Approximate Optimality with Bounded Regret in Dynamic Matching Models

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Joint work with
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and
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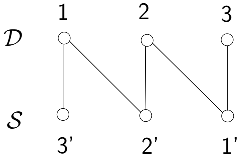
Outline

1. Background
2. Bipartite matching model
3. Optimization
   - Average Cost Criterion
   - Workload
   - Workload Relaxation
   - Asymptotic optimality
4. Final remarks

Bipartite Matching

$(\mathcal{D}, \mathcal{S}, E)$ bipartite graph

$\mathcal{D}(s) = \{d \in \mathcal{D} \ : \ (d, s) \in E\}$
$\mathcal{S}(d) = \{s \in \mathcal{S} \ : \ (d, s) \in E\}$



x_i number of elements of type $i \in \mathcal{D} \cup \mathcal{S}$

Perfect matching: $m \in \mathbb{N}^E$ such that:

$$x_d = \sum_{s \in \mathcal{S}(d)} m_{ds}, \ \forall d \in \mathcal{D}, \quad x_s = \sum_{d \in \mathcal{D}(s)} m_{ds}, \ \forall s \in \mathcal{S}$$

Hall's marriage theorem (1935)

$\exists$ perfect matching if and only if:

$$\sum_{d \in U} x_c \leq \sum_{s \in \mathcal{S}(U)} x_s, \quad \forall U \subset \mathcal{D}$$
$$\sum_{s \in V} x_s \leq \sum_{d \in \mathcal{D}(V)} x_d, \quad \forall V \subset \mathcal{S}$$

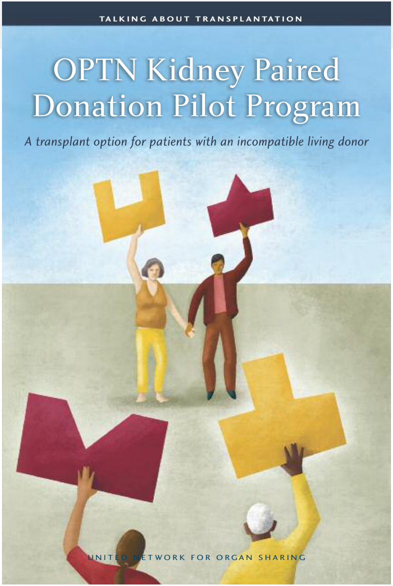
Matching in Health-care

Kidney paired donation

Who can join this program?

For recipients: If you are eligible for a kidney transplant and are receiving care at a transplant center in the United States, you can join ... *You must have a living donor who is willing and medically able to donate his or her kidney ...*

For donors: You must also be willing to take part ...



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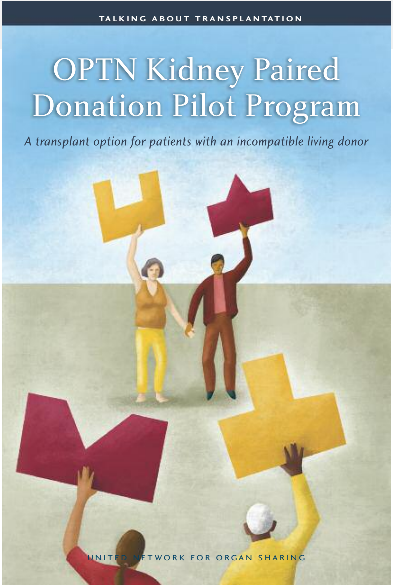


Need for Dynamic Matching Models

Dynamic Matching Model: FCFS

Another Example

Boston area public housing (some 25 years ago):

Households applying for public housing are allowed to specify those housing projects in which they are willing to live; when a public housing unit becomes newly available, of those households willing to live in the associated housing project, the one that has been waiting the longest is offered the unit.

Model

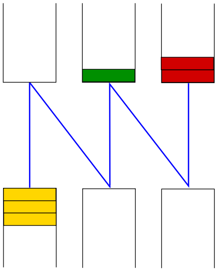
Two independent infinite sequences of items.
Demand / supply i.i.d.

FCFS matching policy admits product-form invariant distribution

Caldentey, Kaplan, Weiss 2009
Adan, Weiss 2012
Adan, B., Mairesse, Weiss 2015

Dynamic Bipartite Matching Model

Multiclass queueing model – Supply/Demand play symmetric roles



- Discrete time queueing model with two types of arrival: "supply" and "demand".
- Discrete time: at each time step there is one customer and one server that arrive into the system, independently of the past.
- Instantaneous matchings according to a bipartite matching graph.
  Unmatched supply/demand stored in a buffer.

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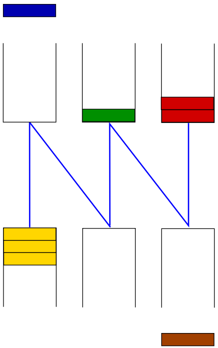

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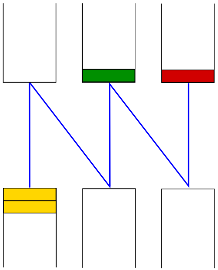

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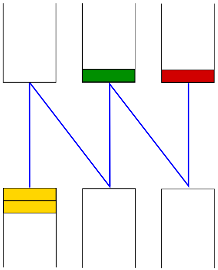

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Given by: a matching graph, a joint probability measure μ for arrivals of demand/supply and a matching policy.

Dynamic Matching Model: Stability

For the dynamic model with i.i.d. arrivals, when is the Markovian model stable? (positive recurrent)

- Necessary condition: generalization of Hall's marriage theorem
- Under this condition, certain policies are stabilizing, such as MaxWeight
- Under this condition, other policies are *not* stabilizing

B., Gupta, Mairesse 2013.

Dynamic Matching Model: Approximate Optimality

Subject of this talk:

- How to define ‘heavy traffic’? This requires a formulation of ‘network load’
- What is the structure of an optimal policy for the model in heavy traffic?
- How do we use this structure for policy design?

B., Meyn 2016

Necessary stability conditions

Assumption: matching graph $(\mathcal{D}, \mathcal{S}, E)$ is connected.

Necessary conditions: If the model is stable then the marginals of μ satisfy

$$\textsc{NCond}: \quad \begin{cases} \mu_{\mathcal{D}}(U) < \mu_{\mathcal{S}}(\mathcal{S}(U)), & \forall U \subsetneq \mathcal{D} \\ \mu_{\mathcal{S}}(V) < \mu_{\mathcal{D}}(\mathcal{D}(V)), & \forall V \subsetneq \mathcal{S} \end{cases}$$

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Sufficient conditions: If $\textsc{NCond}$ holds, then there exists a policy that is stabilizing

Prop. Given $[(\mathcal{D}, \mathcal{S}, E), \mu]$, there exists an algorithm of time complexity $O((|\mathcal{D}| + |\mathcal{S}|)^3)$ to decide if $\textsc{NCond}$ is satisfied.

Optimization

Cost function c on buffer levels.

Average-cost: $$\eta = \limsup_{N\to\infty} \frac{1}{N} \sum_{t=0}^{N-1} \mathsf{E}\big[c(Q(t))\big]$$

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Queue dynamics: $Q(t+1) = Q(t) - U(t) + A(t)\,, \qquad t \geq 0$

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Input process $\mathbf{U}$ represents the sequence of matching activities. Input space:

$$\mathrm{U}_\diamond = \Big\{ \sum_{e\in E} n_e u^e : n_e \in \mathbb{Z}_+ \Big\} \qquad \text{with } u^e = \mathbf{1}^i + \mathbf{1}^j \text{ for } e = (i,j) \in \mathcal{E}.$$

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$X(t) = Q(t) + A(t)$ the state process of the MDP model,

$$X(t+1) = X(t) - U(t) + A(t+1)$$

The state space $\mathrm{X}_\diamond = \{x \in \mathbb{Z}_+^\ell : \xi^0 \cdot x = 0\}$ with $\xi^0 = (1, \dots, 1, -1, \dots, -1)$.

Workload

For any $D \subset \mathcal{D}$, corresponding workload vector ξ^D defined so that

$$\xi^D \cdot x = \sum_{i \in D} x_i^{\mathcal{D}} - \sum_{j \in \mathcal{S}(D)} x_j^{\mathcal{S}}$$

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Necessary and sufficient condition for a stabilizing policy:

NCond: $\delta_D := -\xi^D \cdot \alpha > 0$ for each D

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$$\text{Fluid model,} \qquad \frac{d}{dt} x(t) = -u(t) + \alpha$$

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Heavy-traffic: $\delta_D \sim 0$ for one or more D

Workload Dynamics

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Workload relaxation: take this as the model for control.

Relaxations

A workload relaxation takes this as the model for control:

One Dimensional Workload relaxation,

$$\widehat{W}(t+1) = \widehat{W}(t) - \delta + \underbrace{I(t)}_{\text{Idleness} \geq 0} + \underbrace{\Delta(t+1)}_{\text{Zero mean}}$$

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Control of the relaxation = inventory model of Clark & Scarf (1960)

Hedging policy, with threshold τ^*: *Idling is not permitted unless* $\widehat{W}(t) < -\tau^*$

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Heavy-traffic: For average-cost optimal control, $\tau^* \sim \frac{1}{2}\frac{\sigma_\Delta^2}{\delta}\log(1 + \overline{c}_+/\overline{c}_-)$

Asymptotic optimality

Family of arrival processes $\{A^\delta(t)\}$, parameterized by $\delta \in [0, \bar{\delta}^\bullet]$, $\bar{\delta}^\bullet \in (0,1)$.
Additional assumptions:

(A1) For one set $D \subsetneq \mathcal{D}$ we have $\xi^D \cdot \alpha^\delta = -\delta$, where α^δ denotes the mean of $A^\delta(t)$.
Moreover, there is a fixed constant $\underline{\delta} > 0$ such that $\xi^{D'} \cdot \alpha^\delta \leq -\underline{\delta}$ for any $D' \subsetneq \mathcal{D}$, $D' \neq D$, and $\delta \in [0, \bar{\delta}^\bullet]$.

(A2) The distributions are continuous at $\delta = 0$, with linear rate: For some constant b,

$$\mathsf{E}[\|A^\delta(t) - A^0(t)\|] \leq b\delta.$$

(A3) Graph structure for arrivals and for feasible matches independent of $\delta \geq 0$ $\Longrightarrow$ The matching graph is connected even for $\delta = 0$.
Moreover, there exists $i_0 \in \mathcal{S}(D)$, $j_0 \in D^c$, and $p_I > 0$ such that

$$P\{A^\delta_{i_0}(t) \geq 1 \text{ and } A^\delta_{j_0}(t) \geq 1\} \geq p_I, \qquad 0 \leq \delta \leq \bar{\delta}^\bullet.$$

Asymptotic optimality

- h-MWT (h-MaxWeight with threshold) policy:
  For a differentiable function $h\colon \mathbb{R}^\ell \to \mathbb{R}_+$, and a threshold $\tau \geq 0$,

$$\begin{aligned} \phi(x) = \arg\max \quad & u \cdot \nabla h(x) \\ \text{subject to} \quad & u \text{ feasible} \\ \textit{and} \quad & I(t) \leq \max(-W(t) - \tau, 0), \ \textit{when } X(t) = x \textit{ and } U(t) = u. \end{aligned}$$

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- Thm (Asymptotic Optimality With Bounded Regret) [B., Meyn '16]
  There is an h-MWT policy with finite average cost η, satisfying

$$\hat{\eta}^* \leq \eta^* \leq \eta \leq \hat{\eta}^* + O(1)$$

  where η^* is the optimal average cost for the MDP model, $\hat{\eta}^*$ is the optimal average cost for the workload relaxation, and the term $O(1)$ does not depend upon δ.

Asymptotic optimality

- The average cost for the relaxation satisfies the uniform bound,

$$\hat{\eta}^* = \hat{\eta}^{**} + O(1)$$

where $\hat{\eta}^{**}$ is the optimal cost for the diffusion approx. for the relaxation:

$$\hat{\eta}^{**} = \tau^* \, \bar{c}_- = \tfrac{1}{2} \frac{\sigma_N^2}{\delta} \, \bar{c}_- \log\Big(1 + \frac{\bar{c}_+}{\bar{c}_-}\Big)$$

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- $h(x) = \hat{h}(\xi \cdot x) + h_c(x)$
  - h_c is introduced to penalize deviations between $c(x)$ and $\bar{c}(\xi \cdot x)$.
  - The first term $\hat{h}$ is a function of workload. For $w \geq -\tau^*$, it solves the second-order differential equation,

$$-\delta \hat{h}'(w) + \tfrac{1}{2}\sigma_\Delta^2 \hat{h}''(w) = -\bar{c}(w) + \hat{\eta}^{**}, \qquad (1)$$

There is a solution that is convex and increasing on $[-\tau^*, \infty)$, with $\hat{h}'(-\tau^*) = \hat{h}''(-\tau^*) = 0$. Then extended to get a convex C^2 function on $\mathbb{R}$.

Example

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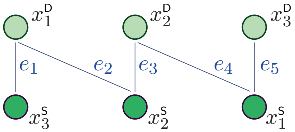
Cost: $c(x) = x_1^{\mathcal{D}} + 2x_2^{\mathcal{D}} + 3x_3^{\mathcal{D}} + 3x_1^{\mathcal{S}} + 2x_2^{\mathcal{S}} + x_3^{\mathcal{S}}$

$\Longrightarrow$ Effective Cost: $\bar{c}(w) = 4|w|$



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Matching of Supply 1 and Demand 2 allowed only if $W(t) < -\tau^*$

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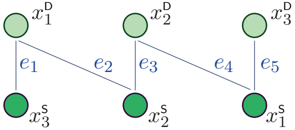
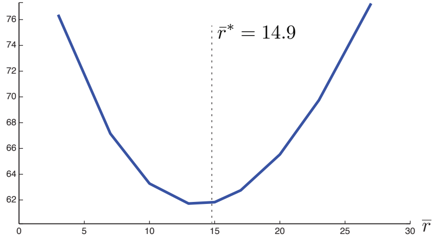


Average Cost Estimated in Simulation:



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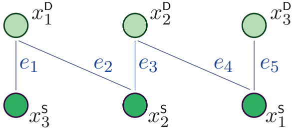

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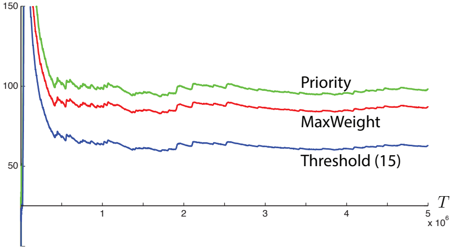
Average Cost Comparisons:



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Simulation with $\tau = 14.9$

Final remarks

- Performance bounds?
- Approximate optimal control for relaxations in higher dimensions?
- More general arrival assumptions. Admission control? Abandonnements?
- Optimization for non-bipartite matching?
- Applications?

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Dynamic bipartite matching models

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