Colouring geometric overlap graphs

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Recently, Pawlik et al. [3] have shown that triangle-free intersection graphs of line segments can have arbitrary large chromatic number, thus answering a long standing conjecture of Erdős [1] in the negative. They show this by constructing a sequence of graphs that have chromatic number tending to infinity. In fact, the chromatic number is $O(\log \log n)$ where n is the number of vertices. Graphs in this sequence can also all be obtained from a set of axis-parallel rectangles as follows.

Take a set $\mathcal{F}$ of axis-parallel (hollow) rectangular frames in the plane so that for any two frames that intersect, the right side of one rectangle intersects the top and bottom of the other rectangle (see Figure 1). Then we can construct a graph G where there is a vertex of G labelled by each frame of $\mathcal{F}$ and there is an edge between two vertices if the corresponding frames intersect.

If $\mathcal{G}$ is the set of all graphs that can be obtained this way then a recent result of Krawczyk et al. [2] show that a complementary upper bound of $O(\log \log n)$ on the chromatic number of all graphs in $\mathcal{G}$. This shows that a "better" sequence of graphs with faster asymptotic growth in the chromatic number is not possible for any construction whose elements remain in $\mathcal{G}$.

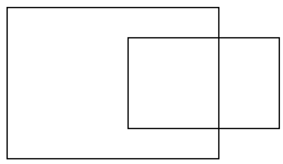

Figure 1: The only intersection allowed.

To attain this upper bound, they exploit a relationship between (offline) coloring of graphs in $\mathcal{G}$ and *online coloring* of another class of intersection graph. In the online colouring problem, vertices (and edges incident to them) are revealed one by one and the algorithm must commit to a colour for that vertex as soon as that vertex is revealed, before knowing the rest of the graph.

The goal of this project is to understand Krawczyk et al.'s [2] work and its relation to online and offline colouring.

References

[1] P. Erdős, *Graph theory and probability*, Canadian Journal of Mathematics **11** (1959), 34–38.

[2] T. Krawczyk, A. Pawlik, and B. Walczak, *Coloring triangle-free rectangular frame intersection graphs with $O(\log \log n)$ colors*, Proceedings of the 39th International Workshop on Graph-Theoretic Concepts in Computer Science (WG 2013), `arXiv:1301.0541`

[3] A. Pawlik, J. Kozik, T. Krawczyk, M. Lasoń, P. Micek, W.T. Trotter, and B. Walczak, *Triangle-free geometric intersection graphs with large chromatic number*, Discrete Comput. Geom. **50** (2013), 714–726.