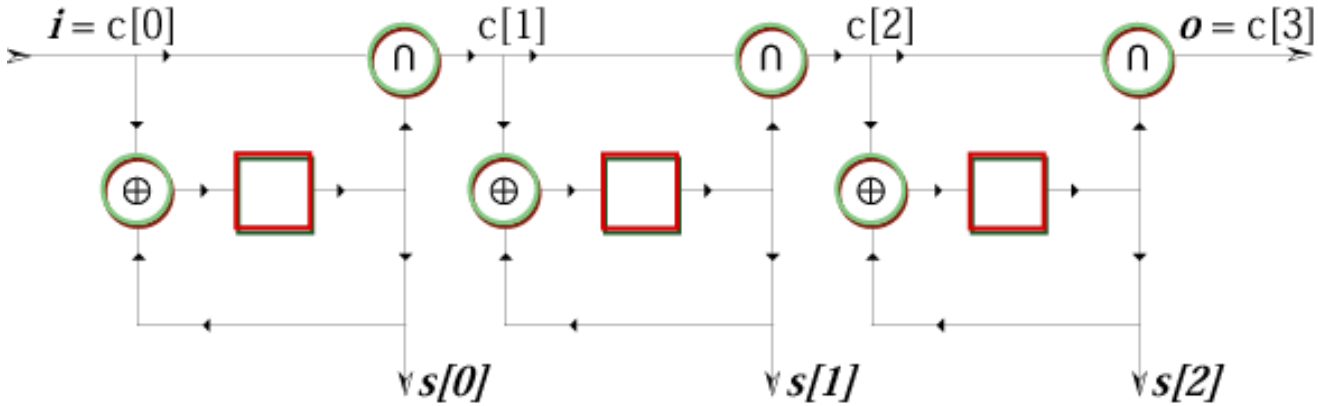
$$s_N = 0$$

$$s_{N+1} = i_N + s_N - 2 i_N s_N$$

$$c_N = i_N s_N$$

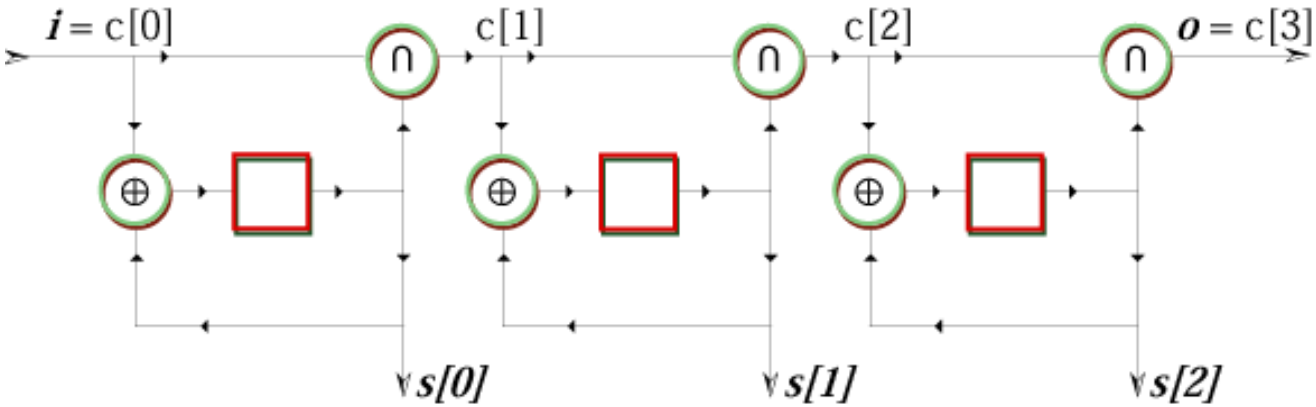
$$\sum_{k<N} i_k = s_N + 2 \sum_{k<N} c_k$$

Counter Modulo 8



cycle	i = c[0]	s[0]	s[1]	s[2]	c[1]	c[2]	x[0]	x[1]	x[2]	o
0	1	0	0	0	0	0	1	0	0	0
1	1	1	0	0	1	0	0	1	0	0
2	1	0	1	0	0	0	1	1	0	0
3	1	1	1	0	1	1	0	0	1	0
4	1	0	0	1	0	0	1	0	1	0
5	1	1	0	1	1	0	0	1	1	0
6	1	0	1	1	0	0	1	1	1	0
7	1	1	1	1	1	1	0	0	0	1
8	1	0	0	0	0	0	1	0	0	0

Counter Modulo 8



$$\sum_{k<N} c_k[0] = s_N[0] + 2\sum_{k<N} c_k[1]$$

$$\sum_{k<N} c_k[1] = s_N[1] + 2\sum_{k<N} c_k[2]$$

$$\sum_{k<N} c_k[2] = s_N[2] + 2\sum_{k<N} c_k[3]$$

$$\sum_{k<N} c_k[0] = S_N + 8\sum_{k<N} c_k[3]$$

$$S_N = s_N[0] + 2s_N[1] + 4s_N[2]$$