Learning on images with segmentation graph kernels

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Outline

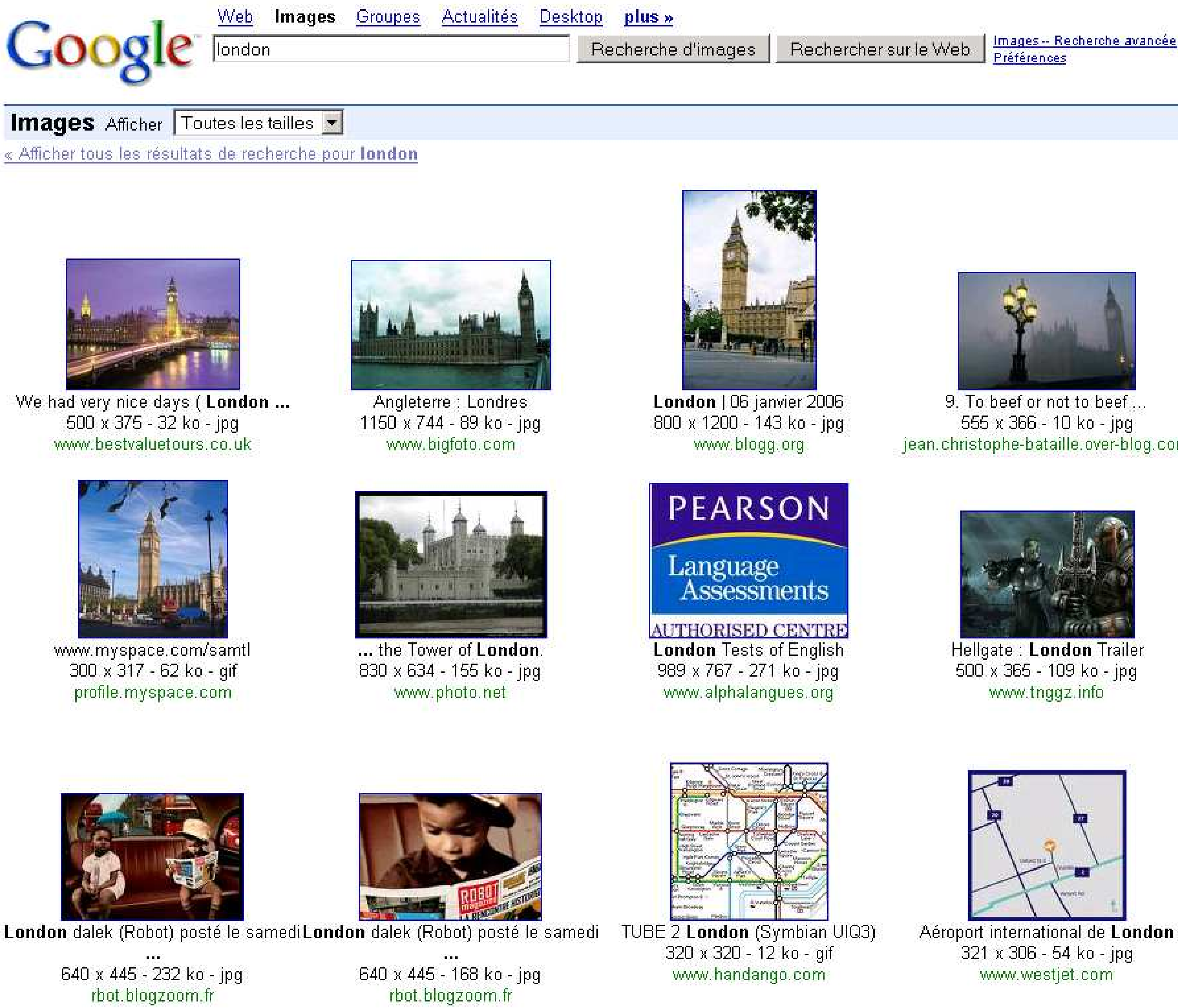
- Learning on images
- Kernel methods
- Segmentation graph kernels
- Experiments
- Conclusion

Learning tasks on images

- Multiplication of digital media
- Many different tasks to be solved
  - Associated with different machine learning problems

Image retrieval

Classification, ranking, outlier detection



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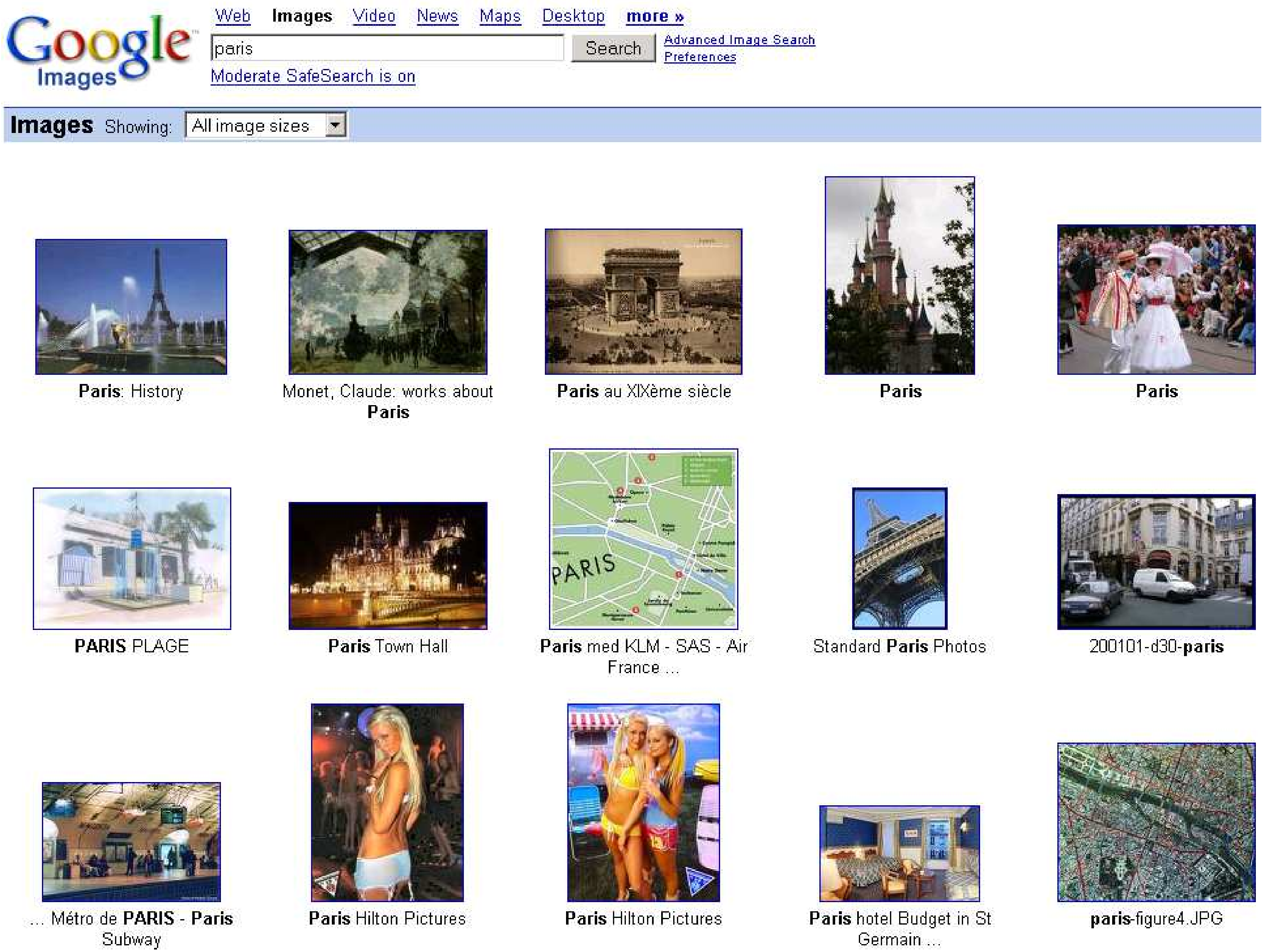
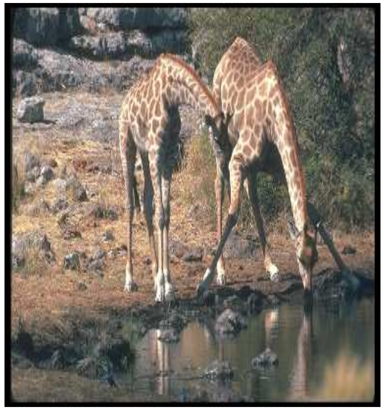


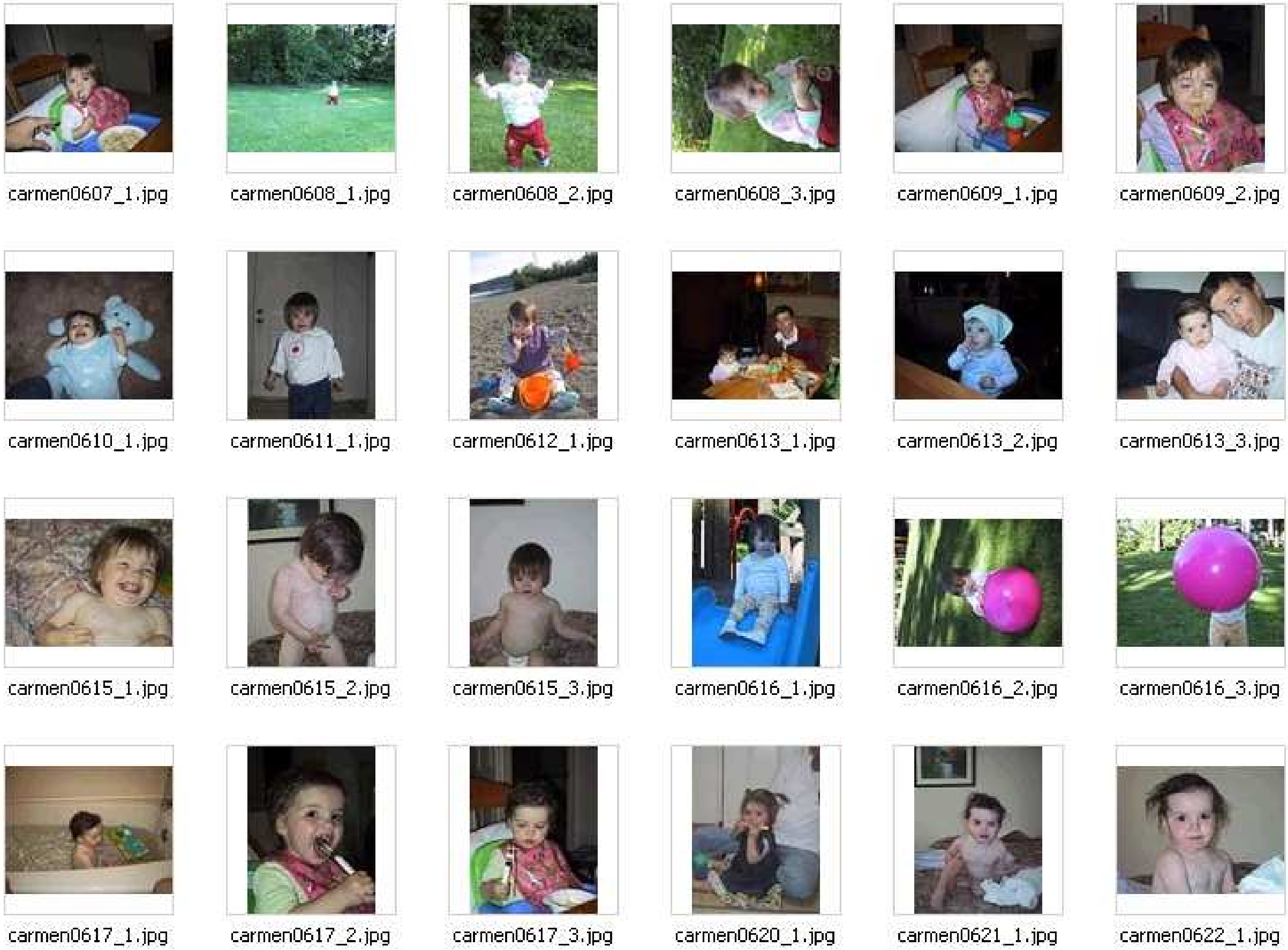
Image annotation
Classification, clustering






Personal photos

Classification, clustering, visualisation



Learning tasks on images

- Multiplication of digital media
- Many different tasks to be solved
  - Associated with different machine learning problems
- Application: retrieval/indexing of images
- Common issues:
  - Complex tasks
  - Heterogeneous data – links with other medias (text and sound)
  - Massive data

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$\Rightarrow$ **Kernel methods**

Kernel methods for machine learning

- **Motivation**:
  - Develop modular and versatile methods to learn from data
  - Minimal assumptions regarding the type of data (vectors, strings, graphs)
  - Theoretical guarantees

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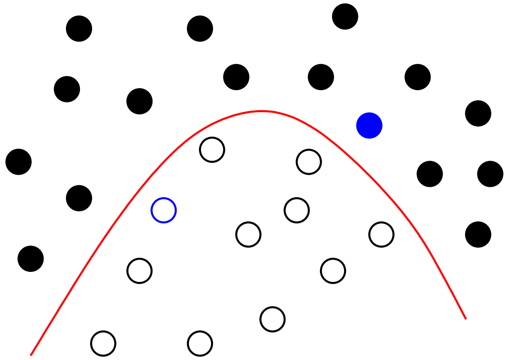
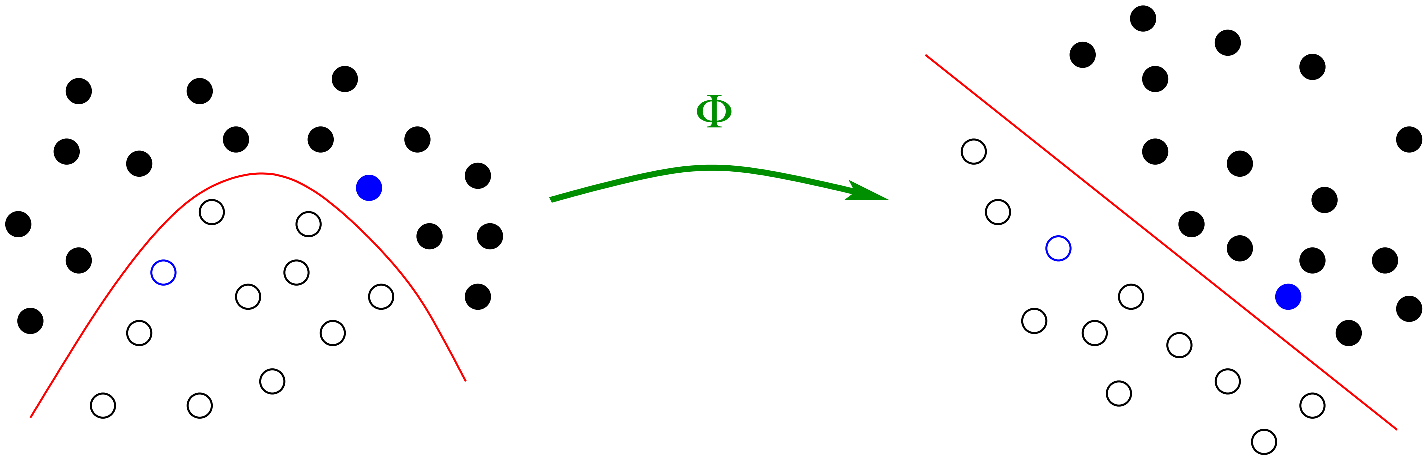
- **Motivation**:
  - Develop modular and versatile methods to learn from data
  - Minimal assumptions regarding the type of data (vectors, strings, graphs)
  - Theoretical guarantees
- **Main idea**:
  - use only pairwise comparison between objects through dot-products
  - use algorithms that depend only on those dot-products ("linear algorithms")

Kernel trick : linear $\Rightarrow$ non linear



Kernel trick : linear $\Rightarrow$ non linear



- Non linear map $\Phi : x \in \mathcal{X} \mapsto \Phi(x) \in \mathcal{F}$
- Linear estimation in "feature space" $\mathcal{F}$
- Assumption: results only depend on dot products $\langle \Phi(x_i), \Phi(x_j) \rangle$ for pairs of data points
- Kernel: $k(x, x') = \langle \Phi(x), \Phi(x') \rangle$
- Implicit embedding!

Kernel methods for machine learning

- **Definition**: given a set of objects $\mathcal{X}$, a positive definite kernel is a symmetric function $k(x, x')$ such that for all finite sequences of points $x_i \in \mathcal{X}$ and $\alpha_i \in \mathbb{R}$,

$$\sum_{i,j} \alpha_i \alpha_j k(x_i, x_j) \geqslant 0$$

  (i.e., the matrix $(k(x_i, x_j))$ is symmetric positive semi-definite)

- **Aronszajn theorem** (1950): k is a positive definite kernel if and only if there exists a Hilbert space $\mathcal{F}$ and a mapping $\Phi : \mathcal{X} \mapsto \mathcal{F}$ such that

$$\forall (x, x') \in \mathcal{X}^2, \; k(x, x') = \langle \Phi(x), \Phi(x') \rangle_{\mathcal{H}}$$

- $\mathcal{X} =$ "input space", $\mathcal{F} =$ "feature space", $\Phi =$ "feature map"
- Functional view: reproducing kernel Hilbert spaces

Kernel trick and modularity

- Kernel trick: any algorithm for finite-dimensional vectors that only uses pairwise dot-products can be applied in the feature space.
  - Replacing dot-products by kernel functions
  - Implicit use of (very) large feature spaces
  - Linear to non-linear learning methods

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- Modularity of kernel methods
  1. Work on new algorithms and theoretical analysis
  2. Work on new kernels for specific data types

Kernel algorithms

- Classification and regression
  - Support vector machine, linear regression, etc...
- Clustering
- Outlier detection
- Ranking
- Integration of heterogeneous data

$\Rightarrow$ Developed independently of specific kernel instances

Kernels : kernels on vectors $x \in \mathbb{R}^d$

- Linear kernel $k(x,y) = x^\top y$
  - Linear functions
- Polynomial kernel $k(x,y) = (r + s x^\top y)^d$
  - Polynomial functions
- Gaussian-RBF kernels $k(x,y) = \exp(-\alpha \|x-y\|^2)$
  - Smooth functions
- Structured objects? Choice of parameters?

Kernels for images

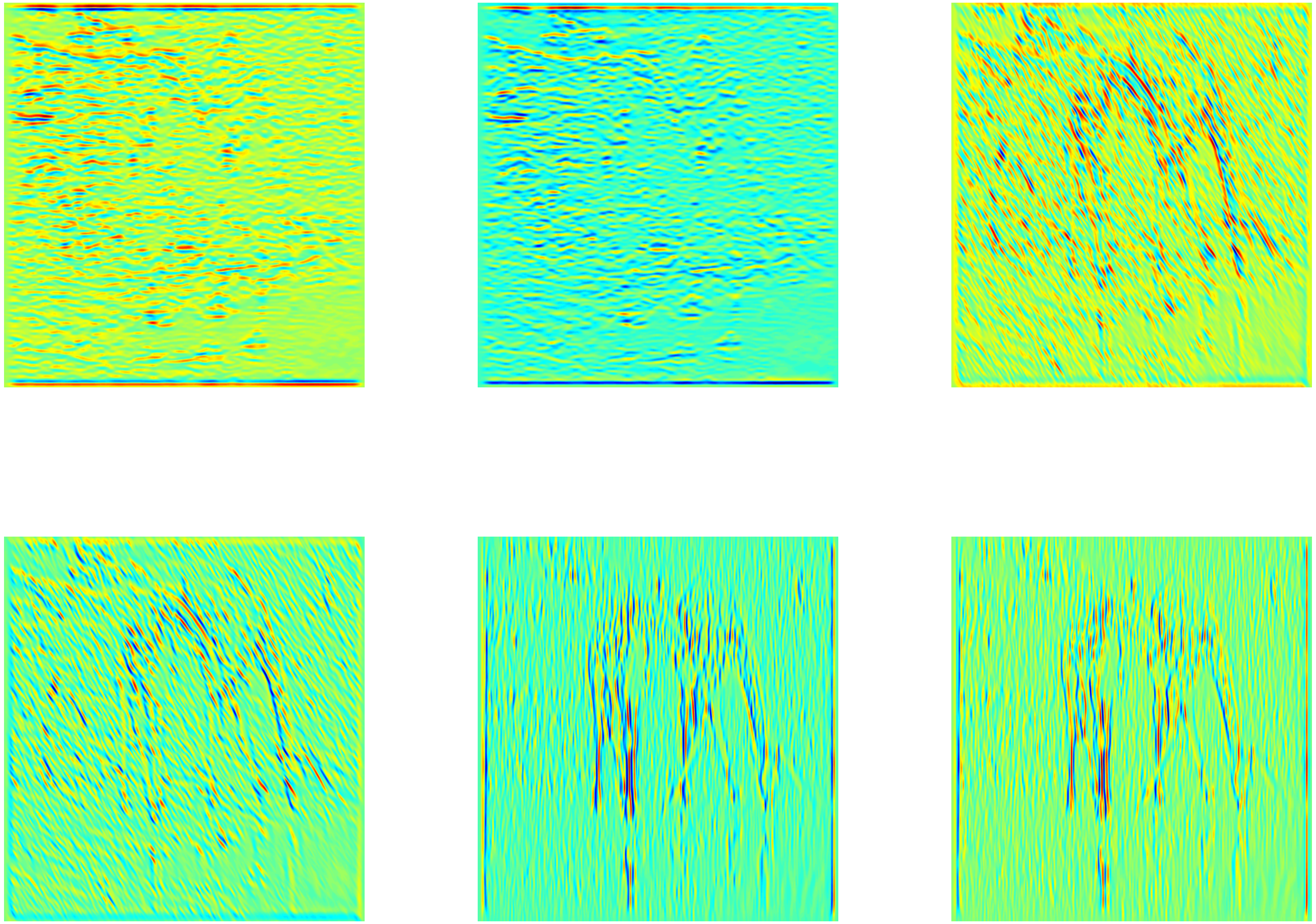
- Most applications of kernel methods to images
  - Compute a set of features (e.g., wavelets)
  - Run an SVM with many training examples
- Why not design specific kernels?
  - Using natural structure of images beyond flat wavelet representations
  - Using prior information to lower the number of training samples

kernel methods for images

- "Natural" representations
  - Vector of pixels + kernels between vectors (most of learning theory!)
  - Bags of pixels: leads to kernels between histograms
    (Chapelle & Haffner, 1999, Cuturi et al, 2006)
  - Large set of hand-crafted features (e.g., Osuna and Freund, 1998)

Input picture

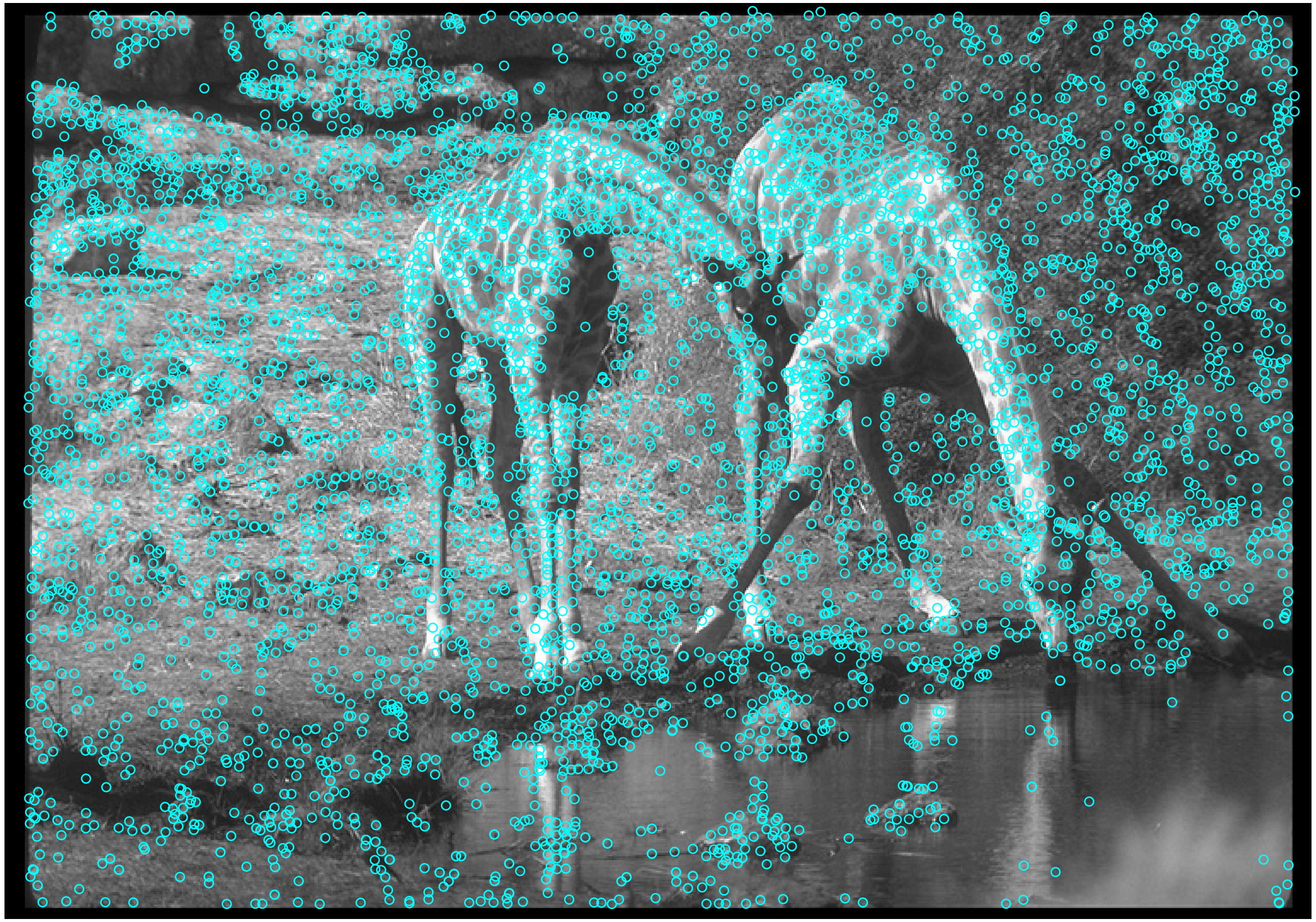
Wavelets



kernel methods for images

- "Natural" representations
  - Vector of pixels
  - Bags of pixels
  - Large set of hand-crafted features
- Loss of natural global geometry
  - Often requires a lot of training examples
- Natural representations
  - Salient points (SIFT features, Lowe, 2004)
  - Segmentation

SIFT features



Segmentation

- Goal: extract objects of interest
- Many methods available, ...
  - ... but, rarely find the object of interest entirely
- Segmentation graphs
  - Allows to work on “more reliable” over-segmentation
  - Going to a large square grid (millions of pixels) to a small graph (dozens or hundreds of regions)

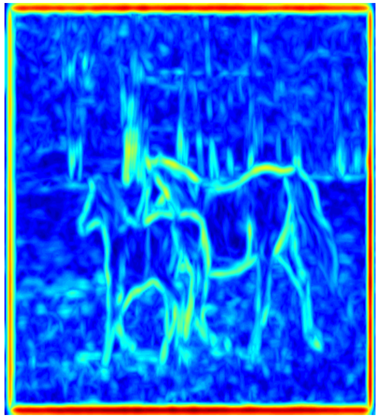
Image as a segmentation graph

- Segmentation method
  - LAB Gradient with oriented edge filters (Malik et al, 2001)
  - Watershed transform with post-processing (Meyer, 2001)
  - Very fast!

Watershed

image

gradient



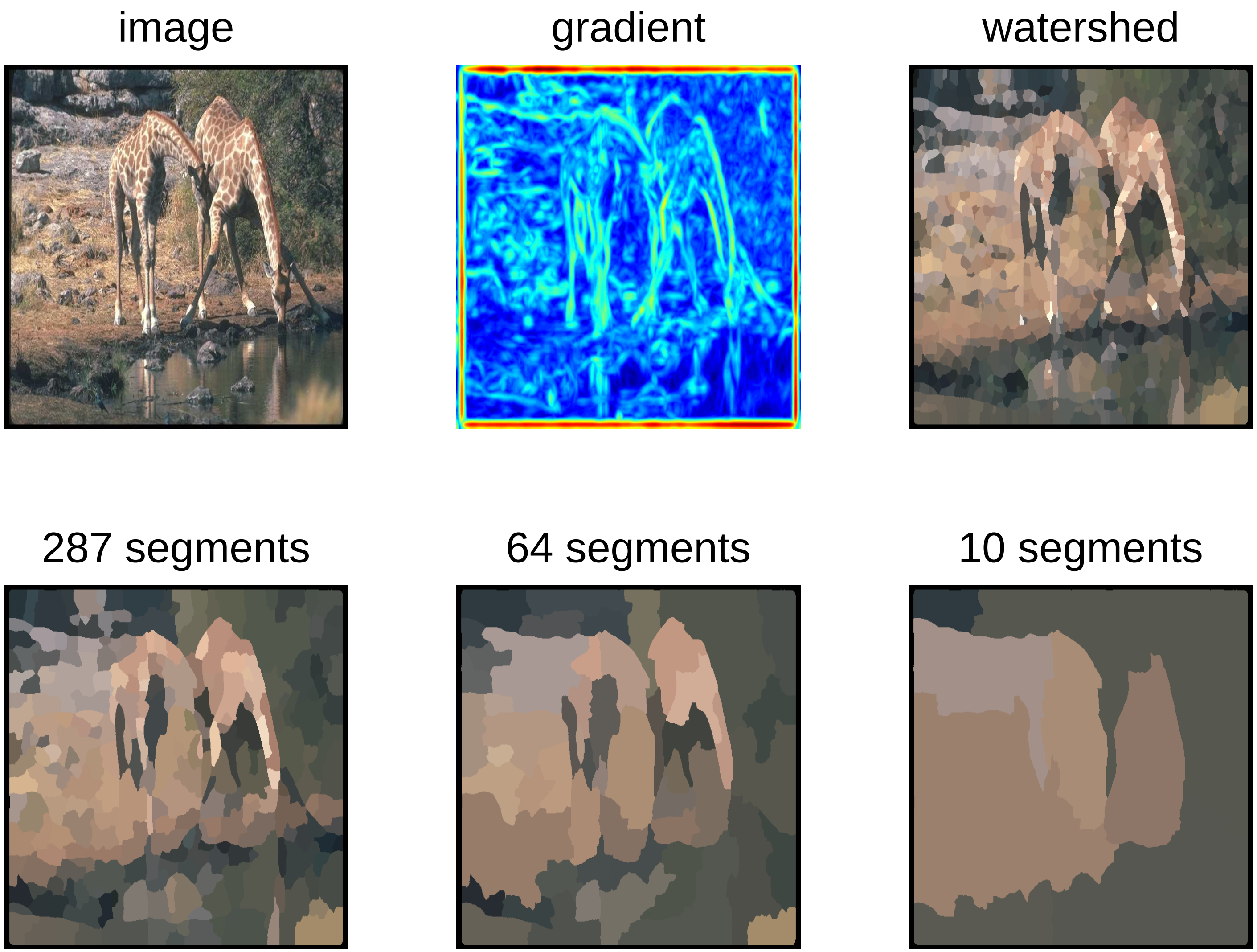
watershed

287 segments

64 segments

10 segments

Watershed



image
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Image as a segmentation graph

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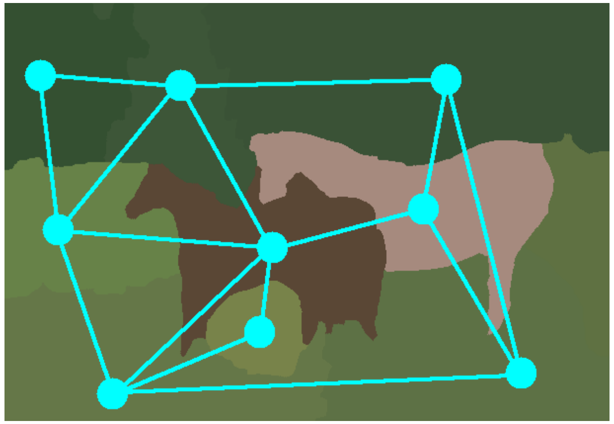
- Labelled undirected Graph
  - Vertices: connected segmented regions
  - Edges: between spatially neighboring regions
  - Labels: region pixels

$\Rightarrow$



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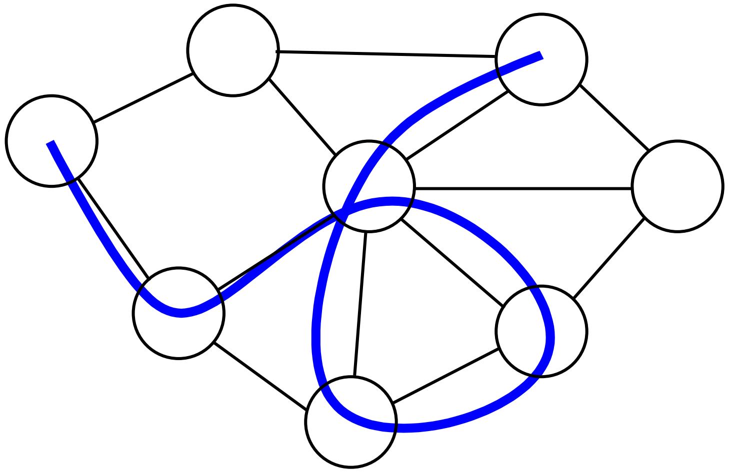
- Difficulties
  - Extremely high-dimensional labels
  - Planar undirected graph
  - Inexact matching

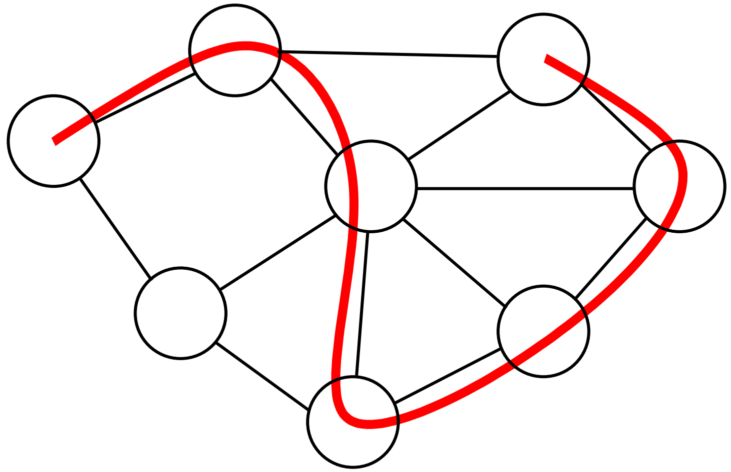
Kernels between structured objects
Strings, graphs, etc...

- Numerous applications (text, bio-informatics)
- From probabilistic models on objects (e.g., Saunders et al, 2003)
- Enumeration of subparts (Haussler, 1998, Watkins, 1998)
  - Efficient for strings
  - Possibility of gaps, partial matches, very efficient algorithms (Leslie et al, 2002, Lodhi et al, 2002, etc...)
- Most approaches fails for general graphs (even for undirected trees!)
  - NP-Hardness results (Gärtner et al, 2003)
  - Need alternative set of subparts

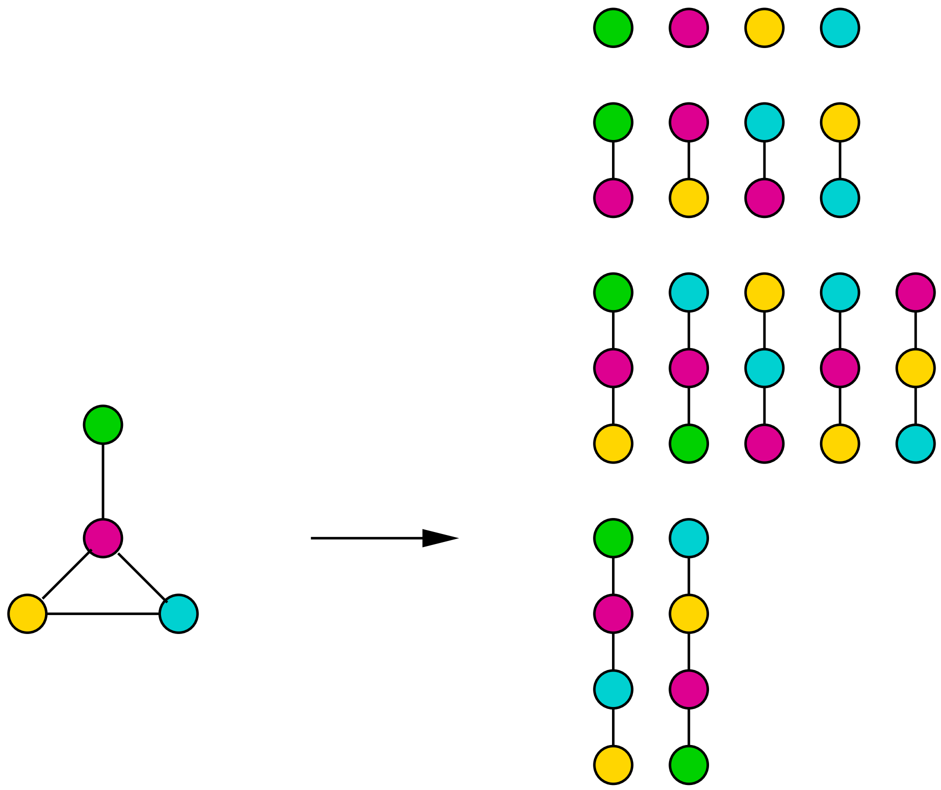
Paths and walks

- Given a graph G,
  - A path is a sequence of distinct neighboring vertices
  - A walk is a sequence of neighboring vertices
- Apparently similar notions

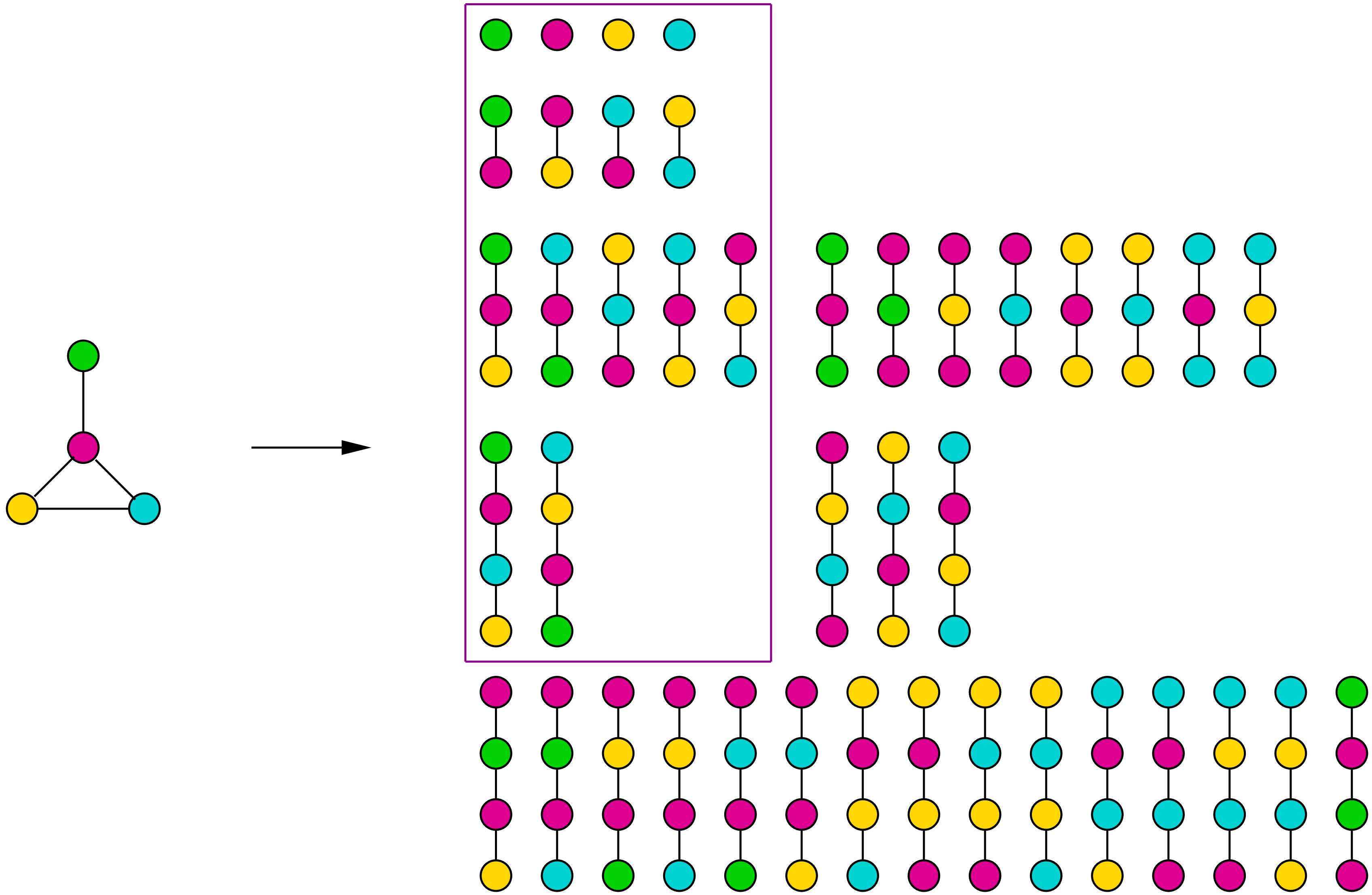




Paths



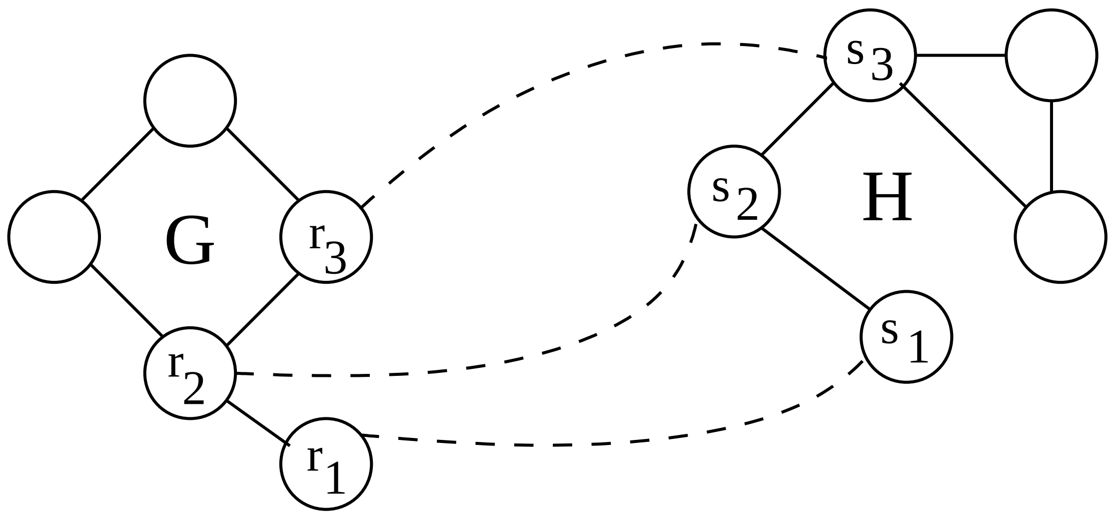
Walks



Walk kernel (Kashima, 2004, Borgwardt, 2005)

- $\mathcal{W}_{\mathbf{G}}^{p}$ (resp. $\mathcal{W}_{\mathbf{H}}^{p}$) denotes the set of walks of length p in $\mathbf{G}$ (resp. $\mathbf{H}$)
- Given *basis kernel* on labels $k(\ell, \ell')$
- p-th order walk kernel:

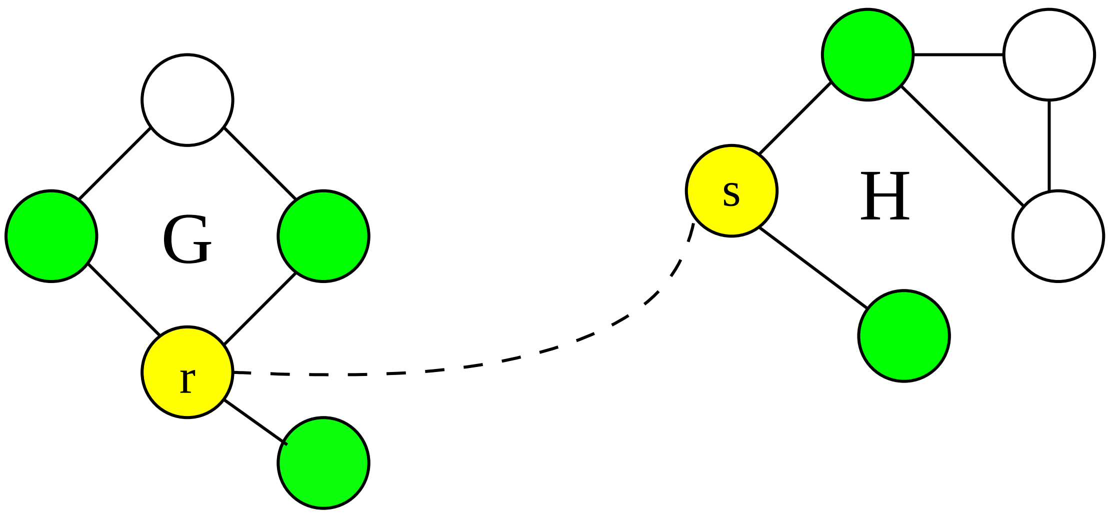
$$k_{\mathcal{W}}^{p}(\mathbf{G}, \mathbf{H}) = \sum_{\substack{(r_1, \ldots, r_p) \in \mathcal{W}_{\mathbf{G}}^{p} \\ (s_1, \ldots, s_p) \in \mathcal{W}_{\mathbf{H}}^{p}}} \prod_{i=1}^{p} k(\ell_{\mathbf{G}}(r_i), \ell_{\mathbf{H}}(s_i)).$$



Dynamic programming for the walk kernel

- Dynamic programming in $O(pd_{\mathbf{G}}d_{\mathbf{H}}n_{\mathbf{G}}n_{\mathbf{H}})$
- $k^p_{\mathcal{W}}(\mathbf{G}, \mathbf{H}, r, s) =$ sum restricted to walks starting at r and s
- recursion between $p-1$-th walk and p-th walk kernel

$$k^p_{\mathcal{W}}(\mathbf{G}, \mathbf{H}, r, s) = k(\ell_{\mathbf{G}}(r), \ell_{\mathbf{H}}(s)) \sum_{\substack{r' \in \mathcal{N}_{\mathbf{G}}(r) \\ s' \in \mathcal{N}_{\mathbf{H}}(s)}} k^{p-1}_{\mathcal{W}}(\mathbf{G}, \mathbf{H}, r', s').$$



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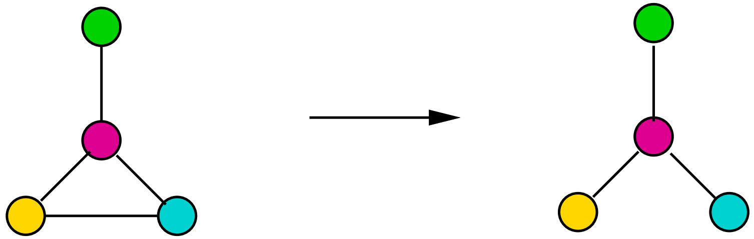
- Kernel obtained as $k_{\mathcal{T}}^{p,\alpha}(\mathbf{G},\mathbf{H}) = \sum_{r\in\mathcal{V}_{\mathbf{G}},s\in\mathcal{V}_{\mathbf{H}}} k_{\mathcal{T}}^{p,\alpha}(\mathbf{G},\mathbf{H},r,s)$
- NB: more flexible than matrix inversion approaches

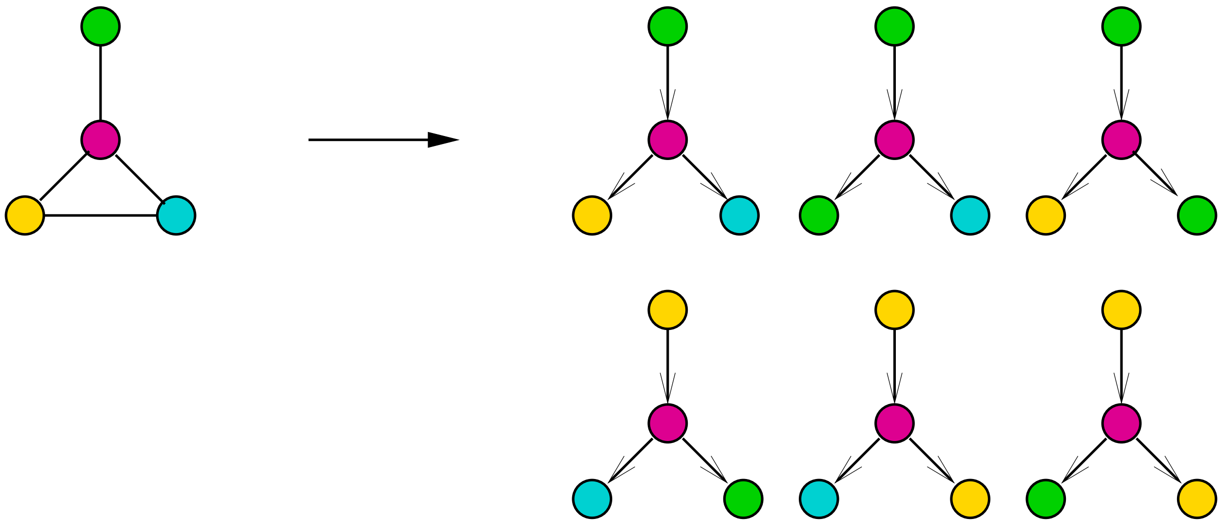
Subtrees and tree patterns

- subtree $=$ subgraph with no cycle
- tree-walks (or tree patterns)
  - natural extensions to subtrees to the "walk world"
  - α-ary tree-walk (a.k.a tree pattern) of $\mathbf{G}$: rooted directed α-ary tree whose vertices are vertices of $\mathbf{G}$, such that if they are neighbors in the tree pattern, they must be neighbors in $\mathbf{G}$ as well

Subtrees



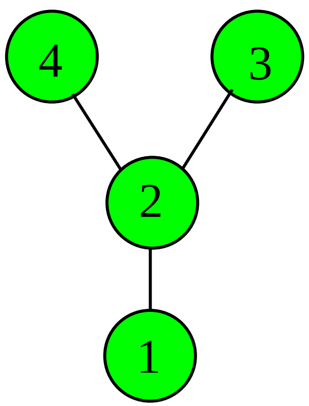
Tree patterns

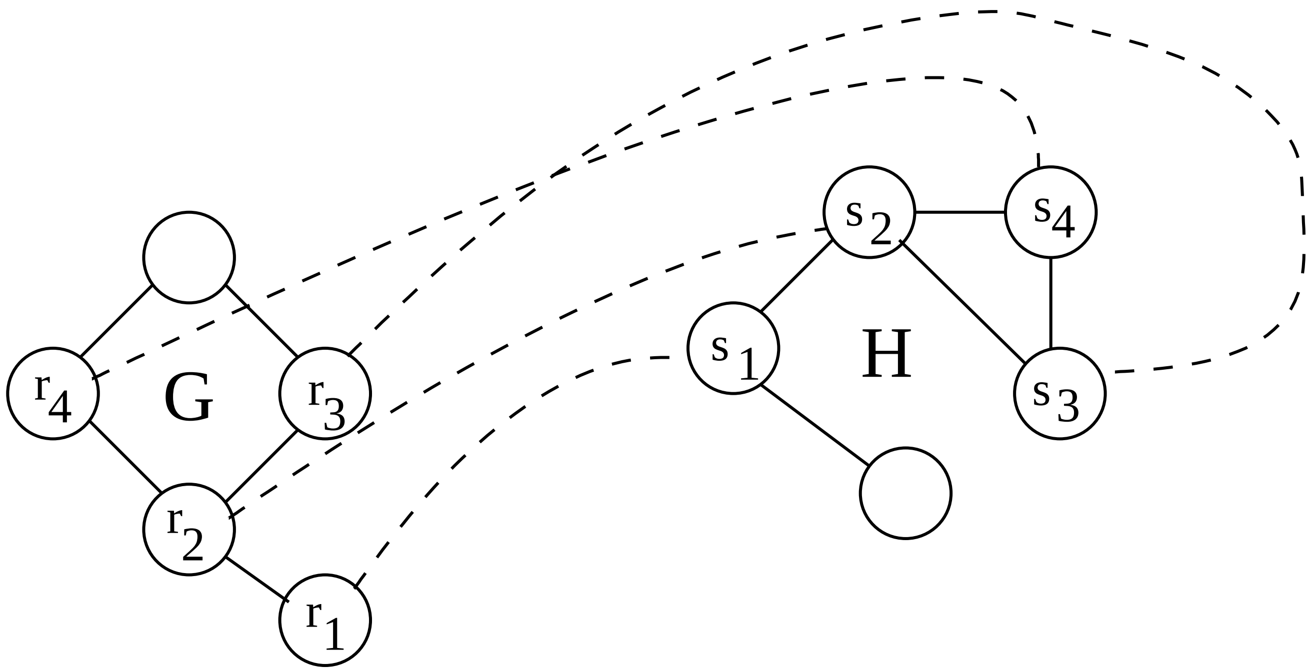


Treewalk kernel

- $\mathcal{T}_{\mathbf{G}}^{p,\alpha}$ (resp. $\mathcal{T}_{\mathbf{H}}^{p,\alpha}$) denotes the set of α-ary tree patterns of $\mathbf{G}$ (resp. $\mathbf{H}$) of depth p

- $k_{\mathcal{T}}^{p,\alpha}(\mathbf{G},\mathbf{H})$ is defined as the sum over all tree patterns in $\mathcal{T}_{p,\alpha}(\mathbf{G})$ and all tree patterns in $\mathcal{T}_{p,\alpha}(\mathbf{H})$ (that share the same tree structure)





Dynamic programming

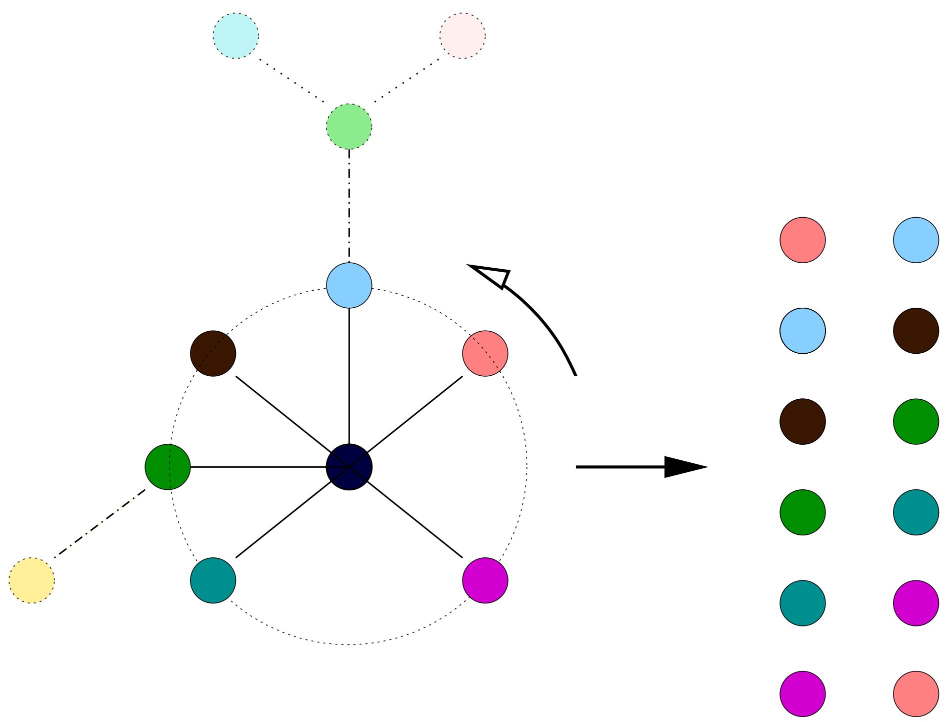
- Dynamic programming in $O(p\alpha^2 d_{\mathbf{G}} d_{\mathbf{H}} n_{\mathbf{G}} n_{\mathbf{H}})$
- NB: need planarity to avoid exponential complexity

$$k_{\mathcal{T}}^{p,\alpha}(\mathbf{G}, \mathbf{H}, r, s) \;=\; k(\ell_{\mathbf{G}}(r), \ell_{\mathbf{H}}(s)) \times \sum_{\substack{I \in \mathcal{I}_{\mathbf{G}}^{\alpha}(r) \\ J \in \mathcal{I}_{\mathbf{H}}^{\alpha}(s)}} \prod_{\substack{r' \in I \\ s' \in J}} k_{\mathcal{T}}^{p-1,\alpha}(\mathbf{G}, \mathbf{H}, r', s').$$

$$k_{\mathcal{T}}^{p,\alpha}(\mathbf{G}, \mathbf{H}) = \sum_{\substack{r \in \mathcal{V}_{\mathbf{G}} \\ s \in \mathcal{V}_{\mathbf{H}}}} k_{\mathcal{T}}^{p,\alpha}(\mathbf{G}, \mathbf{H}, r, s).$$

Planar graphs and neighborhoods

- Natural cyclic ordering of neighbors for planar graphs
- Example: intervals of length 2



Engineering segmentation kernels

- kernels between segments:
  - Chi-square metric: $d_\chi^2(P,Q) = \sum_{j=1}^{N} \frac{(p_i - q_i)^2}{p_i + q_i}$
  - P_ℓ = the histogram of colors of region labelled by ℓ

$$k(\ell,\ell') = k_\chi(P_\ell, P_{\ell'}) = e^{-\mu d_\chi^2(P_\ell, P_{\ell'})}$$

  - Segments weighting scheme $k(\ell,\ell') = \lambda A_\ell^\gamma A_{\ell'}^\gamma e^{-\mu d_\chi^2(P_\ell, P_{\ell'})}$ where A_ℓ is the area of the corresponding region

- Many (?) parameters:

Kernel	free param.	fixed param.
Histogram		μ
Walk	p	$\mu, \lambda, \alpha = 1$
Tree-walk	$p, \alpha > 1$	μ, λ
Weighted tree-walk	$p, \alpha > 1, \gamma$	μ, λ

Multiple kernel learning

- Given set of basis kernels K_j, learn a linear combination

$$K(\eta) = \sum_j \eta_j K_j$$

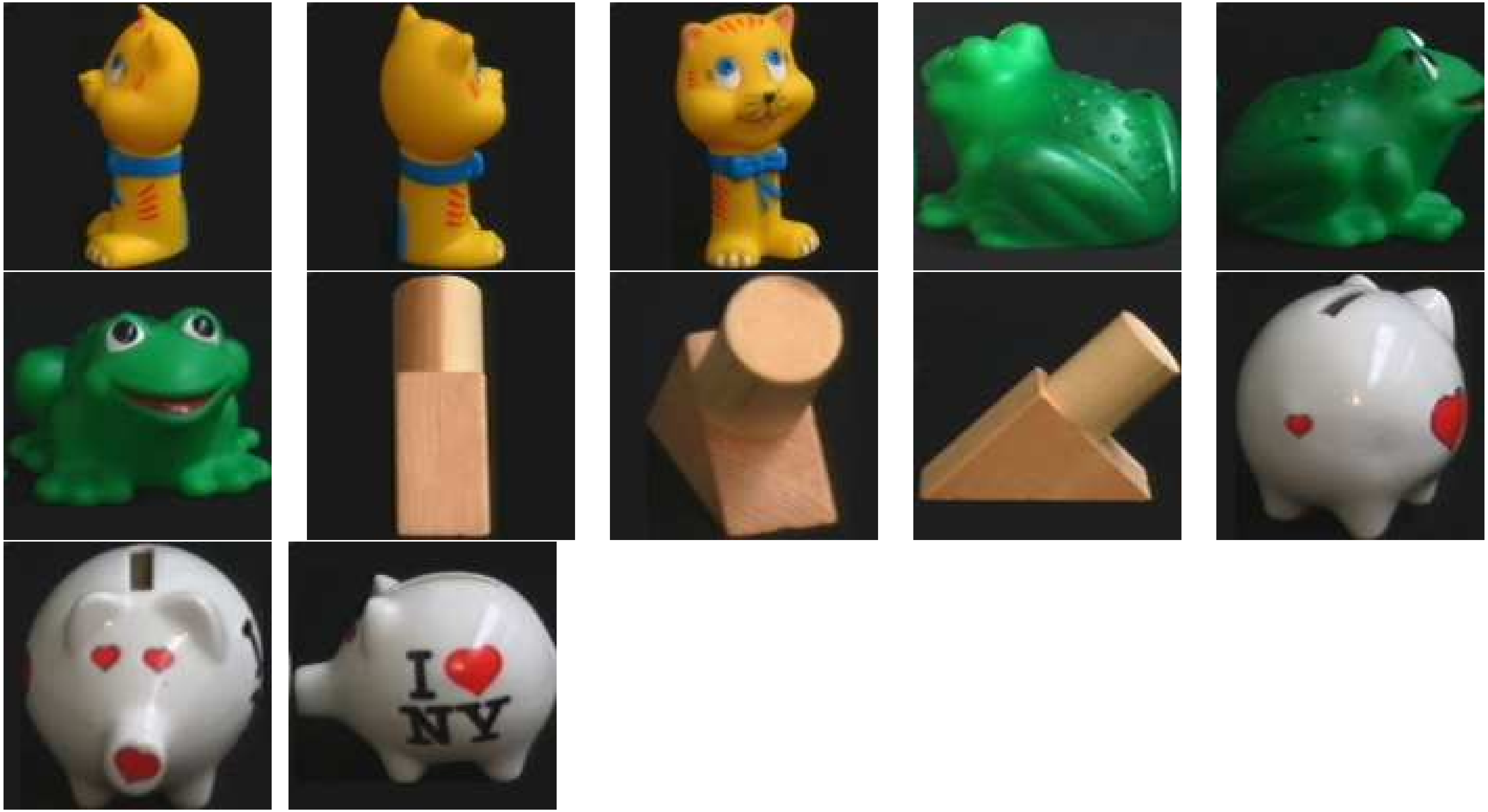
- Convex optimization problem which jointly learns η and the classifier obtained from $K(\eta)$
(Lanckriet et al, 2004, Bach et al, 2004, 2005)

- Kernel selection

- Fusion of heterogeneous kernels from different data sources

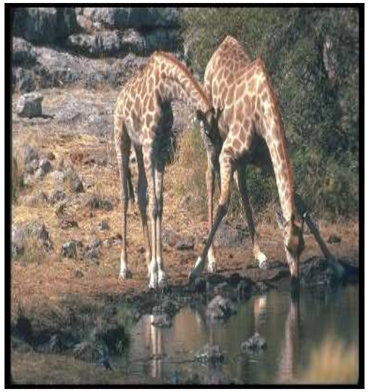
Classification experiments

- `Coil100`: database of 7200 images of 100 *objects in a uniform background*, with 72 images per object.



Classification experiments

- `Corel14` is a database of 1400 *natural images* of 14 different classes






Comparison of kernels

- kernels :
  - histogram kernel (**H**)
  - walk-based kernel (**W**)
  - tree-walk kernel (**TW**)
  - weighted-vertex tree-walk kernel (**wTW**)
  - combination of the above by multiple kernel learning (**M**)

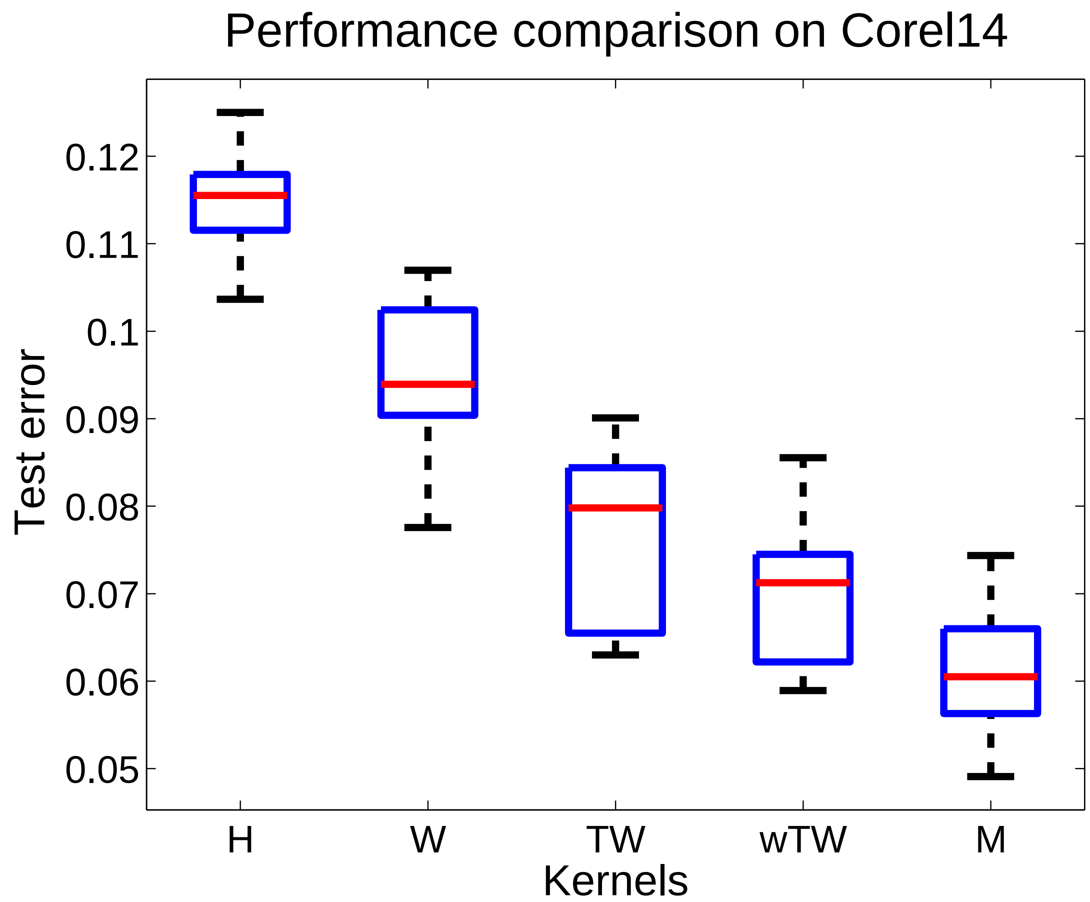
- Hyperparameters selected by cross-validation

- Error rates on ten replications:

	H	W	TW	wTW	M
`Coil100`	1.2%	0.8%	0.0%	0.0%	0.0%
`Corel14`	10.36%	8.52%	7.24%	6.12%	5.38%

Performance on Corel14 dataset

- histogram kernel (**H**)
- walk-based kernel (**W**)
- tree-walk kernel (**TW**)
- weighted-vertex tree-walk kernel (**wTW**)
- combination by MKL (**M**)



Multiple kernel learning

- 100 kernels corresponding to 100 settings of hyperparameters

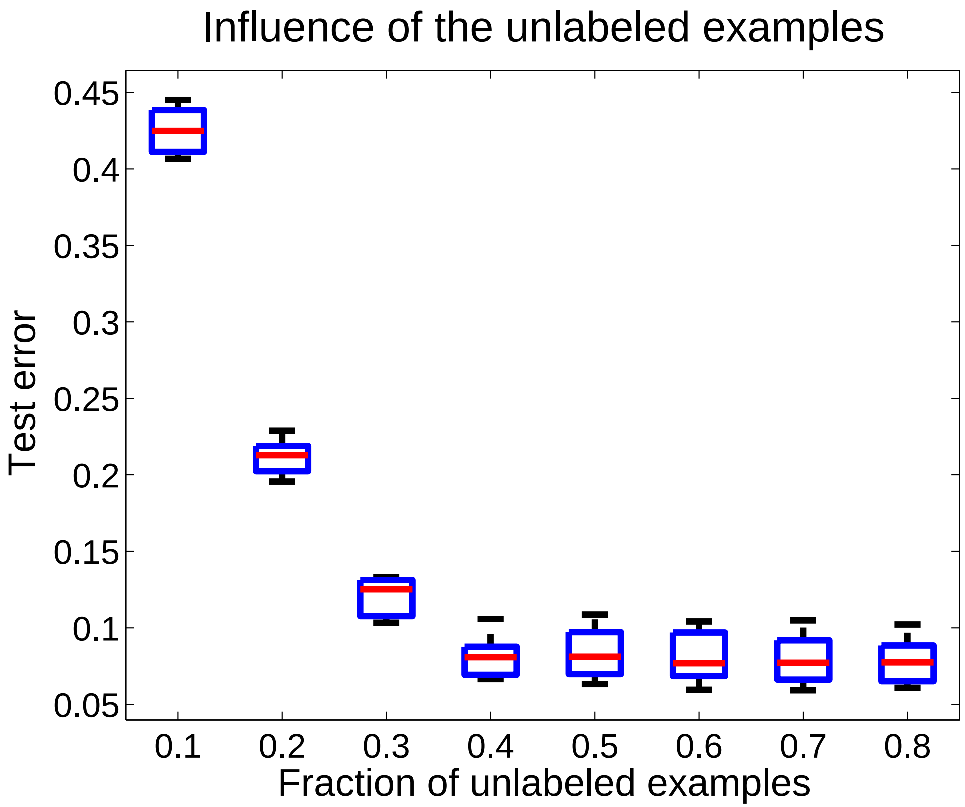
Kernel	free param.	fixed param.
Histogram		μ
Walk	p	$\mu, \lambda, \alpha = 1$
Tree-walk	$p, \alpha > 1$	μ, λ
Weighted tree-walk	$p, \alpha > 1, \gamma$	μ, λ

- Selected kernels

p, α, γ	$10, 3, 0.6$	$7, 1, 0.6$	$10, 3, 0.3$	$5, 3, 0.0$	$8, 1, 0.0$
η	0.12	0.17	0.10	0.07	0.04

Semi-supervised learning

- Kernels give task flexibility
- Example: semi-supervised algorithm of Chapelle and Zien (2004)
- 10% labelled examples, 10% test examples, 10% to 80% unlabelled examples



Conclusion

- Learning on images with kernels on segmentation graphs
  - Based on a natural and still noisy representation of images
  - Prior information allows better generalization performances
  - Modularity
- Current work and natural extensions:
  - Non-tottering trick (Mahé et al, 2005)
  - Allows gaps (Saunders et al, 2001)
  - Shock graphs (e.g., Suard et al., 2005)
  - SIFT features
- Application to image retrieval