« Specification and Abstraction of Semantics »

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Contents

Souvenir, Souvenir 3
Specification and abstraction of semantics
- Motivation 9
- Bi-inductive structural definitions 13
- Example: semantics of the eager λ-calculus 16
- Abstraction 47
- Conclusion 50

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1. Souvenir, Souvenir

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Neil D. Jones

An explorer of automatic semantics-based program manipulation

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A Long Common Professional Interest and Collaboration

– Semantique I;
– Semantique II;
– Atlantique;
– Daedalus;

Many more shared events

– Århus workshop in 81,
– ...
– POPL'97 in Paris,
– ...
– POPL'04 in Venice
– ...
– Decision to start ASTRÉE
– ...
– VMCAI'2009

Happy Souvenirs

2. Specification and abstraction of semantics

Motivation

Motivation

– We look for a formalism to specify abstract program semantics

  from definitional semantics ...

  to static program analysis algorithms

  - coping with termination & non-termination,
  - handling the many different styles of presentations found in the literature (rules, fixpoint, equations, constraints, ...) in a uniform way

– A simple generalization of inductive definitions from sets to posets seems adequate.

On the importance of defining both finite and infinite behaviors

– Example of the *choice operator* $E_1 \mid E_2$ where:

$$E_1 \Longrightarrow a \quad E_2 \Longrightarrow b \qquad \text{termination}$$
$$\text{or} \quad E_1 \Longrightarrow \bot \quad E_2 \Longrightarrow \bot \qquad \text{non-termination}$$

– The *finite behavior* of $E_1 \mid E_2$ is:

$$a \mid b \Longrightarrow a \qquad a \mid b \Longrightarrow b \quad .$$

– But for the case $\bot \mid \bot \Longrightarrow \bot$, the *infinite behaviors* of $E_1 \mid E_2$ depend on the choice method:

Non-deterministic	Parallel	Eager	Mixed left-to-right	Mixed right-to-left
$\bot \mid b \Longrightarrow b$	$\bot \mid b \Longrightarrow b$			$\bot \mid b \Longrightarrow b$
$\bot \mid b \Longrightarrow \bot$		$\bot \mid b \Longrightarrow \bot$	$\bot \mid b \Longrightarrow \bot$	$\bot \mid b \Longrightarrow \bot$
$a \mid \bot \Longrightarrow a$	$a \mid \bot \Longrightarrow a$		$a \mid \bot \Longrightarrow a$	
$a \mid \bot \Longrightarrow \bot$		$a \mid \bot \Longrightarrow \bot$	$a \mid \bot \Longrightarrow \bot$	$a \mid \bot \Longrightarrow \bot$

– Nondeterministic: an internal choice is made initially to evaluate E_1 or to evaluate E_2;
– Parallel: evaluate E_1 and E_2 concurrently, with an unspecified scheduling, and return the first available result a or b;
– Mixed left-to-right: evaluate E_1 and then either return its result a or evaluate E_2 and return its result b;
– Mixed right-to-left: evaluate E_2 and then either return its result b or evaluate E_1 and return its result a;
– Eager: evaluate both E_1 and E_2 and return either results if both terminate.

Bi-inductive Structural Definitions

Over-simplified for the presentation!

Inductive definitions

Set-theoretic [Acz77]

$\langle \wp(\mathcal{U}), \subseteq \rangle$	universe
$\frac{P}{c} \in \mathcal{R} \quad (P \in \wp(\mathcal{U}), c \in \mathcal{U})$	rules
$F(X) \triangleq \left\{ c \;\middle|\; \exists \frac{P}{c} \in \mathcal{R} : P \subseteq X \right\}$	transformer
$\mathsf{lfp}^{\subseteq} F \in \wp(\mathcal{U})$	fixpoint def.
$\subseteq$−least $X : F(X) = X$	equational def.
$\subseteq$−least $X : F(X) \subseteq X$	constraint def.
$\left\{ \frac{X}{c} \;\middle|\; X \subseteq \mathcal{U} \wedge c \in F(X) \right\}$	rules

Inductive definitions

Set-theoretic [Acz77]	Order-theoretic	
$\langle \wp(\mathcal{U}), \subseteq \rangle$	$\langle \mathcal{D}, \sqsubseteq \rangle$	universe
$\frac{P}{c} \in \mathcal{R} \quad (P \in \wp(\mathcal{U}), c \in \mathcal{U})$	$\frac{P}{C} \in \mathcal{R} \quad (P, C \in \mathcal{D})$	rules
$F(X) \triangleq \left\{ c \;\middle|\; \exists \frac{P}{c} \in \mathcal{R} : P \subseteq X \right\}$	$F(X) \triangleq \bigsqcup \left\{ C \;\middle|\; \exists \frac{P}{c} \in \mathcal{R} : P \sqsubseteq X \right\}$	transformer
$\mathsf{lfp}^{\subseteq} F \in \wp(\mathcal{U})$	$\mathsf{lfp}^{\sqsubseteq} F \in \mathcal{D}$	fixpoint def.
$\subseteq$−least $X : F(X) = X$	$\sqsubseteq$−least $X : F(X) = X$	equational def.
$\subseteq$−least $X : F(X) \subseteq X$	$\sqsubseteq$−least $X : F(X) \sqsubseteq X$	constraint def.
$\left\{ \frac{X}{c} \;\middle|\; X \subseteq \mathcal{U} \wedge c \in F(X) \right\}$	$\left\{ \frac{X}{F(X)} \;\middle|\; X \in \mathcal{D} \right\}$	rules

Semantics of the Eager λ-calculus

Syntax

Syntax of the Eager λ-calculus

$$
\begin{array}{rcll}
x, y, z, \ldots & \in & \mathbb{X} & \text{variables} \\
c & \in & \mathbb{C} & \text{constants } (\mathbb{X} \cap \mathbb{C} = \varnothing) \\
c & ::= & 0 \mid 1 \mid \ldots & \\
v & \in & \mathbb{V} & \text{values} \\
v & ::= & c \mid \lambda x \cdot a & \\
e & \in & \mathbb{E} & \text{errors} \\
e & ::= & c\ a \mid e\ a & \\
a, a', a_1, \ldots, b, , \ldots & \in & \mathbb{T} & \text{terms} \\
a & ::= & x \mid v \mid a\ a' &
\end{array}
$$

Trace Semantics

Example I: Finite Computation

function argument

$((\lambda x \cdot x\ x)\ (\lambda y \cdot y))\ ((\lambda z \cdot z)\ 0)$

$\rightarrow$ evaluate function

$((\lambda y \cdot y)\ (\lambda y \cdot y))\ ((\lambda z \cdot z)\ 0)$

$\rightarrow$ evaluate function, cont'd

$(\lambda y \cdot y)\ ((\lambda z \cdot z)\ 0)$

$\rightarrow$ evaluate argument

$(\lambda y \cdot y)\ 0$

$\rightarrow$ apply function to argument

0 *a value!*

Example II: Infinite Computation

function argument

$(\lambda x \cdot x\ x)\ (\lambda x \cdot x\ x)$

$\rightarrow$ apply function to argument

$(\lambda x \cdot x\ x)\ (\lambda x \cdot x\ x)$

$\rightarrow$ apply function to argument

$(\lambda x \cdot x\ x)\ (\lambda x \cdot x\ x)$

$\rightarrow$ apply function to argument

$\ldots$ *non termination!*

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Example III: Erroneous Computation

function argument

$((\lambda x \cdot x\ x)\ ((\lambda z \cdot z)\ 0))\ ((\lambda y \cdot y)\ 0)$

$\rightarrow$ evaluate argument

$((\lambda x \cdot x\ x)\ ((\lambda z \cdot z)\ 0))\ 0$

$\rightarrow$ evaluate function

$((\lambda x \cdot x\ x)\ 0)\ 0$

$\rightarrow$ evaluate function, cont'd

$(0\ 0)\ 0$

a runtime error!

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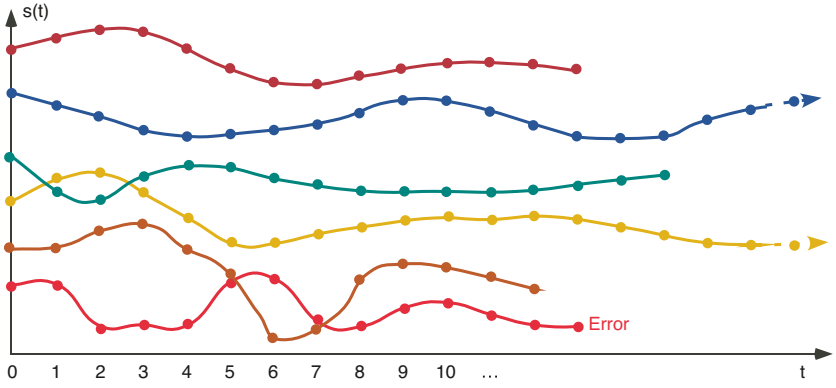
Finite, Infinite and Erroneous Trace Semantics



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Traces

- $\mathbb{T}^\star$ (resp. $\mathbb{T}^+$, $\mathbb{T}^\omega$, $\mathbb{T}^\infty$ and $\mathbb{T}^\infty$) be the set of finite (resp. nonempty finite, infinite, finite or infinite, and nonempty finite or infinite) sequences of terms
- ϵ is the empty sequence $\epsilon \bullet \sigma = \sigma \bullet \epsilon = \sigma$.
- $|\sigma| \in \mathbb{N} \cup \{\omega\}$ is the length of $\sigma \in \mathbb{T}^\infty$. $|\epsilon| = 0$.
- If $\sigma \in \mathbb{T}^+$ then $|\sigma| > 0$ and $\sigma = \sigma_0 \bullet \sigma_1 \bullet \ldots \bullet \sigma_{|\sigma|-1}$.
- If $\sigma \in \mathbb{T}^\omega$ then $|\sigma| = \omega$ and $\sigma = \sigma_0 \bullet \ldots \bullet \sigma_n \bullet \ldots$.

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Operations on Traces

– For a $\in \mathbb{T}$ and $\sigma \in \mathbb{T}^\infty$, we define $a@\sigma$ to be $\sigma' \in \mathbb{T}^\infty$ such that $\forall i < |\sigma| : \sigma'_i = a\ \sigma_i$

$\sigma = \quad \sigma_0 \quad \sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \cdots \quad \sigma_i \quad \cdots$

$a@\sigma = \quad a\ \sigma_0 \quad a\ \sigma_1 \quad a\ \sigma_2 \quad a\ \sigma_3 \quad \cdots \quad a\ \sigma_i \quad \cdots$

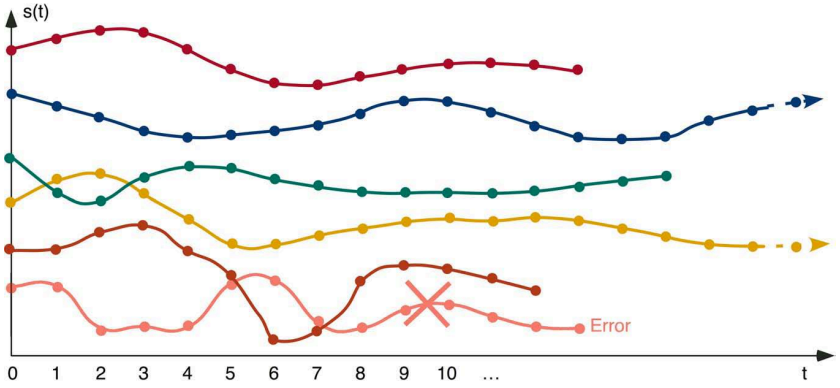
Operations on Traces (Cont'd)

– Similarly for a $\in \mathbb{T}$ and $\sigma \in \mathbb{T}^\infty$, $\sigma@a$ is σ' where $\forall i < |\sigma| : \sigma'_i = \sigma_i\ a$

$\sigma = \quad \sigma_0 \quad \sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \cdots \quad \sigma_i \quad \cdots$

$\sigma@a = \quad \sigma_0\ a \quad \sigma_1\ a \quad \sigma_2\ a \quad \sigma_3\ a \quad \cdots \quad \sigma_i\ a \quad \cdots$

Finite and Infinite Trace Semantics $\vec{\mathbb{S}}$



The Computational Lattice

Given $S, T \in \wp(\mathbb{T}^\infty)$, we define

– $S^+ \triangleq S \cap \mathbb{T}^+$ finite traces

– $S^\omega \triangleq S \cap \mathbb{T}^\omega$ infinite traces

– $S \sqsubseteq T \triangleq S^+ \subseteq T^+ \wedge S^\omega \supseteq T^\omega$ computational order

– $\langle \wp(\mathbb{T}^\infty), \sqsubseteq, \mathbb{T}^\omega, \mathbb{T}^+, \sqcup, \sqcap \rangle$ is a complete lattice

Bifinitary Trace Semantics $\vec{\mathbb{S}}$ of the Eager λ-calculus [1]

$$\mathsf{v} \in \vec{\mathbb{S}},\ \mathsf{v} \in \mathbb{V} \qquad \frac{\mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \bullet \sigma \in \vec{\mathbb{S}}}{(\lambda \mathsf{x} \cdot \mathsf{a})\ \mathsf{v} \bullet \mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \bullet \sigma \in \vec{\mathbb{S}}} \sqsubseteq,\ \mathsf{v} \in \mathbb{V}$$

$$\frac{\sigma \in \vec{\mathbb{S}}^{\omega}}{\mathsf{a}@\sigma \in \vec{\mathbb{S}}} \sqsubseteq,\ \mathsf{a} \in \mathbb{V} \qquad \frac{\sigma \bullet \mathsf{v} \in \vec{\mathbb{S}}^{+},\ (\mathsf{a}\ \mathsf{v}) \bullet \sigma' \in \vec{\mathbb{S}}}{(\mathsf{a}@\sigma) \bullet (\mathsf{a}\ \mathsf{v}) \bullet \sigma' \in \vec{\mathbb{S}}} \sqsubseteq,\ \mathsf{v}, \mathsf{a} \in \mathbb{V}$$

$$\frac{\sigma \in \vec{\mathbb{S}}^{\omega}}{\sigma@\mathsf{b} \in \vec{\mathbb{S}}} \sqsubseteq \qquad \frac{\sigma \bullet \mathsf{v} \in \vec{\mathbb{S}}^{+},\ (\mathsf{v}\ \mathsf{b}) \bullet \sigma' \in \vec{\mathbb{S}}}{(\sigma@\mathsf{b}) \bullet (\mathsf{v}\ \mathsf{b}) \bullet \sigma' \in \vec{\mathbb{S}}} \sqsubseteq,\ \mathsf{v} \in \mathbb{V}$$

[1] Note: $\mathsf{a}[\mathsf{x} \leftarrow \mathsf{b}]$ is the capture-avoiding substitution of b for all free occurences of x within a. We let FV(a) be the free variables of a. We define the call-by-value semantics of closed terms (without free variables) $\overline{\mathbb{T}} \triangleq \{\mathsf{a} \in \mathbb{T} \mid \mathrm{FV}(\mathsf{a}) = \varnothing\}$.

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Fixpoint big-step maximal trace semantics

The bifinitary trace semantics is

$$\vec{\mathbb{S}} = \mathsf{lfp}^{\sqsubseteq}\ \vec{F}$$

where $\vec{F} \in \wp(\overline{\mathbb{T}}^{\infty}) \mapsto \wp(\overline{\mathbb{T}}^{\infty})$ is

$$\begin{aligned}
\vec{F}(S) \triangleq\ & \{\mathsf{v} \in \overline{\mathbb{T}}^{\infty} \mid \mathsf{v} \in \mathbb{V}\} \cup \\
& \{(\lambda \mathsf{x} \cdot \mathsf{a})\ \mathsf{v} \bullet \mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \bullet \sigma \mid \mathsf{v} \in \mathbb{V} \wedge \mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \bullet \sigma \in S\} \cup \\
& \{\sigma@\mathsf{b} \mid \sigma \in S^{\omega}\} \cup \\
& \{(\sigma@\mathsf{b}) \bullet (\mathsf{v}\ \mathsf{b}) \bullet \sigma' \mid \sigma \neq \epsilon \wedge \sigma \bullet \mathsf{v} \in S^{+} \wedge \mathsf{v} \in \mathbb{V} \wedge (\mathsf{v}\ \mathsf{b}) \bullet \sigma' \in S\} \cup \\
& \{\mathsf{a}@\sigma \mid \mathsf{a} \in \mathbb{V} \wedge \sigma \in S^{\omega}\} \cup \\
& \{(\mathsf{a}@\sigma) \bullet (\mathsf{a}\ \mathsf{v}) \bullet \sigma' \mid \mathsf{a}, \mathsf{v} \in \mathbb{V} \wedge \sigma \neq \epsilon \wedge \sigma \bullet \mathsf{v} \in S^{+} \wedge (\mathsf{a}\ \mathsf{v}) \bullet \sigma' \in S\}\ .
\end{aligned}$$

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Relational Semantics

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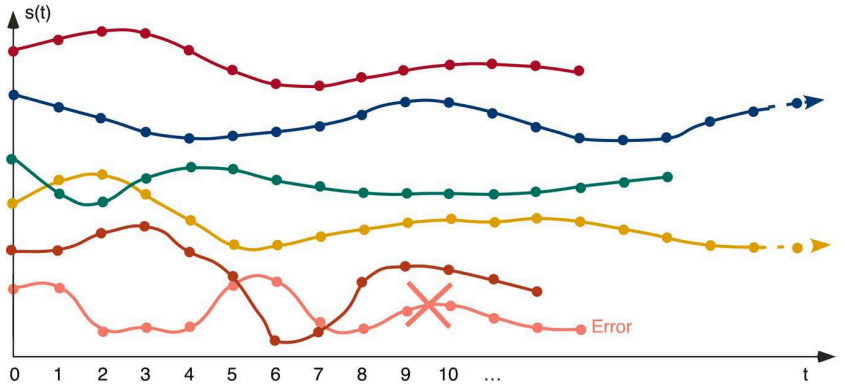
Trace Semantics



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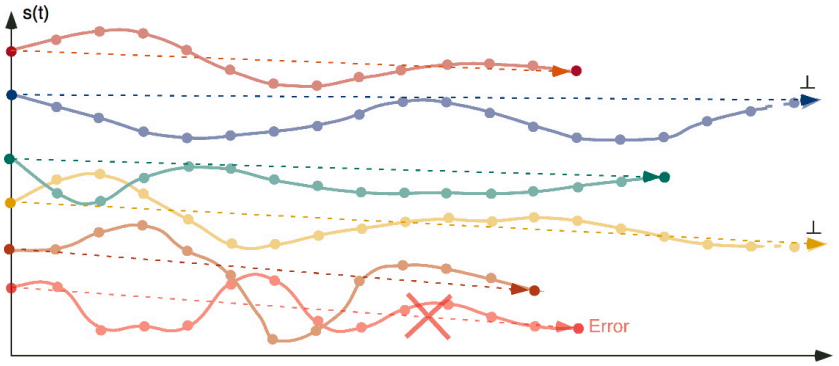
Relational Semantics = α(Trace Semantics)



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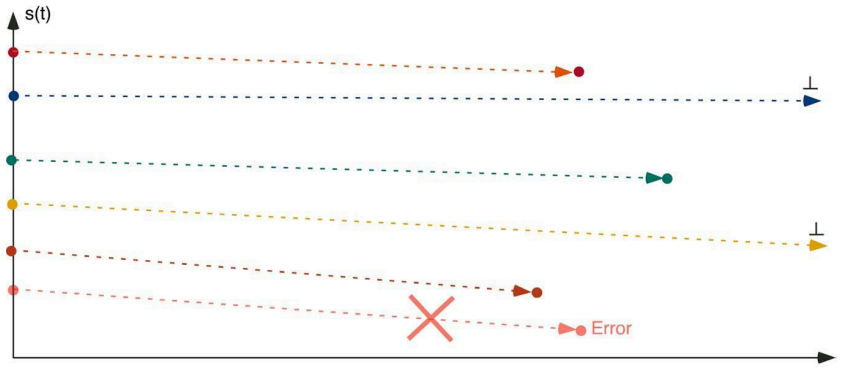
Relational Semantics



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Abstraction to the Bifinitary Relational Semantics of the Eager λ-calculus

remember the input/output behaviors,
forget about the intermediate computation steps

$$\alpha(T) \stackrel{\text{def}}{=} \{\alpha(\sigma) \mid \sigma \in T\}$$

$$\alpha(\sigma_0 \bullet \sigma_1 \bullet \ldots \bullet \sigma_n) \stackrel{\text{def}}{=} \sigma_0 \Longrightarrow \sigma_n$$

$$\alpha(\sigma_0 \bullet \ldots \bullet \sigma_n \bullet \ldots) \stackrel{\text{def}}{=} \sigma_0 \Longrightarrow \bot$$

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Bifinitary Relational Semantics of the Eager λ-calculus

$$\mathsf{v} \Longrightarrow \mathsf{v}, \quad \mathsf{v} \in \mathbb{V}$$

$$\frac{\mathsf{a} \Longrightarrow \bot}{\mathsf{a\ b} \Longrightarrow \bot}\sqsubseteq \qquad \frac{\mathsf{b} \Longrightarrow \bot}{\mathsf{a\ b} \Longrightarrow \bot}\sqsubseteq, \quad \mathsf{a} \in \mathbb{V}$$

$$\frac{\mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \Longrightarrow r}{(\lambda\, \mathsf{x} \cdot \mathsf{a})\quad \mathsf{v} \Longrightarrow r}\sqsubseteq, \quad \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V} \cup \{\bot\}$$

$$\frac{\mathsf{a} \Longrightarrow \mathsf{v}, \quad \mathsf{v\ b} \Longrightarrow r}{\mathsf{a\ b} \Longrightarrow r}\sqsubseteq, \quad \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V} \cup \{\bot\}$$

$$\frac{\mathsf{b} \Longrightarrow \mathsf{v}, \quad \mathsf{a\ v} \Longrightarrow r}{\mathsf{a\ b} \Longrightarrow r}\sqsubseteq, \quad \mathsf{a} \in \mathbb{V},\ \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V} \cup \{\bot\}\ .$$

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On the computational ordering $\sqsubseteq$

– For the bifinitary trace semantics $\vec{\mathbb{S}}$, we could replace the computational ordering $\sqsubseteq$ by $\supseteq$ (thus taking greatest fixpoints for $\subseteq$);

– Impossible for the bifinitary relational semantics!

– Counter-example: the greatest fixpoint starts by assuming that we have the terminating execution

$$(\lambda x \cdot x\ x)(\lambda x \cdot x\ x) \Longrightarrow (\lambda x \cdot x\ x)(\lambda x \cdot x\ x)$$

then the call rule $\dfrac{a[x \leftarrow v] \Longrightarrow r}{(\lambda x \cdot a)\quad v \Longrightarrow r}\sqsubseteq, \quad v \in \mathbb{V},\ r \in \mathbb{V} \cup \{\bot\}$ will preserve this invalid hypothesis!

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Natural Semantics

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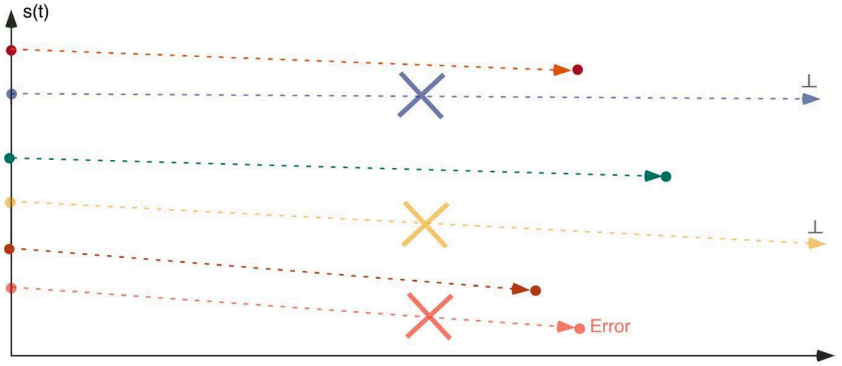
Natural Semantics = α(Relational Semantics)



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Abstraction to the Natural Big-Step Semantics of the Eager λ-calculus

remember the finite input/output behaviors,
forget about non-termination

$$\alpha(T) \stackrel{\text{def}}{=} \bigcup\{\alpha(\sigma) \mid \sigma \in T\}$$

$$\alpha(\sigma_0 \Longrightarrow \sigma_n) \stackrel{\text{def}}{=} \{\sigma_0 \Longrightarrow \sigma_n\}$$

$$\alpha(\sigma_0 \Longrightarrow \bot) \stackrel{\text{def}}{=} \varnothing$$

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Natural Big-Step Semantics of the Eager λ-calculus [Kah88]

$$\mathsf{v} \Longrightarrow \mathsf{v}, \quad \mathsf{v} \in \mathbb{V}$$

$$\frac{\mathsf{a}[\mathsf{x} \leftarrow \mathsf{v}] \Longrightarrow r}{(\lambda \mathsf{x} \cdot \mathsf{a}) \quad \mathsf{v} \Longrightarrow r} \subseteq, \quad \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V}$$

$$\frac{\mathsf{a} \Longrightarrow \mathsf{v}, \quad \mathsf{v}\ \mathsf{b} \Longrightarrow r}{\mathsf{a}\ \mathsf{b} \Longrightarrow r} \subseteq, \quad \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V}$$

$$\frac{\mathsf{b} \Longrightarrow \mathsf{v}, \quad \mathsf{a}\ \mathsf{v} \Longrightarrow r}{\mathsf{a}\ \mathsf{b} \Longrightarrow r} \subseteq, \quad \mathsf{a} \in \mathbb{V},\ \mathsf{v} \in \mathbb{V},\ r \in \mathbb{V}\ .$$

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Transition Semantics

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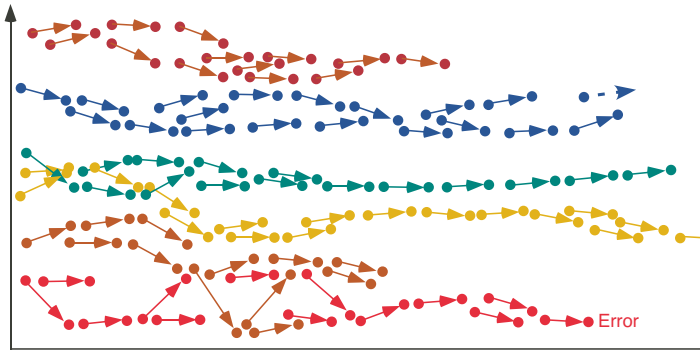
Transition Semantics = α(Trace Semantics)



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Abstraction to the Transition Semantics of the Eager λ-calculus

remember execution steps,
forget about their sequencing

$$\alpha(T) \stackrel{\text{def}}{=} \bigcup \{\alpha(\sigma) \mid \sigma \in T\}$$

$$\alpha(\sigma_0 \bullet \sigma_1 \bullet \ldots \bullet \sigma_n) \stackrel{\text{def}}{=} \{\sigma_i \longrightarrow \sigma_{i+1} \mid 0 \leqslant i \wedge i < n\}$$

$$\alpha(\sigma_0 \bullet \ldots \bullet \sigma_n \bullet \ldots) \stackrel{\text{def}}{=} \{\sigma_i \longrightarrow \sigma_{i+1} \mid i \geqslant 0\}$$

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Transition Semantics of the Eager λ-calculus [Plo81]

$$((\lambda x \cdot a)\ v) \longrightarrow a[x \leftarrow v]$$

$$\frac{a_0 \longrightarrow a_1}{a_0\ b \longrightarrow a_1\ b}\subseteq$$

$$\frac{b_0 \longrightarrow b_1}{v\ b_0 \longrightarrow v\ b_1}\subseteq .$$

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Approximation

$((\lambda x \cdot x\ x)\ ((\lambda z \cdot z)\ 0))\ (\lambda y \cdot y) \rightarrow ((\lambda x \cdot x\ x)\ 0)\ (\lambda y \cdot y)$
$\rightarrow (0\ 0)\ (\lambda y \cdot y)$ *an error!*

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Abstraction

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Kleenian abstraction

- $\langle \mathcal{D}, \sqsubseteq, \bot, \sqcup \rangle$, $\langle \mathcal{D}^\sharp, \sqsubseteq^\sharp, \bot^\sharp, \sqcup^\sharp \rangle$ dcpos
- $F \in \mathcal{D} \mapsto \mathcal{D}$, $F^\sharp \in \mathcal{D}^\sharp \mapsto \mathcal{D}^\sharp$ monotone
- $\alpha \in \mathcal{D} \mapsto \mathcal{D}^\sharp$ strict and continuous on chains of $\mathcal{D}$
- $\alpha \circ F = F^\sharp \circ \alpha$, commutation condition

  $\Longrightarrow \alpha(\mathsf{lfp}^{\sqsubseteq} F) = \mathsf{lfp}^{\sqsubseteq^\sharp} F^\sharp$

OK for abstracting finite behaviors, not infinite ones

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Tarskian abstraction

- $\langle \mathcal{D}, \sqsubseteq, \bot, \sqcup \rangle$, $\langle \mathcal{D}^\sharp, \sqsubseteq^\sharp, \bot^\sharp, \sqcup^\sharp \rangle$ dcpos
- $F \in \mathcal{D} \mapsto \mathcal{D}$, $F^\sharp \in \mathcal{D}^\sharp \mapsto \mathcal{D}^\sharp$ monotone
- $\alpha \in \mathcal{D} \mapsto \mathcal{D}^\sharp$ preserves meets
- $F^\sharp \circ \alpha \sqsubseteq^\sharp \alpha \circ F$, semi-commutation condition
- $\forall y \in \mathcal{D}^\sharp : (F^\sharp(y) \sqsubseteq^\sharp y) \Longrightarrow (\exists x \in \mathcal{D} : \alpha(x) = y \wedge F(x) \sqsubseteq x$
  $\Longrightarrow \alpha(\mathsf{lfp}^{\sqsubseteq} F) = \mathsf{lfp}^{\sqsubseteq^\sharp} F^\sharp$

OK for abstracting infinite behaviors, not finite ones
$\Rightarrow$ abstract by parts.

Conclusion

Conclusion

- Both finite and infinite semantics are needed in static analysis (such as strictness, [Myc80]), typing [Cou97, Ler06], etc;
- Such static analyzes must be proved correct with respect to a semantics chosen at an various level of abstraction (small-step/big-step trace/relational/natural semantics);
- Static analyzes use various equivalent presentations (fixpoints, equational, constraints and inference rules)
- The bifinite extension of SOS *might* satisfy these needs.

THE END, THANK YOU

Neil, for such a long friendship and cooperation

Best wishes for your new constraintless research career

Bibliography

[Acz77] P. Aczel. An introduction to inductive definitions. In J. Barwise, editor, *Handbook of Mathematical Logic*, volume 90 of *Studies in Logic and the Foundations of Mathematics*, pages 739–782. Elsevier, 1977.

[Cou97] P. Cousot. Types as abstract interpretations, invited paper. In *24th POPL*, pages 316–331, Paris, FR, Jan. 1997. ACM Press.

[Kah88] G. Kahn. Natural semantics. In K. Fuchi and M. Nivat, editors, *Programming of Future Generation Computers*, pages 237–258. Elsevier, 1988.

[Ler06] X. Leroy. Coinductive big-step operational semantics. In P. Sestoft, editor, *Proc. 15th ESOP '2006*, Vienna, AT, LNCS 3924, pages 54–68. Springer, 27–28 Mar. 2006.

[Myc80] A. Mycroft. The theory and practice of transforming call-by-need into call-by-value. In B. Robinet, editor, *Proc. 4th Int. Symp. on Programming*, Paris, FR, 22–24 Apr. 1980, LNCS 83, pages 270–281. Springer, 1980.

[Plo81] G.D. Plotkin. A structural approach to operational semantics. Technical Report DAIMI FN-19, Aarhus University, DK, Sep. 1981.

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