

# Open problems

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## 1 Covering a graph with $\sqrt{n}$ balls

*Presented by Marthe Bonamy*

**Problem 1.** *Let  $G$  be a connected graph on  $n$  vertices. Can we find  $k = \lceil \sqrt{n} \rceil$  vertices  $v_1, \dots, v_k$  such that  $\forall u \in V(G), \exists i \in \{0, \dots, k-1\}, d(u, v_i) \leq i$ .*

Problem introduced by Bessy and Rautenbach (2016) and Berger (2016). Proved if we take  $k = 1.3 \lceil \sqrt{n} \rceil$  [BR15].

## 2 Long induced paths in planar graphs

*Presented by Louis Esperet.*

**Problem 2.** *Let  $G$  be a planar graph with a path on  $n$  vertices. What is the minimum size of a longest induced path in  $G$ ?*

Known: the answer is between  $O(\log n / \log \log n)$  and  $\Omega(\sqrt{\log n})$ .

Conjecture:  $\Theta((\log n)^c)$  for some constant  $c$ .

## 3 Odd-girth preserving homomorphisms to 3-trees

*Presented by Reza Naserasr.*

An homomorphism from  $G$  to  $H$  is a mapping  $\phi: V(G) \rightarrow V(H)$  s.t.  $\{x, y\} \in E(G) \Rightarrow \{\phi(x), \phi(y)\} \in E(H)$ . The odd girth of a graph is the length of a smallest odd cycle.

**Problem 3.** *Let  $G$  be a planar graph of odd-girth  $2k+1$ . Is there a partial 3-tree  $H$  of odd-girth  $2k+1$  such that there is an homomorphism from  $G$  to  $H$ ?*

Introduced by Beaudou, Foucaud, Naserasr.

## 4 Knot diagrams with a given knot type

*Presented by Harisson Chapman.*

Let us consider 4-valent maps (4-regular plane graphs). We color edges such that on every vertex, 'opposite edges' receive the same color. Given a graph it is easy to color it. We consider graphs that cannot be colored with more than one color (in this case we say that they have loop number one). Each vertex is decorated with a sign in order to use the graph to represent a knot diagram (the sign says which pair of opposite edges is over the other).

Let  $\mathcal{K}_n(k)$  be all knot diagrams on  $n$  vertices with a given knot type  $k$ .

**Problem 4.** *For two knot types  $k_1, k_2$ , is it true that  $\lim_{n \rightarrow \infty} |\mathcal{K}_n(k_1)|^{1/n} = \lim_{n \rightarrow \infty} |\mathcal{K}_n(k_2)|^{1/n}$ ?*

## 5 Diameter of the coloring graph of 2-degenerate graphs

*Presented by Marthe Bonamy.*

Let  $G$  be a 2-degenerate graph (i.e. every subgraph of  $G$  has a vertex of degree at most 2). This graph is 4-colorable.

Let  $R_4(G) = \{V', E'\}$  be the graph of proper 4-colorings of  $G$ :  $V'$  contains all proper ( $\leq 4$ )-colorings of  $G$  and two colorings are adjacent if they differ on one vertex.

**Problem 5** (Asked by Cereceda 2007). *Is it true that  $\text{diam}(R_4(G)) \leq \text{poly}(|V(G)|)$ ?*

## 6 Chromatic number of graphs of exact distance

*Presented by Konstantinos Stravropoulos.*

Let  $G = (V, E)$ . We define  $G^{\#p}$  as the graph  $(V, E')$  where  $\{u, v\} \in E' \iff d_G(u, v) = p$ .

If  $p = 2k + 1$  and  $G$  belongs to a class of bounded expansion, then it is known that  $\chi(G^{\#p}) = f(p) \leq 2^{2^{\chi_r(G)}}$ .

**Problem 6.** *Is there a constant  $c$  such that the following holds*

$$\forall k \in \mathbb{N}, \chi(G^{\#2k+1}) \leq c?$$

*for every planar graph  $G$ ?*

## 7 Mimiking graphs

*Presented by Dimitrios Thilikos.*

Let  $G$  be a graph with a set of  $2k$  terminals  $T \subseteq V(G)$ . Let  $\chi = (T_1, T_2, \sigma)$  be such that  $(T_1, T_2)$  is a partition of  $T$ ,  $|T_1| = |T_2|$ , and  $\sigma$  is a bijection between  $T_1$  and  $T_2$ .

Let  $\text{DP}$  be a function that, given a graph  $G$  and a triple  $\chi$  as above, answers 1 if there are disjoint paths in  $G$  between the vertices mapped by  $\chi$ , and zero otherwise.

It is known that there is a function  $f$  such that given a graph  $G$  and  $T \subseteq V(G)$ , we can find a graph  $H$  with the same set of terminals  $T \subseteq V(H)$  such that  $|V(H)| \leq f(k)$  and  $\text{DP}(G, \chi) = \text{DP}(H, \chi)$  for every triple  $\chi$ . Intuitively,  $H$  is smaller and behaves the same way as  $G$  with respect to the existence of disjoint paths between terminals of  $T$ .

**Problem 7.** *How big should  $f$  be for planar graphs (when the new graph is also planar)?*

## 8 Pattern-avoiding coloring of plane graphs

*Presented by Daniel Goncalves.*

**Problem 8.** *Can we always color a plane graph with colors  $\{1, 2, 3, 4\}$  in a way such that induced 4-cycles avoid the pattern 1-2-3-4 clockwise?*

Related to intersection graphs of segments on the plane.

## 9 Graphs with no cycles of length 0 mod 3

*Presented by Marthe Bonamy.*

**Problem 9.** *In a non-trivial graph with no induced cycle of length 0 modulo 3, can we always find an edge, the removal of which does not create a cycle of length 0 modulo 3?*

Origin unclear.

## References

[BR15] S. Bessy and D. Rautenbach. Bounds, Approximation, and Hardness for the Burning Number. *ArXiv e-prints*, November 2015.